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線形システムとその電力相補システムにおけるグラミアンと2次モードの新しい性質

Novel Properties of the Gramians and Second-Order Modes of Linear State-Space Systems and Their Power Complementary Systems

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1. Introduction

The Gramians and second-order modes of linear state-space systems are significant factors since they play important roles in many fields of analog and digital signal processing. For example, the problem of quantization effects of digital filters, which can be solved by the Gramians and second-order modes, has attracted many researchers since quantization effects depend highly on filter structures and the state-space approach is the most effective and elegant method that can be used to find the optimal structures.

The theory on power complementary systems is also of considerably practical significance in many fields of digital signal processing such as digital filter design, filter banks, and so on. However, literatures on signal processing address this theory mainly from the viewpoint of the transfer functions and thus there has been few researches on the power complementary systems by the state-space approach.

In this paper, we first derive a novel theorem on continuous-time power complementary systems through the state-space approach. Then, we reveal an important property of the second-order modes by applying the new theorem to the second-order modes. Moreover, we extend these results to discrete-time state-space systems and formulate a new expression for the evaluation of minimum quantization effects.

The organization of this paper is as follows. In Section 2, the continuous-time state-space systems, Gramians and second-order modes are introduced.

In Section 3, the theory on bounded-real systems and power complementary systems is addressed. This theory is frequently used in our main discussion. Especially, The bounded-real Riccati equations play central roles in derivation of our new theorems. In Section 4, our main result is presented; new theorems on the Gramians and second-order modes of continuous-time and discrete-time state-space systems are established. In Section 5, a new formula for evaluating minimum attainable value of coefficient sensitivity is presented as an application of our new theorems. This formula shows the effectiveness and high performance of digital filters with minimum coefficient sensitivity structure. In Section 6, a numerical example is given in order to verify our theorems.

2. The Gramians and Second-Order Modes of Continuous-Time State-Space Systems

Consider the following state-space equations for a stable single-input/single-output continuous-time system of order N with the transfer function $H(s)$:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{1}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t) + du(t) \tag{2}$$

where $u(t)$, $y(t)$ and $\boldsymbol{x}(t)$ are the scalar input, the scalar output and the state vector of size $N \times 1$, and the matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and d are coefficient matrices with appropriate size. The block diagram of this system is given in Figure 1. The system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ is assumed to be a minimal realization of $H(s)$, that is, the system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ is controllable and observable. The coefficient matrices and the transfer function are related as

$$H(s) = d + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}. \tag{3}$$

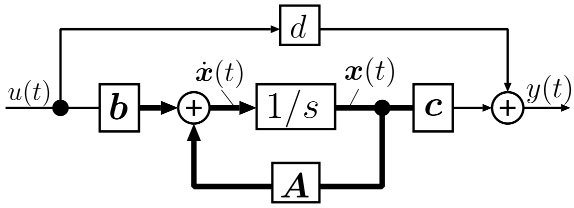

Fig. 1 Continuous-time state-space system.

For the system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$, the solutions $\boldsymbol{K}$ and $\boldsymbol{W}$ of the following Lyapunov equations are called the controllability Gramian and the observability Gramian, respectively:

$$\boldsymbol{A}\boldsymbol{K} + \boldsymbol{K}\boldsymbol{A}^t = -\boldsymbol{b}\boldsymbol{b}^t \tag{4}$$

$$\boldsymbol{A}^t\boldsymbol{W} + \boldsymbol{W}\boldsymbol{A} = -\boldsymbol{c}^t\boldsymbol{c}. \tag{5}$$

The Gramians $\boldsymbol{K}$ and $\boldsymbol{W}$ are symmetric and positive definite, that is, $\boldsymbol{K} = \boldsymbol{K}^t > 0$ and $\boldsymbol{W} = \boldsymbol{W}^t > 0$, because the system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ of $H(s)$ is assumed to be stable, controllable and observable. Then, the eigenvalues $\theta_1^2, \theta_2^2, \cdots, \theta_N^2$ of the matrix product $\boldsymbol{K}\boldsymbol{W}$ are all positive. The positive square roots $\theta_1, \theta_2, \cdots, \theta_N$ of the eigenvalues are called the second-order modes of the system[1, 2].

