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LCAの総合評価：定性的及び定量的な基準に対する拡張AHP法

Comprehensive evaluation of LCA: An extended AHP method with qualitative and quantitative criteria

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1. Introduction

Environment protection has been becoming an important issue for modern industry. LCA is one of the most actively considered techniques for the study and assessment of strategies to meet environmental challenges. LCA presents a multi-attribute decision problem with the goal of arriving at a single measure of performance. Traditional LCA methods can be categorized into two types of conversion: (1)*transformation,* such as *Monetization Method*(MM); (2)*value measurement,* such as *Multi-Attribute Utility Analysis* (MAUA)and *Analytic Hierarchy Process*(AHP) [1]. In traditional LCA methods, all factors are either transformed into money (e.g. MM), or measured into some value(e.g. AHP and MAUA) uniformly(see Figure 1).

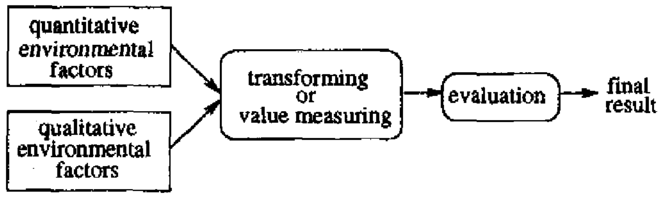

Fig. 1 Traditional LCA method

For quantitative factors MM is an ideal method. Because of the costs of factors can be found in makets, the result of evaluation will be exact. But for qualitative factors with the method of MM, the accuracy and reliability will be affected. In fact, many environmental goals generally do not have well defined or agreed upon costs. Although by examining the social costs, related health costs, and direct market costs incurred by various activities, a value which is meaningful in some situations can be extracted, but techniques for determing dollar equivalents of environmental factors have not yet reached a mature state of development [1,2]. At the same time, for the methods of AHP and MAUA, problems exist in accuracy that all the factors (including quantitative factors) are measured into values uniformly according to human's subjective judgments.

In this paper, the comprehensive evaluation with quantitative and qualitative factors is considered and an extended AHP method with respect to it is proposed. An example of LCA will be used to illustrate the extended AHP method.

2. The Axioms of The AHP

The axioms of AHP [6] can be represented as three parts: (1)decomposing, (2)comparative judging, and (3)ranking.

In part (1), decision problem is decomposed into a series of smaller problems. The goal defined at the highest level is broken down into measures (attributes) which affect it. Each of these attributes can in turn be further decomposed into sub-attributes. At the lowest level of these attributes, known as alternatives, are the proposed products or strategies in question which are the subject of the study in that they are the possible approaches to achieving the goal.

In part (2), a comparison matrix (shown as equation (1)) is set up to carry out pairwise comparisons of the relative importance of the factors with respect to objectives of the higher level.

$$A = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \quad (1)$$

It is interpreted as follows: every element, a_{ij}, of the matrix A shows the relative contribution – to the subject of comparison – of the ith activity as compared to the jth activity, i.e

$$a_{ij} = w_i/w_j \quad 1 \leqq i \leqq n, \quad 1 \leqq j \leqq n. \quad (2)$$

Where $w_i, i = 1, 2, \cdots, n$ presents the weight of factor i. The scale for entering judgments is given by Saaty [6] shown as Table 1.

In a general decision making environment, it is difficult to get the precise values of the w_i/w_j but only estimates of them. Let us consider estimates of these values given by an decision maker (or a group of decision makers) who may make small errors in judgment. According to the eigenvalue theory [6], a small perturbation around a simple eigenvalue, as we have in n when A is consistent, leads to an eigenvalue problem of the form of $Aw = \lambda_{max} w$ where λ_{max} is the principal eigenvalue of A where A may no longer be consistent but is still reciprocal. For the consistency index (CI), we adopt the value $(\lambda_{max} - n)/(n-1)$. It is the negative average

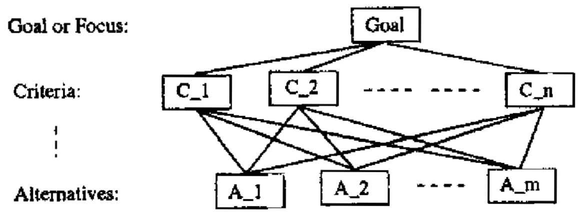

Fig. 2 Decomposing of AHP

Table 1 Comparison Scale

Intensity of importance	Definition
1	Equal importance
3	Moderate importance
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate values between adjacent judgment

of the other roots of the characteristics polynomial of A. This value is compared with the same index obtained as an average over a large number of reciprocal matrices of the same order whose entries are random. The satisfied value of CR is considered as less than 0.1 in the example of this paper.

