計測自動制御学会東北支部 第 303 回研究集会 (2016.7.15)
資料番号 303-3

Improved Block-Structured Sparse Signal Recovery using Iterative-Grouping-based Block-Sparse Adaptive Filtering Algorithms

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キーワード： 圧縮センシング (compressive sensing), スパース信号再構成 (sparse signal reconstruction), ブロックスパース (block-sparsity), 適応フィルタ (adaptive filter), スパース制約 (sparse constraint)

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1. Introduction

Compressive sampling method [1] is viewed as one of most important content in compressive sensing (CS) [2][3], which has attracted a high attention in wide research field, such as computer science, medical science, and imaging processing. In brief, CS signal recovery problem can be formulated as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s}, \tag{1}$$

where $\boldsymbol{s} \in \mathbb{R}^N$ means an original sparse signal, $\boldsymbol{A} \in \mathbb{R}^{M\times N}, (M \ll N)$ stands for a suitable dictionary matrix, and $\boldsymbol{y} \in \mathbb{R}^M$ denotes a transformed down-sampling signal. Rely on the known dictionary matrix $\boldsymbol{A}$ and sampled signal $\boldsymbol{y}$, one can recover the $\boldsymbol{s}$ utilizing sparse signal reconstruction (SSR) algorithms.

Different with conventional sparse signals, whose nonzero atoms appear independently, the signals in block-sparse model have continuous locations as nonzeros or zeros [4][5] as shown in Fig. 1. Usually, the block-structured sparse signals arise in multi-band signals, or gene expression level measurements [4]–[7][18]. Specifically, original block-structured sparse signal can be expressed as below,

$$\boldsymbol{s} = \Big[\underbrace{s_1, \cdots, s_d}_{\boldsymbol{s}^T[1]}, \underbrace{s_{d+1}, \cdots, s_{2d}}_{\boldsymbol{s}^T[2]}, \cdots, \underbrace{s_{N-d+1}, \cdots, s_N}_{\boldsymbol{s}^T[I]}\Big]^T \tag{2}$$

where $N = I \cdot d$, d denotes the grouping distance, and I denotes the number of grouping. Furthermore, according to Eq. (2), $\boldsymbol{s}$ is called block K-sparsity, where $K \in \{1,2, \dots, I\}$ is the most indices of the number of blocks involving nonzero atoms.

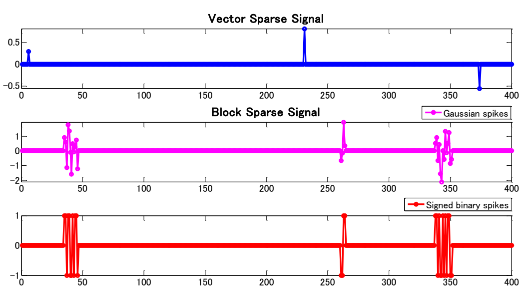


Fig. 1 Block-structured sparse signals.

Recently, the block version of sparse adaptive filtering algorithms, i.e. block zero attracting least mean square (BZA-LMS) algorithm and block ℓ_0-norm LMS (BL0-

LMS) [8], have been proved that can effectively improve recovery performance aim to block-sparse signals under various scenarios. On one hand, block-sparse adaptive filtering algorithms inherit the inherent merits of the conventional sparse adaptive filtering algorithms, such as their moderate computation and well noise elimination capability, moreover the outstanding recovery accuracy [9]. On the other hand, in solving the problems of block-sparse signals recovery, block-sparse adaptive filtering algorithms can adaptively sense partial sparsity information, to obtain evident performance improvement eventually [8].

The grouping pattern of above-mentioned block-sparse adaptive filtering algorithms is based on a fixed form, namely the recovery signal is evenly separated to implement sparse constraint. However, this kind of even grouping pattern may not sufficiently adapt to realistic block-sparse structure of original signals, result in not obtain the full performance gain. To our best of knowledge, this is the first time to employ a novel iterative-grouping pattern into block version of SSR algorithms. The purpose of this paper is to further improve recovery accuracy aim to block-sparse signals.

2. Review of Block-Sparse Adaptive Filtering Algorithms

In this section, the signal recovery method based on adaptive filtering framework, and the block version of adaptive filtering algorithms which apply sparse constraint in fixed groups, will be reviewed.

