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Improved Sparse Adaptive Filtering Algorithm for Multi-path Channel Estimation

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1. Introduction

The broadband wireless communication channel is characterized as a frequency selective fading channel[1). It means that the signal is reflected by the buildings between the transmitter and the receiver, and it is received under the influence of multi-path fading with different delays. Therefore, Inter-symbol interference (ISI) occurs due to the frequency selective fading. In order to solve this problem, accurate channel state information (CSI) is required. A promising approach to obtain accurate CSI is an adaptive channel estimation (ACE) using square error criterion (SEC) based the standard least mean square error (LMS) or the least mean forth error (LMF) algorithm, whose general structure is shown in Fig. 1[2).

Recently, a number of channel measurements have verified that broadband wireless channels often exhibit sparse structure as Fig. 2 [3). For an example, consider the signal whose bandwidth is 7.56MHz at a carrier frequency of 770 MHz. There are only 6 non-zero taps in the channel model. Hence, in this sparse channel, a few taps have non-zero coefficients and most of them have zeroes. To improve the estimation performance, the zero-attracting LMS (ZA-LMS) and zero-attracting LMF (ZA-LMF) algorithms using ℓ_1-norm penalized constraint function have been proposed in paper[5). The ZA-LMF algorithm has a smaller steady-state error for applications. However, its convergence properties are very slower than the ZA-LMS algorithm. In practice, higher order power filters can quickly become unstable unless an extremely small step size is employed[6). To take advantage of the zero attraction and to improve the drawbacks of the LMS and LMF algorithms, Li [4) has proposed the zero-attracting least-mean mixture-norm (ZA-LMMN) algorithm. The ZA-LMMN algorithm uses the square error criterion and the forth error criterion to in-

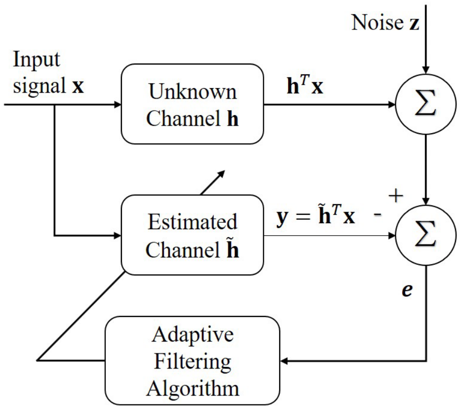

Fig. 1 The structure of channel estimation method based on adaptive filter algorithm.

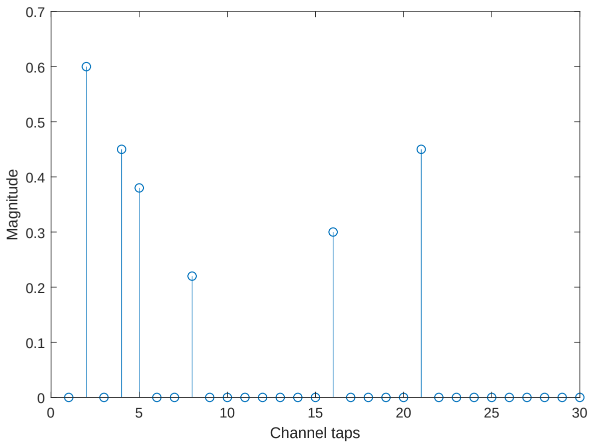

Fig. 2 An example for sparse channel.

tegrate the ZA-LMS and ZA-LMF algorithms. However, the parameter which takes balance between the square error criterion and the forth error criterion is fixed. In this paper, we propose an improved ZA-LMMN algorithm using an adaptive parameter to take balance between the square error criterion and the forth error criterion.

The remainder of this paper is organized as follows. In Section 2, the ZA-LMMN algorithm is described. In Section 3, the improved ZA-LMMN is introduced. In Section 4, numerical simulations are presented to show the effectiveness of the proposed algorithm. The conclusions are given in Section 6.

