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Mobility of Articulated Frame Steering Vehicle from Kinematic Viewpoint

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1. Introduction

Articulated frame steer vehicle consists of two frames, a front frame over the front axle and a rear frame over the rear axle. For any steering action, these two frames bend with respect to each other in a pivot joint C (Fig.1). The bending of two frames is conducted by the elongation and shortening of a hydraulic cylinder. The wheels remain parallel to the vehicle body. Whenever a steering action is made the frames turn, not the wheels.

The main purposes of the articulated design of vehicle frames are (i) to reduce the turning radius, especially in case of the vehicle of long wheelbase, (ii) to provide turning facility when the vehicle is at standstill, and (iii) to achieve on-tracking of the rear wheels so that it faces less running resistance and can move through narrow paths. Articulated frame vehicles are often using in agriculture, industry and some other off-road areas. Therefore, it deserves significant attention to study its handling behavior for realizing the mobility to have better design. But compared with the attention, which conventional off-road vehicle has attracted, there is very little publish work on articulated frame steer vehicle (AFSV).

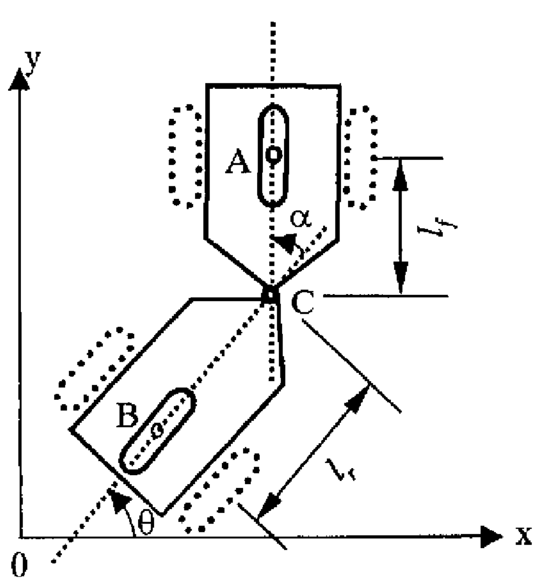

Fig. 1. Schematic diagram of articulated frame tractor

The main objectives of this study are (1) to develop generalized, simple motion equations for articulated frame steer vehicle; (2) to study its mobility or turning behavior in different motion; and (3) to determine the on-tracking and off-tracking conditions.

Articulated frame steer vehicles are usually of slow speed. At low speed or parking lot maneuver, no significant side forces develop in tires. Usually kinematic models are used to describe the ideal behavior of a system. Therefore, kinematic model is considered to use for this study. In order to render the problem amenable to analysis, the vehicle is assumed to be a slow speed bi-cycle model moving on a hard and flat surface, where the slippage or lateral forces are negligible.

2. Theory

2.1 Vehicle Modeling

According to Fig. 2 the wheelbase of front frame is l_f and rear frame is l_r. Considering the principle of rotation of a rigid body[1], and the motion of a disk rolling on a plane, following relationship can be deduced for rear frame *BC*:

$$\dot{x}_B = V\cos\theta$$
$$\dot{y}_B = V\sin\theta$$

Where $\dot{x}_B$ and $\dot{y}_B$ represent the longitudinal and lateral velocity component at B respectively.

For any turn of the vehicle, velocity of any point of its body will be tangential to the arc of the instantaneous turning circle. Therefore, the velocity "V_c" of pivoting point (*C*) is perpendicular to *OC* and it makes angle ϕ with the direction of

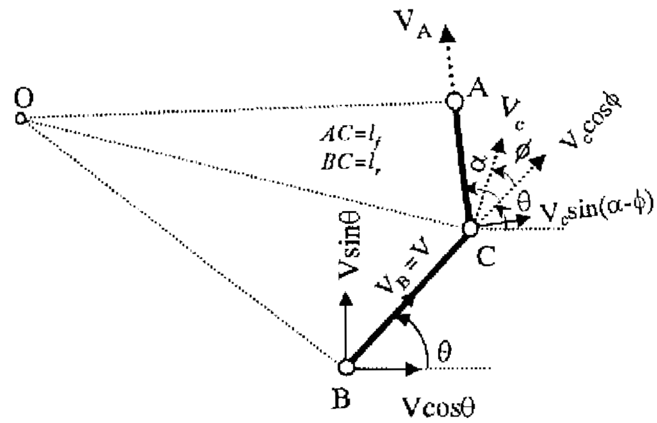

Fig.2. Freebody diagram of AFSV

rear frame. Following relationships, therefore, can be established:

$V_c \cos\phi = V$ and

$$l_r\dot{\theta} = V_c \sin\phi$$
$$l_r\dot{\theta} = \frac{V}{\cos\phi}\sin\phi$$
$$l_r\dot{\theta} = V\tan\phi \qquad (1)$$