In part (3), priorities are synthesized by mutiplying local priorities by the priority of their corresponding criterion in the level above, and adding them for each factor in a level according to the criteria it affects.

The extended AHP method proposed here integrates the evaluation of quantitative factors and qualitative factors into one AHP model. It is based on a view that for quantitative factors it is better as to use their objective costs as possible to make evaluation, and for the other factors use subjective judgment to get an approximate result, finally a comprehensive evaluation is done based on the former intermediate results, so that the accuracy of evaluation result will be improved. However, it must be noted that only by basing on an integrated criterion for various factors can make the evaluation result objective and reliable. The view of comprehensive evaluation with quantitative and qualitative factors is shown in Figure 3.

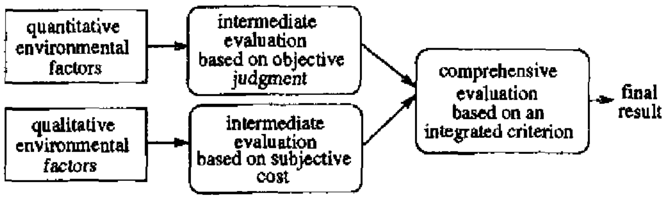

Fig. 3 Comprehensive evaluation proposal

In the part of decomposing of the extended AHP method, the factors are categorized into two types: quantitative factors and qualitative factors. AHP method is used to do evaluation for quantitative factors based on objective costs, and for qualitative factors based on subjective judgment respectively. And then the middle priorities of two type factors are calculated. Finally, the comprehensive evaluation for the two middle priorities is done. The weights to do it are derived from an importance comparison matrix overall factors.

Assume that there are k quantitative factors all of n factors, so that the number of qualitative factors is $(n-k)$. The alternative number is defined as m. The middle priorities of quantitative factors and qualitative factors are defined as follows respectively:

$$R_1 = \begin{bmatrix} r_{11} \\ r_{12} \\ \vdots \\ r_{1m} \end{bmatrix}, \quad R_2 = \begin{bmatrix} r_{21} \\ r_{22} \\ \vdots \\ r_{2m} \end{bmatrix} \tag{3}$$

Also assume that the former k rows (or column) in matrix (1) corresponds to quantitative factors, and the later $(n-k)$ ones corresponds to qualitative factors. The eigenvector with respect to the maximum eigenvalue of matrix (1) are as follows:

$$\begin{bmatrix} e_1 & \cdots & e_i & e_{i+1} & \cdots & e_n \end{bmatrix}^T \tag{4}$$

The average weights of quantitaitve factors and qualitative factors can be obtained from (4) and noemalizing them as follows:

$$w_{quan} = \frac{\sum_{i=1}^{k} e_i}{\sum_{i=1}^{k} e_i + \sum_{i=k+1}^{n} e_i}$$
$$w_{qual} = \frac{\sum_{i=k+1}^{n} e_i}{\sum_{i=1}^{k} e_i + \sum_{i=k+1}^{n} e_i} \tag{5}$$

The final priority of all the m alternatives, $R = [r_1 r_2 \cdots r_m]$, will be calculated by multiplying the middle ranks of two kinds of factors with the average weights of two kinds of factors derived by (5):

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} = \begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ \vdots & \vdots \\ r_{1m} & r_{2m} \end{bmatrix} \begin{bmatrix} w_{quan} \\ w_{qual} \end{bmatrix} \quad (6)$$

As the average weight vector of comprehensive evaluation is derived from the comparison judgment over all factors, the weights of two type factors should be feasible to the decision maker's initial judgment which made over all the factors, and the consistent of final result can be guaranteed.

3. An example

It is an example that uses the extended AHP method to apply in LCA with quantitative and qualitative factors. In step 3 and step 4, AHP is used to get middle ranks of alternatives with quantitative and qualitative factors respectively. In step 5, a comprehensive evaluation on the middle ranks is done. The average weights of quantitative and qualitative factors are calculated from the eigenvector with respect to the maximum eigenvalue of comprehensive comparison matrix created in step 1 and step 2. The part of quantitative problem of this example is refered to [4].