It should be noted that the Gramians depend on realization of the system, while the second-order modes depend only on the transfer function. In the literatures on control system theory, the second-order modes are also called Hankel singular values because the eigenvalues of $\boldsymbol{K}\boldsymbol{W}$ are equal to the singular values of the Hankel operator of $H(s)$.

3. Bounded-Real Systems and Power Complementary Systems

This section introduces the well-known important theory on bounded-real systems and power complementary systems. This theory is frequently

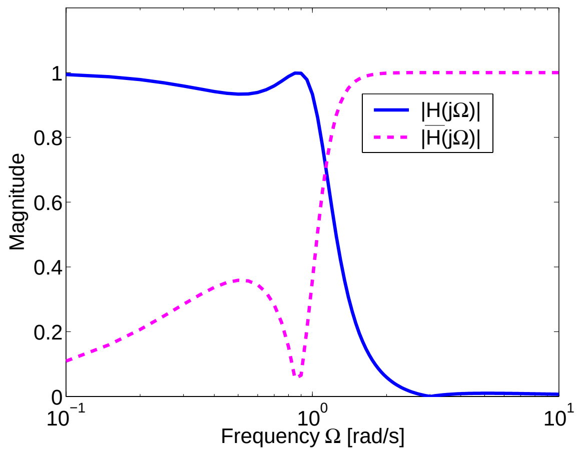

Fig. 2 Power complementary systems. ($\gamma = 1$)

used in the derivation of our result.

3.1 Definition of Bounded-Real Systems and Power Complementary Systems

A linear system $H(s)$ is called bounded-real if and only if

$$|H(j\Omega)| \leqq \gamma, \quad \forall\Omega \in \Re, \tag{6}$$

and a pair of bounded-real linear systems $H(s)$ and $\overline{H}(s)$ is said to be power complementary if

$$|H(j\Omega)|^2 + |\overline{H}(j\Omega)|^2 = \gamma^2, \quad \forall\Omega \in \Re. \tag{7}$$

For example, if $H(s)$ is lowpass, then $\overline{H}(s)$ is highpass as illustrated in Figure 2. While if $H(s)$ is bandstop, then $\overline{H}(s)$ is bandpass.

3.2 Bounded-Real Riccati Equations

The bounded-real systems satisfy the so-called bounded-real Riccati equations. The following lemmas are well-known results relating bounded-real systems to the Riccati equations[3, 4].

Lemma 1 *A state-space system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ of $H(s)$ with minimal realization is bounded-real if and only if there exists $\boldsymbol{P} = \boldsymbol{P}^t > 0$ such that*

$$\begin{aligned}&\boldsymbol{A}^t\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} + \boldsymbol{c}^t\boldsymbol{c}\\ &\quad + (\boldsymbol{P}\boldsymbol{b} + \boldsymbol{c}^t d)r^{-1}(\boldsymbol{P}\boldsymbol{b} + \boldsymbol{c}^t d)^t = \boldsymbol{0}\end{aligned} \tag{8}$$

where $r = \gamma^2 - d^2 > 0$.

Lemma 2 *A state-space system $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ is bounded-real if and only if its dual system $(\boldsymbol{A}^t, \boldsymbol{c}^t, \boldsymbol{b}^t, d)$ is bounded-real. Moreover, if $\boldsymbol{P} = \boldsymbol{P}^t > 0$ is a solution to (8), then $\boldsymbol{Q} = \gamma^2\boldsymbol{P}^{-1}$ is a solution to the dual equation*

$$\begin{aligned}&\boldsymbol{A}\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{A}^t + \boldsymbol{b}\boldsymbol{b}^t\\ &\quad + (\boldsymbol{Q}\boldsymbol{c}^t + \boldsymbol{b}d)r^{-1}(\boldsymbol{Q}\boldsymbol{c}^t + \boldsymbol{b}d)^t = \boldsymbol{0}.\end{aligned} \tag{9}$$

We call Eqs. (8) and (9) the bounded-real Riccati equations. Note that solutions of the bounded-real Riccati equations yield a state-space representation of power complementary systems as follows.