For the front frame *AC*, following relationship can be derived:

$$l_f(\dot{\theta}+\dot{\alpha}) = V_c\sin(\alpha-\phi)$$
$$l_f(\dot{\theta}+\dot{\alpha}) = \frac{V}{\cos\phi}[\sin\alpha\cos\phi - \cos\alpha\sin\phi]$$
$$l_f(\dot{\theta}+\dot{\alpha}) = V\sin\alpha - V\tan\phi\cos\alpha$$
$$l_f(\dot{\theta}+\dot{\alpha}) = V\sin\alpha - l_r\dot{\theta}\cos\alpha$$
$$\dot{\theta}(l_f + l_r\cos\alpha) = V\sin\alpha - l_f\dot{\alpha}$$
$$\dot{\theta} = \frac{V\sin\alpha - l_f\dot{\alpha}}{l_f + l_r\cos\alpha} \qquad (2)$$

If the front and rear wheelbases are of equal lengths, *i.e.*, $l_f = l_r = l$, then equation (2) becomes:

$$\dot{\theta} = \frac{V \sin \alpha - l\dot{\alpha}}{l(1 + \cos \alpha)}$$

$$\dot{\theta} = \frac{2V \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - l\dot{\alpha}}{2l \cos^2 \frac{\alpha}{2}}$$

$$\dot{\theta} = \frac{V}{l} \tan \frac{\alpha}{2} - \frac{\dot{\alpha}}{2} \sec^2 \frac{\alpha}{2}$$

$$\dot{\theta} = \frac{V}{l} \tan \frac{\alpha}{2} - \frac{\dot{\alpha}}{2} (1 + \tan^2 \frac{\alpha}{2}) \quad (3)$$

Yaw velocity of the rear frame of vehicle can be expressed as:

$\dot{\alpha} = u(t)$

Where $u(t)$ is the displacement of hydraulic cylinder.

Then eq. (3) will become:

$$\dot{\theta} = \frac{V}{l} \tan \frac{\alpha}{2} - \frac{u}{2} (1 + \tan^2 \frac{\alpha}{2})$$

The generalized kinematic model of articulated frame steer vehicle can now be expressed as:

$$\begin{bmatrix} \dot{\alpha}_A = u(t) \\ \dot{\theta} = \dfrac{V \sin\alpha - l_f \dot{\alpha}}{l_f + l_r \cos\alpha} \\ \dot{x}_B = V \cos\theta \\ \dot{y}_B = V \sin\theta \end{bmatrix}$$

2.2 Analysis of turning behavior:

To minimize slippage, at any stage of turn, all the wheels should be tangential

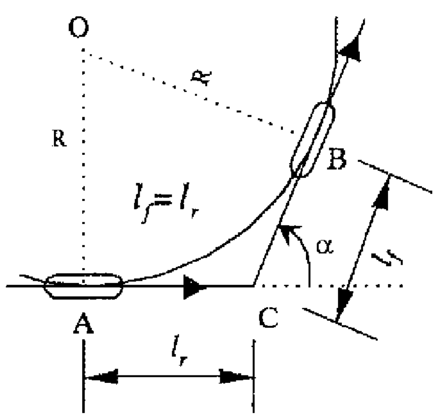

Fig. 3a. On-tracking condition

to its arc of the instantaneous turning circle. Figure 3 shows that on tracking can be achieved if both front and rear wheelbases are of equal lengths. Therefore, to analysis the on-tracking behavior of AFSV, equal lengths of wheelbases are considered.

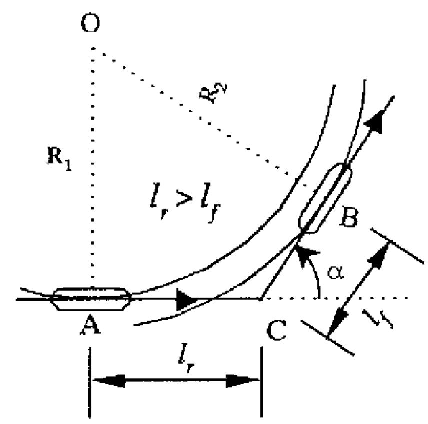

Fig. 3b. Off-tracking condition

Equation (3) represents the yaw velocity of the rear frame of vehicle. Right hand side of this equation has two distinct portions; $\frac{V}{l}\tan\frac{\alpha}{2}$ and $\frac{\dot{\alpha}}{2}(1+\tan^2\frac{\alpha}{2})$. For any steering action, the former portion of equation will cause rotation of the rear frame in the same direction of steering. But due to the negative sign, the later part of equation causes to turn the same frame in opposite to the steering direction.