2.1 Adaptive Framework to Recover CS Signal

Frankly, in practical transmission process, ambient interferences cannot be neglected. Recalling mentioned CS problem Eq. (1), an updated underdetermined equation considering harmful noises is denoted by

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{v}. \tag{3}$$

The recursion error of popular adaptive filtering algorithms [10] can be denoted by

$$e(n) = y_j - \boldsymbol{a}_j \boldsymbol{s}(n), \tag{4}$$

where y_j is viewed as desired signal, $\boldsymbol{a}_j$ is utilized as input training-sequence signal, and $\boldsymbol{s}(n)$ denotes recovery signal in adaptive framework shown in Table 1. Through iteratively minimizing $e(n)$, $\boldsymbol{s}(n)$ is recovered more and more accurately, the illustration of solving CS problem by adaptive framework is shown in Fig. 2.

Table 1 Corresponding variable meanings of adaptive algorithms in solving CS problem.

Adaptive algorithms	CS problem
Input signal	$\boldsymbol{a}_j,\ j \in \{1,2,\ldots,M\}$
Recovery signal	$\boldsymbol{s}(n)$
Desired signal	$y_j = \boldsymbol{a}_j\boldsymbol{s} + v_j$

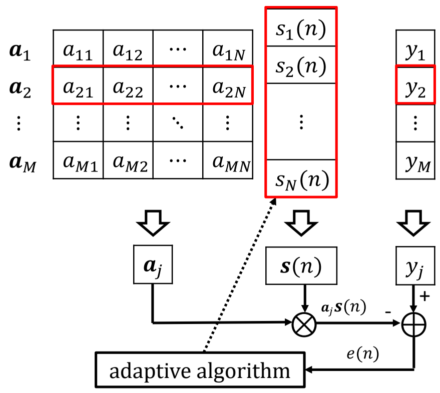


Fig. 2 Solve CS problem in adaptive framework.

2.2 Block Version of Sparse Constraint

Note that standard adaptive filtering algorithms cannot exploit signal sparsity, while actually many effective sparse adaptive filtering algorithms have been presented to obtain sparse solutions [9][11]–[16]. Through separately sensing the sparsity information of each group in recovery signal as shown at the top of Fig. 3, block-sparse adaptive filtering algorithms obtained better recovery effects aim to block-structured sparse signals.

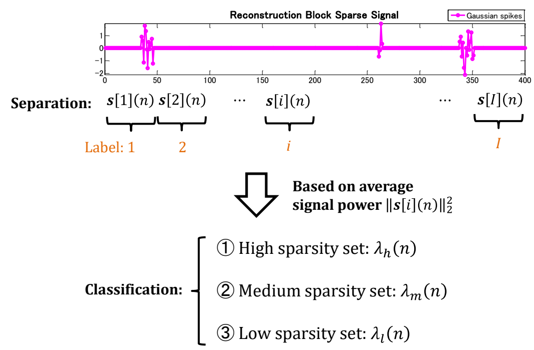


Fig. 3 Grouping separation and grouping classification.

2.3 Block-Sparse Adaptive Filtering Algorithms

The cost function of BZA-LMS algorithm is defined as below,

$$G_{BZA}(n) = e^2(n) + \lambda_i(n)\left\|\boldsymbol{s}[i](n)\right\|_1, \tag{5}$$

where $i \in \{1,2,\dots,I\}$. In Eq. (5), each adaptive regularization parameter (AREPA) is responsible for adaptively regularizing sparse penalty strength to each group of recovery signal, where the formula of AREPA series is defined as

$$\lambda_i(n) = \lambda \cdot \left(\delta + \left\| \boldsymbol{s}[i](n) \right\|_2^2\right) \Big/ \left\| \boldsymbol{s}[i](n) \right\|_2^2 \cdot \sigma^2 \cdot d, \quad (6)$$

where λ means initial REPA parameter. In fact, the AREPA is determined by following four variables,

- $\|\boldsymbol{s}[i](n)\|_2^2$: Average power of recovery signal in groups, which plays the most critical role in adaptive regulation against signal-sparsity levels;
- δ: Threshold of AREPA, which works for guaranteeing the application stability of AREPA;
- σ: Standard deviation of noise interferences, AREPA can adaptively regulate according to different noise levels by involving variance σ^2;
- d: Grouping distance, direct ratio to $\|\boldsymbol{s}[i](n)\|_2^2$ in general.