Lemma 3 *Given a stable bounded-real state-space system $H(s) = d + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$ with minimal realization, its power complementary system $\overline{H}(s)$ is described by*

$$\overline{H}(s) = w + \boldsymbol{l}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} \tag{10}$$

where w and $\boldsymbol{l}$ are given as

$$w = r^{\frac{1}{2}} \tag{11}$$

$$\boldsymbol{l} = -(\boldsymbol{P}\boldsymbol{b} + \boldsymbol{c}^t d)^t / w. \tag{12}$$

Considering the dual system, we can also see that $\overline{H}(s)$ has the following representation

$$\overline{H}(s) = w + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{m} \tag{13}$$

where

$$\boldsymbol{m} = -(\boldsymbol{Q}\boldsymbol{c}^t + \boldsymbol{b}d)/w. \tag{14}$$

4. Novel Properties of the Gramians and Second-Order Modes

In this section, our main result is presented. The theorems presented in this section reveal novel properties of the Gramians and second-order modes.

Theorem 1 *Let* $\boldsymbol{K}$ *and* $\boldsymbol{W}$ *be the controllability and observability Gramians of a stable linear system* $H(s) = d + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$ *with minimal realization. Let* $\overline{H}(s)$ *be the power complementary system of* $H(s)$*. Then, there exist matrices* $\boldsymbol{P}, \boldsymbol{Q}, \overline{\boldsymbol{W}}$ *and* $\overline{\boldsymbol{K}}$ *such that*

$$\boldsymbol{P} = \boldsymbol{W} + \overline{\boldsymbol{W}} \tag{15}$$

$$\boldsymbol{Q} = \boldsymbol{K} + \overline{\boldsymbol{K}} \tag{16}$$

where $\boldsymbol{P}$ *and* $\boldsymbol{Q}$ *are positive definite solutions of Eqs. (8) and (9), respectively,* $\overline{\boldsymbol{W}}$ *is the observability Gramian of* $\overline{H}(s) = w + \boldsymbol{l}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$*, and* $\overline{\boldsymbol{K}}$ *is the controllability Gramian of* $\overline{H}(s) = w + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{m}$*. The coefficients* $w, \boldsymbol{l}$ *and* $\boldsymbol{m}$ *in* $\overline{H}(s)$ *are obtained from Eqs. (11), (12) and (14).*

Proof: Since $H(s)$ is stable, a solution $\boldsymbol{P} = \boldsymbol{P}^t > 0$ of Eq. (8) is represented as

$$\boldsymbol{P} = \boldsymbol{X} + \boldsymbol{Y} \tag{17}$$

where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are given as the solutions of the following Lyapunov equations:

$$\boldsymbol{A}^t\boldsymbol{X} + \boldsymbol{X}\boldsymbol{A} = -\boldsymbol{c}^t\boldsymbol{c} \tag{18}$$

$$\boldsymbol{A}^t\boldsymbol{Y} + \boldsymbol{Y}\boldsymbol{A} = -(\boldsymbol{P}\boldsymbol{b} + \boldsymbol{c}^t d)r^{-1}(\boldsymbol{P}\boldsymbol{b} + \boldsymbol{c}^t d)^t \tag{19}$$

From Eqs. (5) and (18), it is obvious that $\boldsymbol{X} = \boldsymbol{W}$. Moreover, from Eqs. (11), (12) and (19), it follows that $\boldsymbol{Y} = \overline{\boldsymbol{W}}$. These relationships show Eq. (15). The proof of Eq. (16) is completely dual to the proof of Eq. (15). □

The above theorem yields an important property of the second-order modes as follows.