Problem

Suppose that the decision maker wishes to choose a best car among three types according to 7 factors (attributes): OA (Outward Appearance), NP (Noise Pollution), GE (Gaseous Emission); PP (Purchase Price), MC (Maintenance Cost per year), GC (Gallons per 10,000 miles of city driving), GR (Gallons per 10,000 miles of rural driving).

Additional conditions: he will use the car for 3 years and drive 30,000 city miles and 20,000 rural miles over the 3 years; expected gasoline cost is \$1.50/gal.

Step 1: Analyzing the hierarchy of the example

According to the axiom of AHP [3,6], we get the hierarchy of this example shown as Figure 4.

The former three factors are intangible factors, no market exist that can give their cost values. The other four factors are quantitative factors, their cost value can be calculated based on market information.

Step 2: Getting comprehensive comparison matrix

In order to do comprehensive evaluation, an integrated criterion for all the factors is necessary. The comprehensive comparison matrix is created acording to the decision maker's pairwise comparison judgment of all the factors:

	OA	*NP*	*GE*	*PP*	*MC*	*GC*	*GR*
OA	1	1/3	1/3	5	5	1/3	5
NP	3	1	3	1	7	3	9
GE	3	1/3	1	5	7	3	9
PP	1/5	1/5	1/5	1	3	1/5	3
MC	1/5	1/7	1/7	1/3	1	1/5	3
GC	3	1/3	1/3	5	5	1	7
GR	1/5	1/9	1/9	1/3	1/3	1/7	1

(7)

The consistency index (CI) of this pairwise comparison matrix is 0.0947, and the consistency ratio of it is 0.0717, they are all less than 0.10, so that we consider the judgment of decision maker is consistency roughly.

Step 3: Evaluating with qualitative factors

The pairwise comparison matrices of cars on each qualitative criterion (OA, NP and GE) are obtained by a questionnaire for decision maker as follows:

$$\underset{\text{criterion OA}}{\begin{bmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 9 \\ 1 & 1/9 & 1 \end{bmatrix}}, \quad \underset{\text{criterion NP}}{\begin{bmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 1 \\ 1/9 & 1 & 1 \end{bmatrix}}, \quad \underset{\text{criterion GE}}{\begin{bmatrix} 1 & 7/9 & 7 \\ 9/7 & 1 & 9 \\ 1/7 & 1/9 & 1 \end{bmatrix}} \quad (8)$$

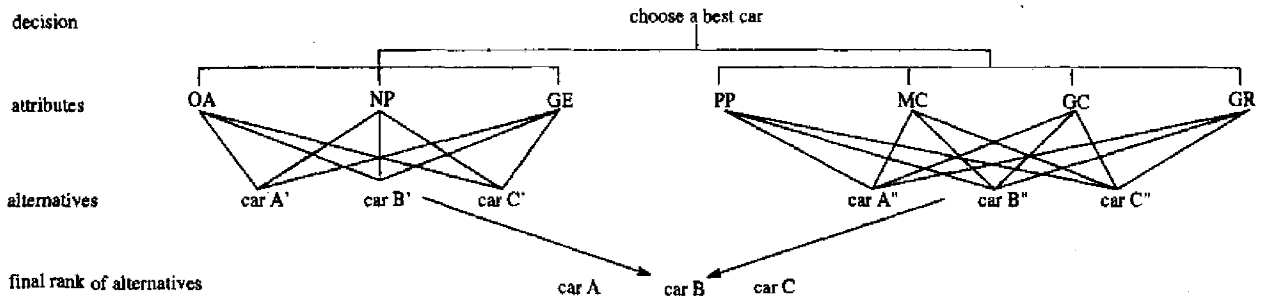

Fig. 4 The hierarchy of the example

The normalized eigenvectors with respect to the maximum eigenvalues of the above three matrices are as follows.

$$\begin{matrix}\text{criterion OA} & \text{criterion NP} & \text{criterion GE}\\ \begin{bmatrix}0.0909\\0.8181\\0.0909\end{bmatrix}, & \begin{bmatrix}0.8181\\0.0909\\0.0909\end{bmatrix}, & \begin{bmatrix}0.4118\\0.5294\\0.0588\end{bmatrix}\end{matrix} \quad (9)$$

A judgment matrix can be constructed from them.