The update recursion equation derived from the cost function (5) is given as

$$\boldsymbol{s}(n+1) = \boldsymbol{s}(n) + \mu e(n)\boldsymbol{x}(n) - \gamma_i(n)\,\mathrm{sgn}\left(\boldsymbol{s}[i](n)\right), \quad (7)$$

where $\gamma_i(n) = \mu\lambda_i(n)$ denotes adaptive zero attraction series. Furthermore, the equation derivation of BL0-LMS algorithm is similar to BZA-LMS algorithm except implementing the optimal ℓ_0-norm sparse penalty, correspondingly the update recursion equation of BL0-LMS algorithm is given as

$$\begin{aligned} s_l[i](n+1) = s_l[i](n) + \mu e(n) x[i](n+1-l) \\ -\gamma_i(n)\, g\left(s_l[i](n)\right), \end{aligned} \quad (8)$$

where $l \in \{1,2,\dots,d\}$, and the imposed approximation function for simplification is defined as

$$g\left(s_l[i](n)\right) = \begin{cases} \alpha + \alpha^2 s_l[i](n), & s_l[i](n) \in [-1/\alpha, 0) \\ \alpha - \alpha^2 s_l[i](n), & s_l[i](n) \in (0, 1/\alpha] \\ 0, & \text{elsewhere} \end{cases}. \quad (9)$$

The updating procedure of mentioned two adaptive algorithms BZA-LMS and BL0-LMS is illustrated in the left portion of Fig. 4.

3. Proposed Iterative-Grouping Algorithms

For the problem of block-sparse signal recovery, block-sparse adaptive algorithms exhibit performance superiority [8]. However, usually the distribution of nonzero atoms in block-sparse signal yields explicit block form, as a result that grouping signals basically can be classified into several distinct sparsity levels, which inspires us to optimize fixed grouping pattern.

Table 2 BZA-LMS-I algorithm and BL0-LMS-I algorithm.

① Initialize $\boldsymbol{s}(0)=\boldsymbol{0}$, n=1, set suitable μ, λ, δ, d, α by trial and error method;

② While below termination condition is unsatisfied,

$$\left\|\boldsymbol{s}(n) - \boldsymbol{s}(n-1)\right\|_2 < \varepsilon \text{ or } n > C;$$

③ Select input signal $\boldsymbol{a}_j$, and desired signal y_j contaminated by additive noise v_j;

④ Calculate recursion error,

$$e(n) = y_i - \boldsymbol{a}_j \boldsymbol{s}(n);$$

⑤ Recursion update by LMS,

$$\boldsymbol{s}(n+1) = \boldsymbol{s}(n) + \mu e(n)\boldsymbol{a}_j^T;$$

⑥ Grouping classification into three sparsity levels, by fixed lower threshold P_h and upper threshold P_l of recovery signal average power $\|\boldsymbol{s}[i](n)\|_2^2$, separately;

⑦ Design three AREPAs, i.e. $\lambda_h(n)$ for high sparsity, $\lambda_m(n)$ for medium sparsity, and $\lambda_l(n)$ for low sparsity, separately. Then obtain corresponding zero attraction parameter $\gamma_h(n)$, $\gamma_m(n)$, and $\gamma_l(n)$. For instance,

$$\lambda_h(n) = \lambda \cdot \left(\delta + \left\|\boldsymbol{s}_h[i](n)\right\|_2^2\right) \Big/ \left\|\boldsymbol{s}_h[i](n)\right\|_2^2 \cdot \sigma^2 \cdot d,$$

$$\gamma_h(n) = \mu\lambda_h(n);$$

⑧ Sparse penalty implement by BZA or BL0,

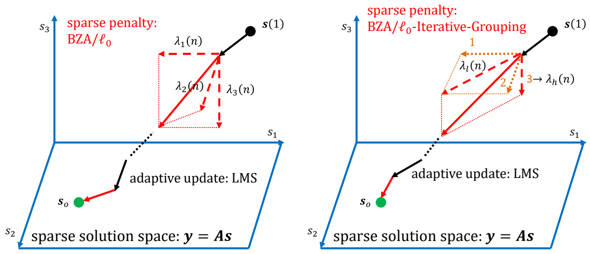
$$\boldsymbol{s}_{BZA}(n+1) = \boldsymbol{s}(n+1) - \gamma_i(n)\,\mathrm{sgn}\left(\boldsymbol{s}[i](n)\right),$$

$$s_{BL0,l}[i](n+1) = s_l[i](n) + \mu e(n) x[i](n+1-l) - \gamma_i(n)\, g\left(s_l[i](n)\right);$$

⑨ The number of iteration increases by one,

$$n = n+1;$$

⑩ End while.