Since the trajectory for any turn forms curved shape, it is useful to explain the turning behavior from the concept of curvature. A curved shape arbitrary line AD is illustrated in Fig. 4, whose curvature is in the xy plane. The normals to the curve at B and C intersect in the point O'. The change in the slope angle between B and C

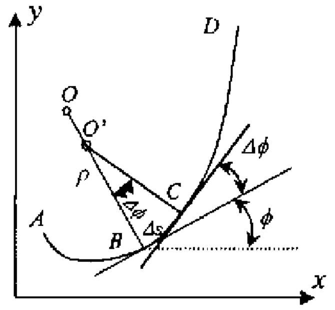

Fig. 4.Curvature in *xy plane*

is $\Delta\phi$. When $\Delta\phi$ is small, the arc Δs is approximately $O'B\ \Delta\phi$. In the limit as point C approaches B, that is, as $\Delta s \to o$, the curvature at point B can be defined as:

$$\kappa = \frac{d\phi}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\phi}{\Delta s} = \lim_{\Delta s \to 0} \frac{1}{O'B} = \frac{1}{\rho}$$

Where $\rho = OB$ is the radius of curvature at point B.

From this definition, curvature of vehicle trajectory can be expressed as:

$$\begin{aligned} \kappa &= \frac{d\theta}{ds} \\ &= \frac{d\theta}{dt}\frac{dt}{ds} \\ &= \dot{\theta}\frac{1}{V} \end{aligned}$$

$$\frac{1}{\rho} = \frac{1}{V}\left[\frac{V}{l}\tan\frac{\alpha}{2} - \frac{\dot{\alpha}}{2}\left(1 + \tan^2\frac{\alpha}{2}\right)\right] \qquad (4)$$

Equation (4) shows that when steering angle α changes with time, *i.e.*, , $\frac{d\alpha}{dt} \neq 0$, the curvature or radius of turning trajectory also will vary. In that case the rear wheel will not be able to follow the path of front wheel. But, when the steering angle will remain constant for arbitrary long time, *i.e.*, $\frac{d\alpha}{dt} = 0$, the value of later portion of equation (4) will be zero and the right hand side of the equation will be a constant. In other words, there will be no change in the curvature of the trajectory or the radius of turn. On tracking will be attained if and only if this condition exists for equal lengths of wheelbases.

3. Simulation Results:

Figure 5a shows the trajectories of the center of wheel axles of an AFSV in circular motion, when the lengths of front and rear wheelbases are equal. At the beginning of moving, front and rear wheelbases were in the same direction. A, B and C indicate the center of front axle, rear axle and bending point respectively. Figure 5b shows the time history of the steering angle, where it is found that the steering angle started to increase gradually and when it reached its maximum (45°), it became constant until the end of move. From this two figures it can be seen that when the steering angle was increasing gradually rear wheel could not follow the track of front wheel. But when the steering angle became fixed, full on-tracking was attained for the rear wheel.

Figure 6a and 6b show the wheel trajectories and time history of steering angle respectively, where front wheelbase is smaller than rear wheelbase. Other conditions were same as mentioned for Fig. 5a and 5b. It is found that when steering angle became fixed, still on-tracking was not attained for rear wheel. This result proves that on-tracking is not possible for unequal lengths of wheelbases.

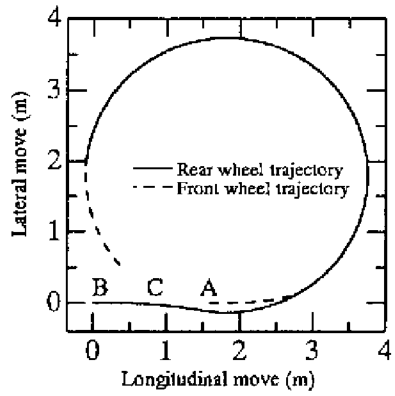

Fig. 5a. Trajectories of front and rear wheels of AFSV when the wheelbases are of equal lengths

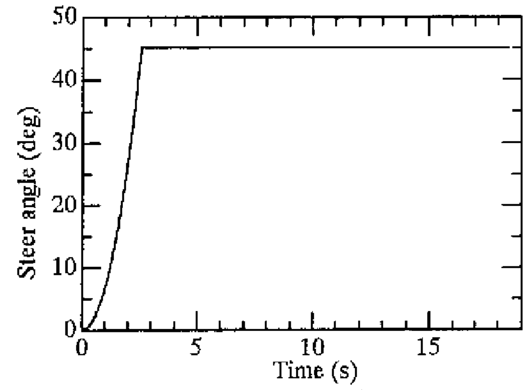

Fig. 5b. Time history of steering angle for AFSV in circular motion

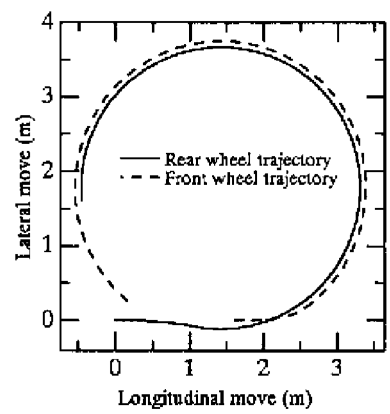

Fig. 6a. Trajectories of front and rear wheelbases of AFSV when rear frame is larger than front frame