From (7) the pairwise comparison matrix of qualitative factors (OA, NP and GE) is gotten. It is a sub matrix of (7). And the normalized eigenvector with respect to its maximum eigenvalue is as follows:

$$\begin{bmatrix}1 & 1/3 & 1/3\\ 3 & 1 & 3\\ 3 & 1/3 & 1\end{bmatrix} \rightarrow \begin{bmatrix}0.2039\\0.8823\\0.4242\end{bmatrix} \rightarrow \begin{bmatrix}0.1350\\0.5841\\0.2809\end{bmatrix} \quad (10)$$

The importance of each option is obtained by multiplying the matrix formed by the vectors of option weights (equation (9)) by the vector of criteria weights (equation (10)). So that the middel priority vector about the qualitative factors is (0.6058, 0.3122, 0.0819), i.e. $C' \rightarrow B' \rightarrow A'$.

Step 4: Evaluating with quantitative factors

Relevant data on these quantitative attributes are known as below:

	PP($)	MC($)	GC	GR
A	10000	200	250	200
B	8000	400	500	250
C	6000	600	1000	500

(11)

The relative weights of alternatives on quantitative criteria PP, MC, GC and GR can be got as follows:

$$\begin{matrix}\text{criterion PP} & \text{criterion MC} & \text{criterion GC} & \text{criterion GR}\\ \begin{bmatrix}0.4167\\0.3333\\0.2500\end{bmatrix}, & \begin{bmatrix}0.1667\\0.3333\\0.5000\end{bmatrix}, & \begin{bmatrix}0.1429\\0.2857\\0.5714\end{bmatrix}, & \begin{bmatrix}0.2105\\0.2632\\0.5263\end{bmatrix}\end{matrix} \quad (12)$$

From (7) we can get the comporison matrix of quantitative factors (PP, MC, GC, GR) as follows, it is a sub matrix of (7).

$$\begin{bmatrix}1 & 3 & 1/5 & 3\\ 1/3 & 1 & 1/5 & 3\\ 5 & 5 & 1 & 7\\ 1/3 & 1/3 & 1/7 & 1\end{bmatrix} \rightarrow \begin{bmatrix}0.3027\\0.1731\\0.9329\\0.0904\end{bmatrix} \rightarrow \begin{bmatrix}0.2019\\0.1155\\0.6223\\0.0603\end{bmatrix} \quad (13)$$

The importance of each alternative on quantitaitve factors (0.2050, 0.2995, 0.4955) is obtained by multiplying equation (12) by vector (13). As a result, the middle rank about quantitative factors is as follows: $A'' \rightarrow B'' \rightarrow C''$.

Step 5: Comprehensive evaluation

The comprehensive comparison matrix (7) got at step 2 is the basis of comprehensive evaluation with quantitative and qualitative factors. Because that it is derived from the comparison judgment over all factors, the matrix is consistent about both quantitative factors and qualitative factors. So that the result of final evaluation done by it will be reliable. From it we can get the eigenvector with respect to the maximum eigenvalue as follows.

$$\begin{bmatrix} 0.2566 & 0.6928 & 0.5461 & 0.1110 & 0.0727 & 0.3691 & 0.0461 \end{bmatrix}^T \quad (14)$$

It represents the relative importances of all the factors including quantitative factors and qualitative factors.

Because the middle ranks got at step 3 and step 4 is about qualitative factors and quantitative factors separately, the weights that to be used to do the comprehensive evaluation should also correspond to quantitative factors and qualitative factors respectively. Therefore, we choose the averages of importances of the two kinds of factors to be comprehensive evaluation weights.

The average weighting with respect to qualitative factors is: 0.4985.

i.e. 0.4985=(0.2566+0.6928+0.5461)/3;

The average weighting with respect to quantitative factors is: 0.1497.

i.e. 0.1497=(0.1110+0.0727+0.3691+0.0461)/4.