Updating procedure in 3-D domain

Fig. 4 Update procedures of fixed grouping and iterative-grouping sparse adaptive algorithms.

In this study, we propose a novel iterative-grouping pattern in signal process to more adapt to block-sparsity structure of original signal, obtaining two proposed Iterative-Grouping-based BZA-LMS (BZA-LMS-I) algorithm and BL0-LMS-I algorithm. Both our proposed algorithms will further improve the recovery accuracy of block-sparse signals, by additional grouping classification iteratively based on three sparsity levels: high sparsity, medium sparsity, and low sparsity, which is shown at the bottom of Fig. 3. Moreover, the detailed recovery procedure of algorithms BZA-LMS-I and BL0-LMS-I is presented in Table 2. The updating procedure of proposed adaptive algorithms BZA-LMS-I and BL0-LMS-I is illustrated in the right portion of Fig. 4.

4. Simulation Experiments

In this experiments section, the signal recovery performances are evaluated by mean square deviation (MSD). For global parameters, block-sparse signal $\boldsymbol{s}$ is modeled according to [8]: original signal length is $N = 400$, down-sampling dimension is $M = 100$, the size of nonzero atoms in $\boldsymbol{s}$ is $S \in \{10, 20, 30\}$ separately, block sparsity is $K \in \{1, 2, 3\}$ accordingly, also the locations distribution of nonzero blocks and the nonzero coefficients yield discrete uniform. Magnitude of nonzero coefficients yield standard Gaussian distribution $\mathcal{CN}(0,1)$, each entry of dictionary matrix $\boldsymbol{A}$ is independently generated from Gaussian distribution $\mathcal{N}(0,1/M)$. Note that $\boldsymbol{s}$ is normalized in our experiments, and additive noise $\boldsymbol{v}$ is simulated by common Gaussian noise s.t. $\mathcal{N}(0,1/\sigma^2)$, where standard deviation is $\sigma \in \{1.7 \times 10^{-2}, 3 \times 10^{-2}\}$, and signal-to-noise ratio (SNR) is defined as $10\log_{10}(\|\boldsymbol{As}\|_2^2/\sigma^2)$, result in SNR are set as $\{10\text{dB}, 15\text{dB}\}$ separately. Monte-Carlo trials are set as 200 times.

For private parameters, step-size μ is set as 0.05, tolerance error $\varepsilon = 1 \times 10^{-4}$, and iteration runs upper limit $C = 1 \times 10^{6}$ for the six sparse adaptive algorithms. Additionally, the threshold δ of AREPA is set as 0.8, and initial grouping distance is designed as 20 for the four block-sparse adaptive algorithms. Approximation equation parameter is $\alpha = 10$ for the three ℓ_0-norm sparse penalty algorithms. Eventually, the lower threshold of recovery signal average power is $P_h = 5 \times 10^{-3}$ (high sparsity) and upper threshold is $P_l = 5 \times 10^{-2}$ (low sparsity) for proposed two BZA-LMS-I and BL0-LMS-I algorithms. The following three experiments are separately versus various aspects, namely different levels of block-sparsity, noise strengths, and computation complexity.

Experiment ①: In this experiment, the recovery performances of proposed two iterative-grouping-based adaptive algorithms BZA-LMS-I and BL0-LMS-I, are verified comparing with two block version of greedy pursuit algorithms BOMP [4] and BStOMP [17], and existing sparse adaptive algorithms and their block versions under different block-sparsity. In Fig. 5, total number of nonzero atoms $S = 10$, the number of blocks of nonzero atoms $K = 1$ in original signal, SNR is set as 15dB, and step-size of all six sparse adaptive algorithms is set as $\mu = 0.05$ to conduct signal process. It is evident to find that fixed grouping adaptive algorithms BZA-LMS and BL0-LMS much outperform greedy pursuit algorithms, which is consistent with the conclusion in [8]. Rely on the explicit adaption to block-sparse structure of original signal by proposed iterative-grouping algorithms, the MSD performance is further evidently improved without reducing obvious convergence speed. In Fig. 6, the block-sparsity is extended $S = 30, K = 3$, the fixed grouping adaptive algorithms BZA-LMS and BL0-LMS exhibit good performance even under less sparsity condition. In addition, proposed BZA-LMS-I algorithm and BL0-LMS-I algorithm can more accurately sense block-sparse information to obtain improved effects.