Theorem 2 *If a stable system* $H(s) = d + \boldsymbol{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$ *with minimal realization is bounded-real, then the second-order modes are all bounded by* γ*, that is,*

$$\theta_i \leqq \gamma \quad \textit{for } 1 \leqq i \leqq N. \tag{20}$$

Proof: The proof is straightforward. From Theorem 1, it is easily seen that $\boldsymbol{K} \leqq \boldsymbol{Q}$ and $\boldsymbol{W} \leqq \boldsymbol{P}$. Hence we have $\boldsymbol{K}\boldsymbol{W} \leqq \gamma^2\boldsymbol{I}$. This result shows $\theta_i \leqq \gamma$. If $|H(j\Omega)| = \gamma$ for all ω, $\boldsymbol{K}$ and $\boldsymbol{W}$ are equal to $\boldsymbol{Q}$ and $\boldsymbol{P}$, respectively, and therefore $\theta_i = \gamma$ for $1 \leqq i \leqq N$. □

These new theorems can be extended to discrete-time systems. That is, the following two theorems can be established.

Theorem 3 *Let* $\boldsymbol{K}_d$ *and* $\boldsymbol{W}_d$ *be the controllability and observability Gramians of an* N*-th order stable discrete-time system* $H_d(z) = d_d + \boldsymbol{c}_d(z\boldsymbol{I} - \boldsymbol{A}_d)^{-1}\boldsymbol{b}_d$ *with minimal realization, and let* $\overline{H}_d(z)$ *be the power complementary system of* $H_d(z)$*.* $\boldsymbol{K}_d$ *and* $\boldsymbol{W}_d$ *are given as the positive definite solutions of the following discrete-time Lyapunov equations:*

$$\boldsymbol{K}_d = \boldsymbol{A}_d\boldsymbol{K}_d\boldsymbol{A}_d^t + \boldsymbol{b}_d\boldsymbol{b}_d^t \tag{21}$$

$$\boldsymbol{W}_d = \boldsymbol{A}_d^t\boldsymbol{W}_d\boldsymbol{A}_d + \boldsymbol{c}_d^t\boldsymbol{c}_d. \tag{22}$$

Then, there exist matrices $\boldsymbol{P}_d, \boldsymbol{Q}_d, \overline{\boldsymbol{W}}_d$ *and* $\overline{\boldsymbol{K}}_d$ *such that*

$$\boldsymbol{P}_d = \boldsymbol{W}_d + \overline{\boldsymbol{W}}_d \tag{23}$$

$$\boldsymbol{Q}_d = \boldsymbol{K}_d + \overline{\boldsymbol{K}}_d. \tag{24}$$

In the above equations, $\boldsymbol{P}_d$ *and* $\boldsymbol{Q}_d$ *are given as positive definite solutions of the following discrete-time bounded-real Riccati equations*

$$\boldsymbol{P}_d - \boldsymbol{A}_d^t\boldsymbol{P}_d\boldsymbol{A}_d - \boldsymbol{c}_d^t\boldsymbol{c}_d - (\boldsymbol{A}_d^t\boldsymbol{P}_d\boldsymbol{b}_d + \boldsymbol{c}_d^t d_d)r_1^{-1}(\boldsymbol{A}_d^t\boldsymbol{P}_d\boldsymbol{b}_d + \boldsymbol{c}_d^t d_d)^t = \boldsymbol{0} \tag{25}$$

$$\boldsymbol{Q}_d - \boldsymbol{A}_d\boldsymbol{Q}_d\boldsymbol{A}_d^t - \boldsymbol{b}_d\boldsymbol{b}_d^t - (\boldsymbol{A}_d\boldsymbol{Q}_d\boldsymbol{c}_d^t + \boldsymbol{b}_d d_d)r_2^{-1}(\boldsymbol{A}_d\boldsymbol{Q}_d\boldsymbol{c}_d^t + \boldsymbol{b}_d d_d)^t = \boldsymbol{0} \tag{26}$$