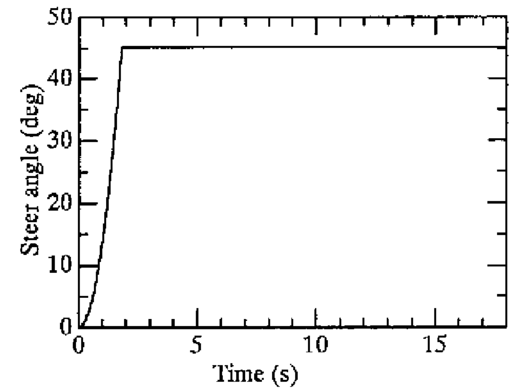

Fig. 6b.Time history of steering angle for AFSV in circular motion

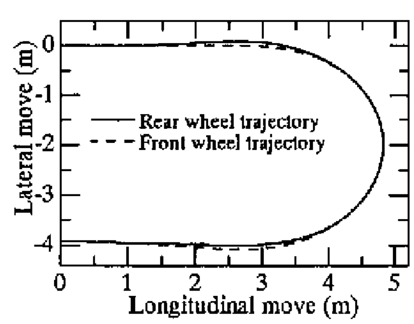

Fig. 7a. U-turn of AFSV when the length of both wheel bases are same

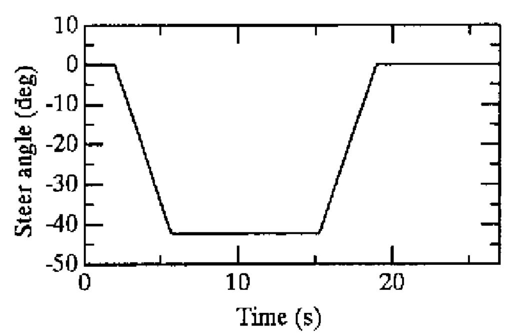

Fig. 7b. Time history of steering angle for U-turn of AFSV

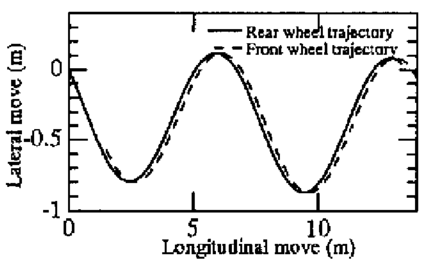

Fig. 8. Slalom motion of AFSV when the lengthf of both wheelbases are equal

Wheel trajectories, and time history of steering angle for AFSV in U-turn is shown in Fig. 7a and 7b respectively. Front and real wheelbases were of equal lengths. These figures show that the portion of turn, where steering angle was not fixed, *i.e.*, $\frac{d\alpha}{dt} \neq 0$, there was no on-tracking. But for the portion of fixed steering, full on-tracking was attained.

Figure 8 shows the trajectories of AFSV in slalom motion, for equal lengths of wheelbases. Although the deviation between the front and rear wheel trajectories is not so significant, there was no on-tracking throughout this motion, because the steering angle was not fixed.

4. Discussion:

Oida[2] mentioned that the articulated tractor varies the position of its center of gravity according to the turning angle, so that the motion equations become very complex and difficult to solve. He developed a complicated set of equations to describe the turning behavior of AFSV. Dudzinski[3] developed another set of lengthy equations to describe turning process of AFSV when the vehicle is at standstill. But in this paper, a set of very simple equations is introduced to describe the behavior of AFSV, which is applicable either for the vehicle at standstill or in motion. Simulation results showed the performances of these equations to represent the AFSV.

Oida also mentioned that the ruts (trajectories) of front and rear wheels did not coincide when the bending point was not located at the center of the wheelbase. But this study showed that on-tracking can be attained only for the AFSV of equal length wheelbases and when the steering angle is fixed. Other than these conditions, on-tracking will not be possible.

Conclusion:

A kinematic model for an articulated frame steer vehicle is developed, which can describe the kinematic behavior of vehicle, either in motion or in standstill. Simulations were done with this model for circular, U-turn and slalom motions. Results showed that on-tracking can be achieved only for the AFSV of equal wheelbases and when the steering angle is fixed.

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