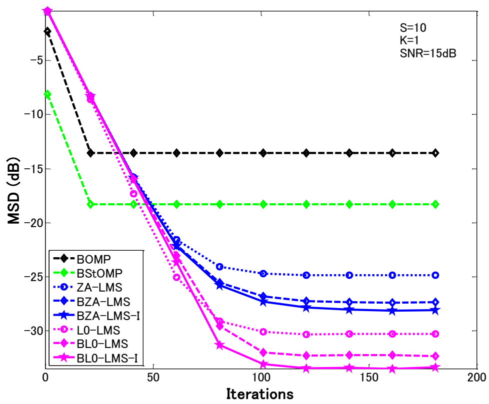


Fig. 5 MSD comparisons vs. Block sparsity (S=10, K=1).

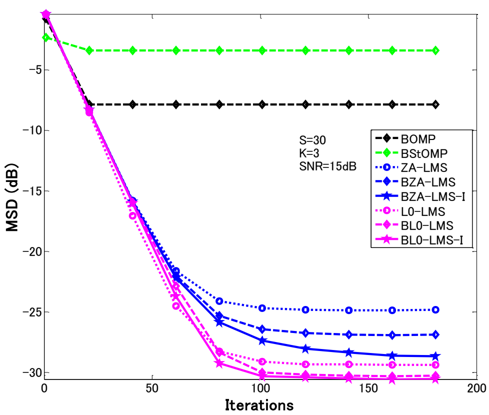


Fig. 6 MSD comparisons vs. Block sparsity (S=30, K=3).

Experiment ②: With ambient noise strength increases, results in SNR decreases to 10dB in this experiment. In this experiment, the sparsity level is set a moderate value, i.e. $S = 20, K = 2$, when the fixed grouping adaptive algorithms BZA-LMS and BL0-LMS still obtain good performance gain as shown in Fig.7, and proposed BZA-LMS-I algorithm and BL0-LMS-I algorithm exhibit better recovery properties in robustly solving block-sparse CS problem.

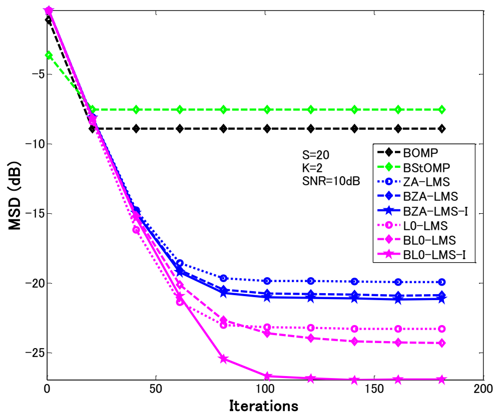


Fig.7 MSD comparisons vs. SNR (10dB).

Experiment ③: In this section, we will explore the computation consumption of proposed iterative-grouping-based sparse adaptive algorithms BZA-LMS-I and BL0-LMS-I. One can easily find that both proposed algorithms moderately consume increased computation time within an acceptable range (approximate 3 times of block version of sparse adaptive algorithms), under different sparsity, which is literally listed in Table 3. The running time is measured using the Matlab (R2013a) program under Core i5-4120U 64-bit processor and Windows 10 environment.

Table 3 Running time comparisons (unit: 1×10^{-2}s).

ALGORITHMS	S=10, K=1	S=20, K=2	S=30, K=3
BOMP	0.18	0.23	0.34
Block-StOMP	0.96	1.28	1.57
ZA-LMS	9.93	9.47	9.88
ℓ_0-LMS	10.61	10.19	10.58
Block-ZA-LMS	11.59	11.07	11.58
Block-ℓ_0-LMS	13.46	12.96	13.41
Block-ZA-LMS-I	38.46	36.56	38.26
Block-ℓ_0-LMS-I	39.16	37.20	38.74

5. Conclusion

In this paper, we optimized the fixed grouping pattern of block-sparse adaptive filtering algorithms to iterative-grouping pattern in solving block-sparse signal recovery problem, to propose two novel adaptive algorithms BZA-LMS-I and BL0-LMS-I. Simulation experiments verified their improved effects, firstly evident performance improvement under various block-sparsity levels without sacrificing immoderate convergence speed, meanwhile reliable robustness against strong noises. At last, the computation complexity of our proposed algorithms is verified as moderate increase, which also inspires us to further improve algorithms efficiency in the future.

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