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# ロバスト PID 制御系の設計

# Robust PID Design via Error-compensated IMC

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## Abstract

A new design method of robust control system is proposed on the basis of a kind of modified internal model control framework. The paper also shows how to extend the relative stability and the derivation of conventional PID controller from correlative internal model controller.

## 1. Introduction

The conventional feedback control system is shown in figure 1, where G is the process and C is the controller.

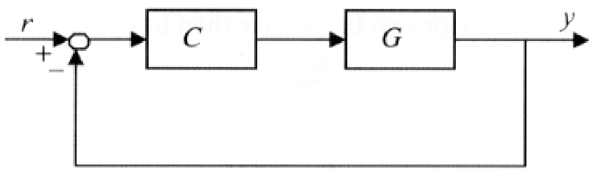


**Figure 1** Feedback control

$$C(s) = \frac{P(s)}{1 - G(s)P(s)} \qquad (1)$$

Where $$P(s) = \frac{C(s)}{1 + C(s)G(s)} \qquad (2)$$

If the process G is stable, then the feedback system in figure 1 is equivalent transform to internal model control (IMC) which is shown in figure 2. The conventional controller C is given by (1) and (2) which imply that the conventional PID controller can be derived from the correlative IMC controller.

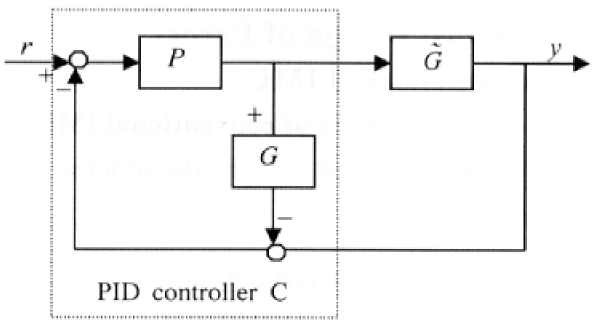


**Figure 2** IMC framework

The transfer function from r to y is

$$\frac{y}{r} = G(s)P(s) \quad (3)$$

The sensitivity function and the complementary sensitivity function are given by

$$S(s) = 1 - G(s)P(s)$$
$$T(s) = G(s)P(s) \quad (4)$$

Where $\quad P(s) = G^{-1}(s) f(s) \quad (5)$

The filter $f(s)$ makes the controller proper and causal.

Then $\quad \frac{y}{r} = f(s) \quad (6)$

$$S(s) = 1 - f(s)$$
$$T(s) = f(s) \quad (7)$$

Therefore, the control system can be designed through adjustment of the filter. However, we hope that we can design the controller without considering the filter to make the design easier.

Since the sensitivity function and the complementary sensitivity function appear linear in the IMC system, this implies that the internal model control provides a much easier framework for the design of robust controller.

For these reasons, we present a new design method of robust control system on the basis of the internal model control philosophy and PID controller will be achieved by rearrangement of the structure.

## 2. Robust Design of Error-compensated IMC

### 2.1 robust stability of conventional IMC

In robust control system, the process can be denoted by

$$\tilde{G}(s) = G(s)(1 + \Delta(s)) \quad (8)$$

It is assumed that $G(s)$ does not contain unstable poles or zeros, $1 + \Delta(s)$ is stable and is given by $\quad |\Delta(s)| \leq 1 \quad (9)$

The transfer function becomes to

$$\frac{y}{r} = \frac{G(s)P(s)(1+\Delta(s))}{1+G(s)P(s)(1+\Delta(s)-1)} = \frac{G(s)P(s)(1+\Delta(s))}{1+G(s)P(s)\Delta(s)} \quad (10)$$

The necessary and sufficient condition for robust stability is that $G(s)P(s)\Delta(s)$ does not encircle the point(-1,0).

Therefore, the bound of model error should be less than unity.

$$|\Delta(s)| \leq 1$$

### 2.2 definitions of Error-compensated IMC

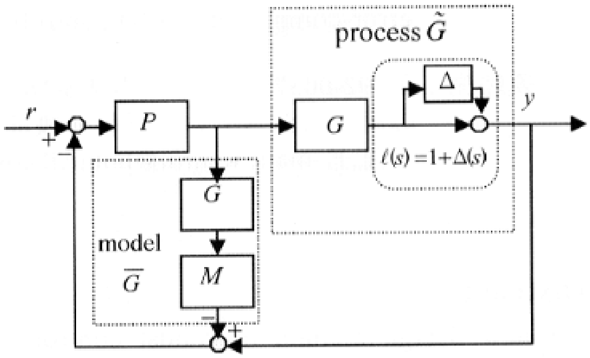


**Figure 3** Error-compensated IMC

To break through this restriction on robust stability, an error-compensator M is added in figure 3. Assume that $P$ is the transfer function of a controller that achieves nominal internal stability. Also assume that $G$, $\tilde{G}$ and process model $\overline{G}$ have the same number of unstable poles. In error-compensated IMC, assume multiplicative uncertainty is less than unity, therefore the model error can be greater than unity.