where $r_1 = r_2 = \gamma^2 - d^2 - \boldsymbol{b}_d^t \boldsymbol{P}_d \boldsymbol{b}_d = \gamma^2 - d^2 - \boldsymbol{c}_d \boldsymbol{Q}_d \boldsymbol{c}_d^t$. $\overline{\boldsymbol{W}}_d$ *is the observability Gramian of* $\overline{H}_d(z) = w_d + \boldsymbol{l}_d (z\boldsymbol{I} - \boldsymbol{A}_d)^{-1} \boldsymbol{b}_d$ *and* $\overline{\boldsymbol{K}}_d$ *is the controllability Gramian of* $\overline{H}_d(z) = w_d + \boldsymbol{c}_d (z\boldsymbol{I} - \boldsymbol{A}_d)^{-1} \boldsymbol{m}_d$. *The coefficients* $w_d, \boldsymbol{l}_d$ *and* $\boldsymbol{m}_d$ *are obtained as*

$$w_d = r_1^{\frac{1}{2}} = r_2^{\frac{1}{2}} \tag{27}$$

$$\boldsymbol{l}_d = -(\boldsymbol{A}_d^t \boldsymbol{P}_d \boldsymbol{b}_d + \boldsymbol{c}_d^t d_d)^t / w_d \tag{28}$$

$$\boldsymbol{m}_d = -(\boldsymbol{A}_d \boldsymbol{Q}_d \boldsymbol{c}_d^t + \boldsymbol{b}_d d_d) / w_d. \tag{29}$$

Theorem 4 *Let the second-order modes of* $H_d(z)$ *be* θ_{d1},
$\theta_{d2}, \cdots, \theta_{dN}$. *If* $H_d(z)$ *is bounded-real, then the second-order modes of the discrete-time system are all bounded by* γ, *that is,*

$$\theta_{di} \leqq \gamma \quad \textit{for } 1 \leqq i \leqq N. \tag{30}$$

Proof: It is easy to prove the above two theorems with the help of the bilinear transformation $s = \alpha(z-1)/(z+1)$. □

5. A New Formula for Evaluating Minimum Attainable Value of Quantization Effects

In this section, a new formula for evaluating minimum attainavle value of quantization effects is presented as an application of Theorem 4.

As we stated in Section 1, the Gramians and second-order modes play crucial roles in analysis and minimization of quantization effects such as roundoff noise and coefficient sensitivity of digital filters[1, 5, 6]. It should be particularly noted that the second-order modes determine the minimum attainable value of quantization e. For example, the coefficient sensitivity S of an N-th order digital filter $H(z)$ and its minimum value $S_{\min}$ can be obtained from the Gramians and second-order modes as follows[6]:

$$S = (\mathrm{tr}[\boldsymbol{K}_{\mathrm{d}}] + 1)\,(\mathrm{tr}[\boldsymbol{W}_{\mathrm{d}}] + 1) \tag{31}$$

$$S_{\min} = \left(\sum_{i=1}^{N} \theta_{di} + 1 \right)^2. \tag{32}$$

We are now ready to construct a new formula for the evaluation of the minimum coefficient sensitivity $S_{\min}$. Applying Theorem 4 to Eq. (32) gives the upper bound of $S_{\min}$ as follows:

$$S_{\min} \leqq (N\gamma + 1)^2. \tag{33}$$

This new formula shows the effectiveness of minimum coefficient sensitivity structure, especially in the case of narrowband digital filters. As is well known, the coefficient sensitivity of digital filters with direct form tends to be infinity as filter bandwidth approaches zero. On the other hand, we can see from Eq. (33) that the coefficient sensitivity of digital filters with minimum coefficient sensitivity structure is at most $(N\gamma+1)^2$, where γ is generally equal to 1. This fact emphasizes that the minimum coefficient sensitivity structure has excellent noise performance and that it is very suitable to implementation of digital filters on finite wordlength signal processors.

The similar discussion can be made for the evaluation of minimum roundoff noise. Details are omitted here.