Normalising them, we can get an average weighting vector with respect to qualitative factors and quantitative factors as follows:

$$\begin{bmatrix} 0.7691 \\ 0.2309 \end{bmatrix} \quad (15)$$

From the middle-ranks of step 3 and step 4, the middle evaluation result matrix can be constructed as follows.

$$\begin{bmatrix} 0.6058 & 0.4955 \\ 0.3122 & 0.2995 \\ 0.0819 & 0.2050 \end{bmatrix} \quad (16)$$

Multiplying the middle evaluation result matrix by the weights of the two kinds of factors, we can get the final rank of comprehensive evaluation.

$$\begin{bmatrix} 0.6058 & 0.4955 \\ 0.3122 & 0.2995 \\ 0.0819 & 0.2050 \end{bmatrix} \begin{bmatrix} 0.7691 \\ 0.2309 \end{bmatrix} = \begin{bmatrix} 0.5803 \\ 0.3093 \\ 0.1103 \end{bmatrix} \quad (17)$$

From the above result it is known that the final priority vector about all the factors is: (0.5803, 0.3093, 0.1103), and the final rank of three cars is: $C \rightarrow B \rightarrow A$. That is, considering all the 7 factors, the most ideal choice is car C.

Discussion

Figure 4 gives an analysis on the weights of all the factors overall ($w0$), the weights of qualitative factors ($w1$) and the weights of quantitative factors inmiddle evaluation ($w2$), the weights of two type factors in comprehensive evaluation are pesented in line *Final*. And Figure 5 express the comparison of priorities of three alternatives decided by the weights above in different levels – evaluation of quantitative factors (*qual.*), evaluation of qualitative factors (*quan.*) and comprehensive evaluation of two type factors (*Final*).

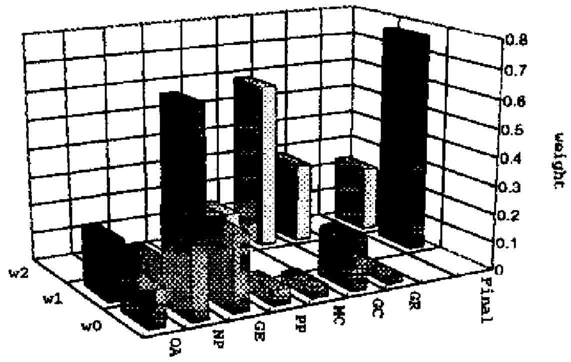

Fig. 5 Weight analysis

In the evaluation of qualitative factors, the middle rank is: $C \rightarrow B \rightarrow A$, and the priority differences between three cars are big. In the evaluation of quantitative factors, the middle rank is: $A \rightarrow B \rightarrow C$, and have small differences between three cars. But the final rank is: $C \rightarrow B \rightarrow A$, keeping the trend of qualitative evaluation. It is the result of effecting from the weights of two type factors in comprehensive evaluation. And this weights (*Final*, Figure 4) reflected the inicial judgment over all the factors ($w0$, Figure 4) of the decision maker.

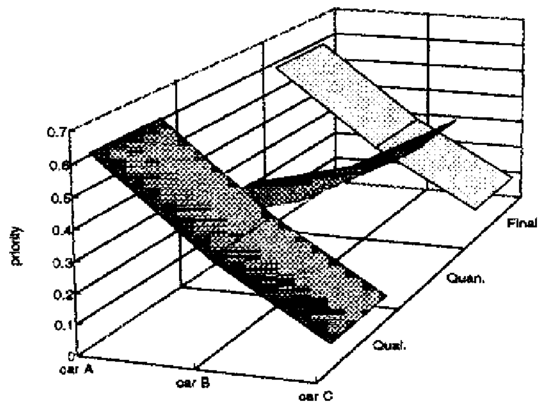

Fig. 6 Priority comparison

4. Conclusion

In the past 15 years, AHP has been applied to many fields successfully. Although it has some shortcomings, AHP has been accepted as a significant contribution to the study of multicriteria decision making. It can solve not only intangible problems but also give the difference degree of the priorities of alternatives intuitively.

Multicriteria decision making in actual is a very complicated problem. They are almost included with both quantitative and qualitative factors simutanousely. In order to ensure the evaluation to be done objectively and reliability, the human opinion must be considered to tarticipate in the decision making process. For the sake of improving the accuracy and reliability of the evaluation results, it is necessary that for quantitative factors as using their objective costs as possible to do evaluation, and for qualitative factors to use human's subjective judgments to get an approximate result. The key to ensure the extended AHP method be objective and reliable is the consistent of weights of comprehensive evaluation. Because that the average weight vector of comprehensive evaluation is derived from the comparison judgment over all factors, the consistent of final result can be guaranteed. The extended AHP method has advantages of simplicity, flexibility, intuitive appeal, and the ability to integrate quantitative and qualitative factors in one model.

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