2. The ZA-LMMN algorithm

2.1 The traditional LMMN algorithm

The traditional LMMN algorithm based on the framework of a channel estimation system is briefly revisited. Consider a broadband multi-path wireless communication channel whose finite impulse response (FIR) channel vector is defined as $\mathbf{h} = [h_0, h_1, ..., h_{N-1}]^T$, where N is the number of channel coefficients. It is supported by only K ($K \ll N$) dominated channel coefficients which are not zeros. The input training signal $\mathbf{x}(t) = [x(t), x(t-1), ..., x(t-N-1)]^T$ is used to probe the unknown sparse channel $\mathbf{h}$ at the time t. At the receiver side, observed signal $y(t)$ is given by

$$y(t) = \mathbf{h}^T\mathbf{x}(t) + z(t), \tag{1}$$

where $z(t)$ is additive white Gaussian noise (AWGN) variable satisfying $\mathcal{CN}(0, \sigma^2)$, which is assumed to be mutually independent with the input training signal $\mathbf{x}(t)$. The task of the channel estimation based on the LMMN algorithm is to obtain $\mathbf{h}$ by minimizing the instantaneous error between the received signal $y(t)$ and the channel estimation output $\tilde{y}(t)$ which is given by $\tilde{y}(t) = \tilde{\mathbf{h}}^T\mathbf{x}(t)$. $\tilde{\mathbf{h}}$ denotes the estimated channel vector. The cost function of the LMMN algorithm is given as

$$J(t) = \frac{\delta_2}{2}J_2(t) + \frac{\delta_4}{4}J_4(t), \tag{2}$$

where $J_2(t) \triangleq E\{e^2(t)\}$, $J_4(t) \triangleq E\{e^4(t)\}$, $E\{\cdot\}$ is the mathematical expectation operator, $\delta_2 \in [0, 1]$ and $\delta_4 = 1 - \delta_2$. Therefore, δ_2 and δ_4 is to give a balance of the combination between $J_2(t)$ and $J_4(t)$. The gradient of $J(t)$ with respect to $\tilde{\mathbf{h}}$ can be written as follows

$$\begin{aligned} \nabla J(t) &= \frac{\partial J(t)}{\partial \tilde{\mathbf{h}}(t)} \\ &= -E\left\{e(t)\left[\delta_2 + \delta_4 e^2(t)\right]\mathbf{x}(t)\right\}. \end{aligned} \tag{3}$$

Using the gradient of $J(t)$ with respect to $\tilde{\mathbf{h}}$, the updating equation of the traditional LMMN algorithm can be written as

$$\begin{aligned}\tilde{\mathbf{h}}(t+1) &= \tilde{\mathbf{h}}(t) + \mu\nabla J(t) \\ &= \tilde{\mathbf{h}}(t) + \mu e(t)\left[\delta_2 + \delta_4 e^2(t)\right]\mathbf{x}(t),\end{aligned} \tag{4}$$

where μ is a step size to control the convergence of the traditional LMMN algorithm. When $\delta_2 = 1$, Eq. 4 reverts to the error norm for the LMS algorithm, whereas when $\delta_2 = 1$, Eq. 4 reverts to the error norm for the LMF algorithm.

2.2 The ZA-LMMN algorithm

The traditional LMMN algorithm does not make use of the channel sparsity because $J(t)$ adopts neither sparse constraint nor penalty function. A modified cost function is proposed in paper [7] to give rise to the ZA-LMMN algorithm. The modified cost function which is given as follows is termed as the ZA-LMMN algorithm.

$$J_{ZA}(t) = \underbrace{\frac{\delta_2}{2}J_2(t) + \frac{\delta_4}{4}J_4(t)}_{J} + \underbrace{\lambda\left\|\tilde{\mathbf{h}}(t)\right\|_1}_{\text{Sparse constraint}}, \tag{5}$$

where $\left\|\tilde{\mathbf{h}}(t)\right\|_1$ is the ℓ_1-norm of $\tilde{\mathbf{h}}(t)$, and λ is regularization parameter to balance the update error term J and sparsity of the channel estimator $\left\|\tilde{\mathbf{h}}(t)\right\|_1$. Thus, the updating equation of the ZA-LMMN algorithm can be written as

$$\tilde{\mathbf{h}}(t+1) = \underbrace{\underbrace{\tilde{\mathbf{h}}(t) + \mu e(t)\left[\delta_2 + \delta_4 e^2(t)\right]\mathbf{x}(t)}_{\text{Traditional LMMN}} - \underbrace{\rho \cdot \operatorname{sgn}\left(\tilde{\mathbf{h}}(t)\right)}_{\text{Sparse constraint}}}_{ZA-LMMN}, \tag{6}$$

where $\rho = \mu \times \lambda$ is the sparse factor, and sgn is a sign function which is given by

$$\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0; \\ -1, & \text{if } x < 0; \\ 0, & \text{if } x = 0. \end{cases} \tag{7}$$