6. A Numerical Example

This section gives a numerical example to verify our theorems. Consider the following 2nd-order continuous-time transfer function $H(s)$:

$$H(s) = \frac{0.0032s^2 + 0.0063}{s^2 + 0.1123s + 0.0063}. \tag{34}$$

This transfer function has an lowpass magnitude response with the gain $\gamma = 1$ which is indicated in

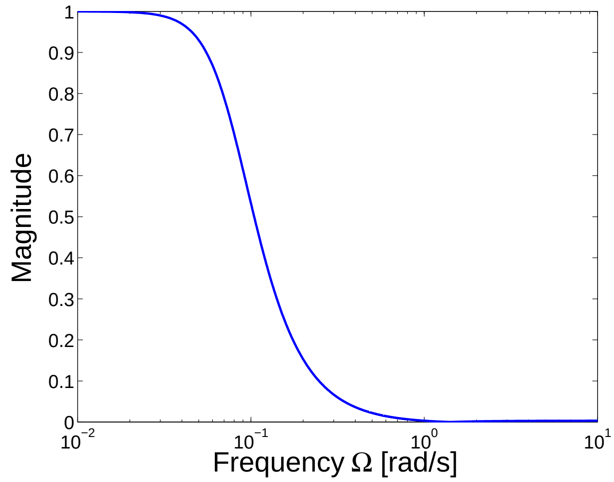

Fig. 3 Magnitude response of $H(s)$. ($\gamma = 1$)

Figure 3. A state-space realization $(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, d)$ of this system is given as follows:

$$\left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{b} \\ \hline \boldsymbol{c} & d \end{array}\right) = \left(\begin{array}{cc|c} -0.1123 & -0.0063 & 1 \\ 1 & 0 & 0 \\ \hline -0.0004 & 0.0063 & 0.0032 \end{array}\right). \quad (35)$$

The Gramians $\boldsymbol{K}, \boldsymbol{W}$ and a solution $\boldsymbol{P}$ of the bounded-real Riccati equation (8) are computed as

$$\boldsymbol{K} = \begin{pmatrix} 4.4527 & 0 \\ 0 & 704.0407 \end{pmatrix} \quad (36)$$

$$\boldsymbol{W} = \begin{pmatrix} 0.0280 & 0.0031 \\ 0.0031 & 0.0005 \end{pmatrix} \quad (37)$$

$$\boldsymbol{P} = \begin{pmatrix} 0.1123 & 0.0063 \\ 0.0063 & 0.0007 \end{pmatrix}. \quad (38)$$

From Eqs. (11), (12) and (38), a power complementary system of the above system is obtained as

$$\overline{H}(s) = \frac{1.0000s^2}{s^2 + 0.1123s + 0.0063} \quad (39)$$

and the observability Gramian of this system is computed as

$$\overline{\boldsymbol{W}} = \begin{pmatrix} 0.0843 & 0.0032 \\ 0.0032 & 0.0002 \end{pmatrix}. \quad (40)$$

Consequently, from (37), (38) and (40) it is verified that $\boldsymbol{W} + \overline{\boldsymbol{W}} = \boldsymbol{P}$. In a similar way, $\boldsymbol{K} + \overline{\boldsymbol{K}} = \boldsymbol{Q}$ can be confirmed. Hence Theorem 1 is verified to be valid.

From Eqs. (36) and (37), the second-order modes of $H(s)$ are obtained as follows:

$$\theta_1 = 0.6822 < 1, \quad \theta_2 = 0.1838 < 1. \quad (41)$$

Therefore, the validity of Theorem 2 is confirmed.

Theorems 3 and 4 for discrete-time systems can be verified in a similar way.

7. Conclusion

This paper has established novel theory on the Gramians and second-order modes of linear state-space systems. It has been shown that a positive definite solution of the bounded-real Riccati equation consists of the Gramians of linear systems and their power complementary systems. Using this result, the upper bound of the second-order modes of linear systems has been described. This upper bound of the second-order modes has given a new formula for evaluating minimum attainable value of coefficient sensitivity and shown the excellent performance of digital filters with minimum coefficient sensitivity structure.

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