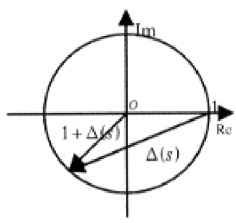


**Figure 4** $\Delta(s)$ in error-compensated IMC

$$\tilde{G}(s) = G(s)(1+\Delta(s))$$

$$\overline{G}(s) = G(s)M(s)$$

The transfer function of the Error-compensated IMC system is given by

$$\frac{y}{r} = \frac{G(s)P(s)(1+\Delta(s))}{1+G(s)P(s)(1+\Delta(s)-M(s))} \quad (11)$$

### 2.3 robust stability of Error-compensated IMC

***Theorem:*** *In the error-compensated IMC system, assume* $\left|1+\Delta(s)-M(s)\right| \le \delta$

*If* $\left|G(s)P(s)\right| \le \dfrac{1}{\delta}$

*Then, the system is robust stable.*

Since (5) and (11)

$$\frac{y}{r} = \frac{f(s)(1+\Delta(s))}{1+f(s)(1+\Delta(s)-M(s))} \quad (12)$$

Notice that the dominator of (12) is

$$1+f(s)(1+\Delta(s)+M(s))$$

If the system is stable, it is necessary and sufficient that Nyquist plot of

$$f(s)(1+\Delta(s)-M(s))$$

not encircle the point (-1,0).

Since $f(s) = \dfrac{1}{(1+\lambda s)^n} \qquad \forall\omega \quad (\lambda \ge 0)$

Then $\qquad \left|f(s)\right| \le 1 \qquad (13)$

therefore, if $\left|1+\Delta(s)-M(s)\right| \le 1 \qquad (14)$

then $\qquad \left|f(s)(1+\Delta(s)-M(s))\right| \le 1 \qquad (15)$

the vector does not encircle the point (-1,0), and the Error-compensated IMC system is robust stable.

For this reason, the filter can be designed arbitrarily to improve the performance of the system if we design the controller satisfying (14). In fact, Error-compensated IMC is designed on the basis of this idea.

### 2.4 work example

Assume

$$1+\Delta(s) = \frac{1}{(1+\frac{L}{\gamma}s)^{\gamma}} \qquad \begin{array}{l} 0 \le \gamma \le 5 \\ L \ge 0 \end{array} \qquad (16)$$

Since the phase lag increases with the order of the process. In practice, we assume the order of it less than five to guarantee the stability.

If the error-compensator is given by

$$M(s)\frac{1}{1+Ls} \qquad (17)$$

then we can prove that the system is stable.

Notice that both the magnitude of multiplicative uncertainty (16) and the error-compensator (17) are less than unity. Furthermore, both of them decrease with increasing of frequency. These imply that M not only can be regarded as the compensator of the phase lag but the magnitude.

To illustrate the idea, consider the following example.

$$G(s) = \frac{1}{s+1} \qquad 1+\Delta(s) = \frac{1}{(1+0.2s)^5}$$

the parameter of compensator L=1

$$M(s) = \frac{1}{1+s}$$

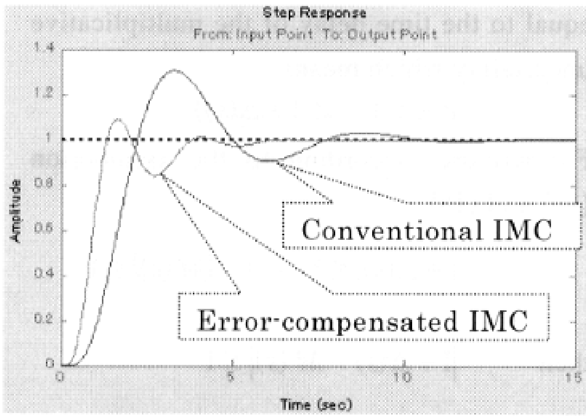


**Figure 5** Step response

## 3. Extension of Relative Stability by Improved Error-compensated IMC

In particular, it is usually describe the relative stability by the distance from the nominal open-loop frequency response to the critical stability point (-1,0). This implies that the relative stability can be extended by decreasing $|1+\Delta(s)-M(s)|$.