Compared with the traditional LMMN algorithm, the ZA-LMMN algorithm provides an additional term (sparse constraint) which attracts the channel coefficients to zeros. It speeds up the convergence for the ZA-LMMN algorithm to estimate a sparse multi-path channel. When $\delta_2 = 1$, Eq. 6 reverts to ZA-LMS algorithm, while when $\delta_2 = 0$, Eq. 6 reverts to ZA-LMF algorithm.

3. The improved ZA-LMMN algorithm

The misadjustment of the LMMN algorithm is given by [8]:

$$M = \frac{\mu N}{2\sigma_w^2} \times \frac{\delta^2\sigma_w^2 + 2\delta(1-\delta)\xi_w^4 + (1-\delta)\xi_w^6}{\delta + 3(1-\delta)\sigma_w^2} E\left\{\mathbf{x}^2(t)\right\} \tag{8}$$

where $\sigma_w^2 = E\left\{z(t)^2\right\}$, $\delta = \delta_2 = 1 - \delta_4$, ξ_w denotes the expectation of the effective measurement noise. The step-size which controls the convergence speed which is given by [2]:

$$0 < \mu < \frac{1}{NE\left\{\mathbf{x}^2(t)\right\}}\left(\delta + (1-\delta)\frac{1}{6E\left\{z^2(t)\right\}}\right) \tag{9}$$

In Eq. 6, the parameters δ is a fixed value. The paper [8] shows that M whose $\delta = 0$ (LMF) is much lower than M whose $\delta = 1$ (LMS), which means that the LMF algorithm has a lower misadjustment than the LMS algorithm. However, μ with $\delta = 0$ (LMF) is much smaller than μ with $\delta = 1$ (LMS), which means that the LMF algorithm has a slower convergence speed than the LMS algorithm.

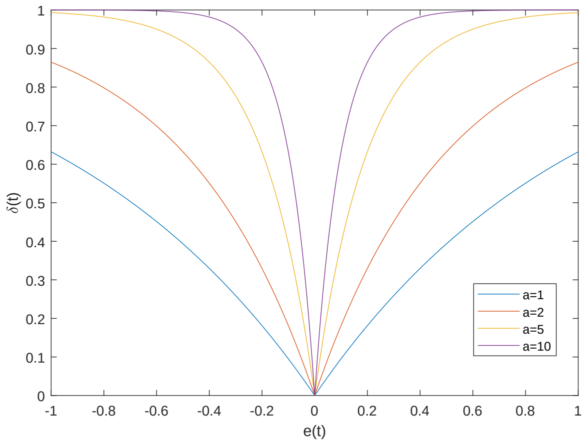

Fig. 3 The function curves of $\delta(t)$ with different parameter a.

In this paper, we propose a variable $\delta(t)$ to adjust the convergence speed and the misadjustment adaptively. The cost function of the improved ZA-LMMN algorithm is given by

$$J(t) = \frac{\delta(t)}{2} J_2(t) + \frac{1-\delta(t)}{4} J_4(t) + \lambda \left\| \tilde{\mathbf{h}}(t) \right\|_1, \quad (10)$$

The updating equation of the improved ZA-LMMN algorithm can be written as

$$\begin{aligned} \tilde{\mathbf{h}}(t+1) &= \tilde{\mathbf{h}}(t) \\ &\quad + \mu e(t) \left[\delta(t) + (1-\delta(t)) e^2(t) \right] \mathbf{x}(t) \\ &\quad - \rho \cdot \operatorname{sgn}\left(\tilde{\mathbf{h}}(t) \right) \end{aligned} \quad (11)$$

where $\delta(t)$ is a variable parameter decided by the residual $e(t)$, which is shown as

$$\delta(t) = 1 - \exp\left(-a \left| e(t) \right|\right) \quad (12)$$

Fig. 3 shows the function curves of $\delta(t)$ with different parameter a. The function of $\delta(t)$ is a symmetric function with return values from 0 to 1. Larger $|e(t)|$ returns closer to 1, while $|e(t)| \to 0$ results in $\delta(t) \to 0$. The parameter $a > 0$ is used to control the slope. When a is larger, the return value around $|e(t)| = 0$ changes faster, and the number of return values close to 1 increases.