For this reason, it is better to make the error-compensator more approach to the multiplicative uncertainty for extension of the relative stability margin, because of the decreasing of $|1+\Delta(s)-M(s)|$.

Assume that improved error-compensator

$$M(s)=\frac{1}{1+Ls+\frac{1}{2}L^2s^2+\cdots\cdots+\frac{1}{n!}L^ns^n} \quad (18)$$

where L is equal to the time delay of the multiplicative uncertainty.

***Proof***

Since the model error usually is time delay element which can be described as:

$$e^{-\theta s}=\frac{1}{1+\theta s+\frac{1}{2}\theta^2s^2+\cdots\cdots+\frac{1}{n!}\theta^ns^n} \quad (19)$$

If we set $L=\theta$, then the value of L is equal to the time delay of the multiplicative uncertainty which means

$$\angle M(s)\approx\angle(1+\Delta(s))$$

Furthermore, according to the assumption (16) and (18)

$$|1+\Delta(s)|\le 1 \quad \text{and} \quad |M(s)|\le 1$$

then $\quad |1+\Delta(s)-M(s)|\le 1$

Therefore the system is robust stable. □

Figure6 shows that $|1+\Delta(s)-M(s)|$ decreases with increasing the order of the error-compensator in the range of high frequency. However, when the order of the error-compensator is greater than 1, there is not much difference among them.

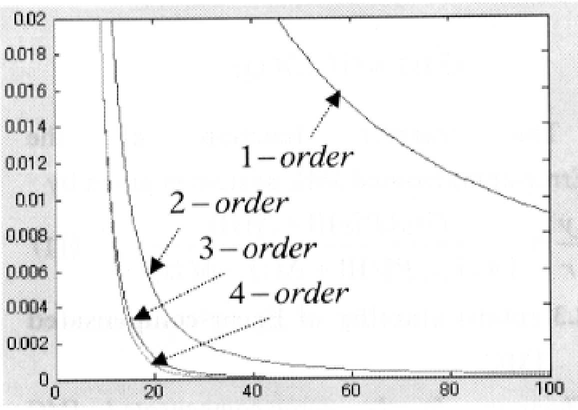


**Figure 6** $|1+\Delta(s)-M(s)|$ with different order

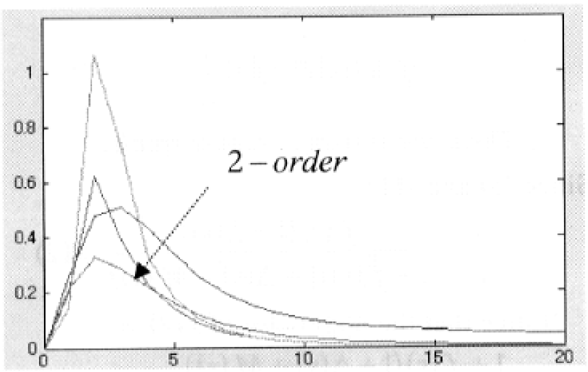


**Figure 7** $|1+\Delta(s)-M(s)|$ of 2-order

Figure 7 shows that the magnitude of 2-roder is a better choice in the range of low frequency. Furthermore, since the 2-order always is less than the 1-order in all the range of frequency, it means that if the 1-order error-compensated IMC system is table, then, the 2-order compensator system is stable too. Therefore usually we select n=2 for the error-compensator.

Thus $$M(s)=\frac{1}{1+Ls+\frac{1}{2}L^2s^2} \quad (20)$$

Consider the work example

$$G(s)=\frac{1}{s+1} \qquad 1+\Delta(s)=\frac{1}{(1+0.2s)^5}$$

formula (20) causes that

$$M(s)=\frac{1}{1+1.1s+0.605s^2}$$

**Table 1** Comparison of the relative stability and ISE of the IMC, EC-IMC (error-compensated IMC) and IEC-IMC (improved error-compensated IMC)

| | IMC | EC-IMC | IEC-IMC |
|---|---|---|---|
| **Gm** | 1.2972 | 1.2682 | 2.0239 |
| **Pm** | 16.7683 | 23.2253 | -180 |
| **Wcg** | 1.6248 | 3.1790 | 3.4922 |
| **Wcp** | 1.4643 | 2.8675 | 0 |
| **ISE** | 1.372 | 0.7295 | 0.778 |

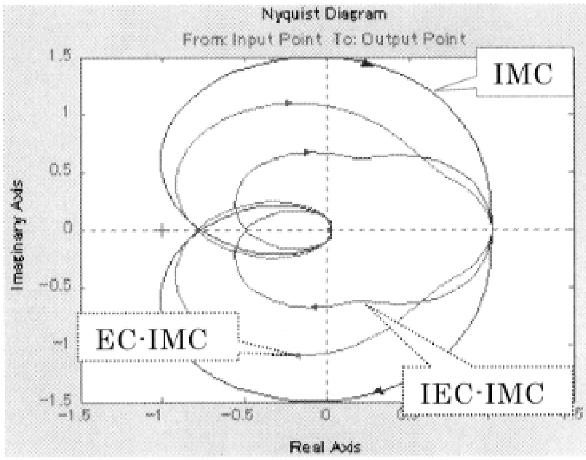


**Figure 8** Nyquist Diagram

*Gm: gain margin; Pm: phase margin; Wcg: the frequency at which the magnitude is 0dB; Wcp: the frequency at which the phase is* $-180°$

Figure 8 shows that the stability margin of IEC-IMC is larger than IMC and EC-IMC, therefore, relative stability has been extended.

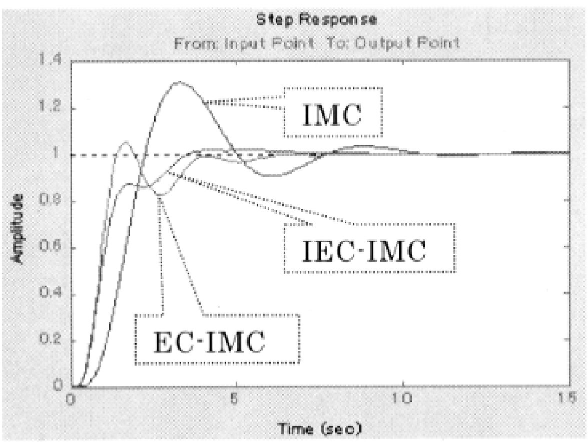


**Figure 9** step response

## 4. Implementing conventional PID controller

PID controllers are widely used in the industry due to their simplicity and ease of re-tuning online. The Error-compensated IMC philosophy can also be used to derive the conventional PI or PID controllers. The dotted diagram in figure10 can be reduced to a conventional PID controller by rearranging the framework.

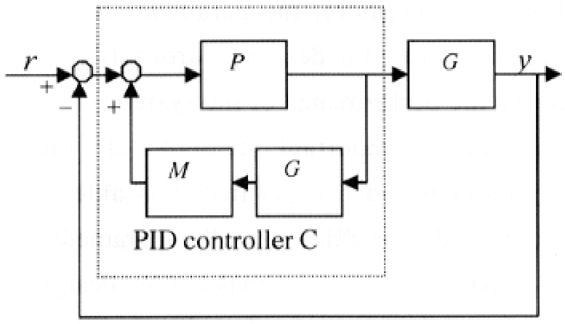


**Figure 10** Rearrangement of EC-IMC to PID controller

Consider the transfer function of process

$$G(s) = \frac{k}{1+\tau s}$$

PID controller $$C(s) = \frac{P(s)}{1-P(s)G(s)M(s)}$$

where

$$M(s) = \frac{1}{1+Ls}$$

therefore

$$C(s) = \frac{\frac{1}{1+\lambda s}\frac{1+\tau s}{k}}{1-\frac{1}{1+\lambda s}\frac{1}{1+Ls}} = \frac{1+\tau s}{ks}\frac{1+Ls}{L\lambda s^2+(L+\lambda)s}$$

Hence, the error-compensated IMC controller has been converted into PI controller $\frac{1+\tau s}{ks}$ with a filter

$$\frac{Ls+1}{L\lambda s^2+(L+\lambda)s}$$

## 5. Conclusions

The paper has presented a robust controller design method which called error-compensated IMC. By employing the algorithm from internal model control, an error-compensator has been introduced to the process model and then yielded the robust IMC controller parameter. The design method which has been proposed can be used to extend the margin of relative stability. Furthermore, the parameter of the robust filter can be designed arbitrarily to improve the performance of the system.

The most important feature of the error-compensated IMC is that it is able to be converted into PID controller parameter. This characteristic can make the design method easier to apply in practice applications.

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