Therefore, a large $\delta(t)$ is chosen due to the large residual $e(t)$ at the beginning of the ZA-LMMN algorithm. The large $\delta(t)$ makes the ZA-LMMN algorithm more biased towards the ZA-LMS algorithm, which can achieve a fast convergence speed. When the ZA-LMMN algorithm is close to convergence, a small $\delta(t)$ is chosen due to the small residual $e(t)$. The small $\delta(t)$ makes the ZA-LMMN algorithm more biased towards the ZA-LMF algorithm, which can achieve a smaller steady-state error.

4. Simulations

The performance of our proposed improved ZA-LMMN algorithm is compared with other sparse adaptive filtering algorithms (ZA-LMS, ZA-LMF and ZA-LMMN). In the experiments, each point for all of the used adaptive filtering algorithms is set to 1000 Monte Carlo runs. In this paper, we use a multi-path channel with $N = 20$ taps to evaluate the channel estimation behavior of each algorithm. The number of dominant channel coefficients is marked by K. The parameter is given in Table.1. In all of the simulations, the values of the dominant channel coefficients are created under a standard normal distribution, and the positions of the K taps are chosen randomly within the length of the designated sparse channel, which is subjected to $\|\mathbf{h}\|_2^2 = 0.1$. The random Bernoulli signal is used as the training signal $\mathbf{x}(t)$, and the white Gaussian noise signal $z(t)$ is independent to $\mathbf{x}(t)$. The noise amplitude is set as 0.01. The mean square error (MSE) is defined as

$$\text{MSE} = 10 \times \log \left\| \mathbf{h} - \tilde{\mathbf{h}} \right\|_2 \quad (13)$$

is used to evaluate the steady-state performance of each algorithms.

In the first simulation, we set $K = 2$, the other parameters and the results are shown in Table. 1. The ZA-LMS algorithm can be considered as the ZA-LMMN algorithm with $\delta = 1$, while The ZA-LMF algorithm can be considered as the ZA-LMMN algorithm with $\delta = 0$. In the ZA-LMMN algorithm, we choose a fixed $\delta = 0.5$ to take a balance between LMS and LMF. In the proposed algorithm, we set $a = 10$. It can be found that the MSE of the ZA-LMF algorithm is smaller than the ZA-LMS algorithm after 5000 iterations, and the MSE of the ZA-LMS is smaller than 20dB after 168 iterations while the MSE of the ZA-LMF is smaller than 20dB after 1094 iterations. When $\delta = 0.5$, the ZA-LMMN algorithm has a smaller MSE after 5000 iterations and less iterations when MSE$<$ 20dB than the ZA-LMS and ZA-LMF algorithms. Our proposed algorithm has a smallest MSE and least iterations than ZA-LMMN algorithm. The convergence curves of all algorithms are shown in Fig. 4. On the one hand, the ZA-LMS, ZA-LMMN algorithms and our proposed algorithm achieve convergence fast, while the final MSEs of the ZA-LMS and ZA-LMMN algorithms are larger than the ZA-LMF algorithm and our proposed algorithm. On the other hand, the convergence of the ZA-LMF algorithm is much slower than other algorithms. The ZA-LMMN algorithm combine the advantages of the ZA-LMS and ZA-LMF algorithms. Our proposed algorithm uses a variable $\delta(t)$ to makes better use of their advantages than the ZA-LMMN algorithm.

The function curve of each $\delta(t)$ in the adaptive filtering algorithms are shown in Fig. 5. The parameters $\delta(t)$ of the ZA-LMS, ZA-LMF and ZA-LMMN algorithms are fixed at 1, 0

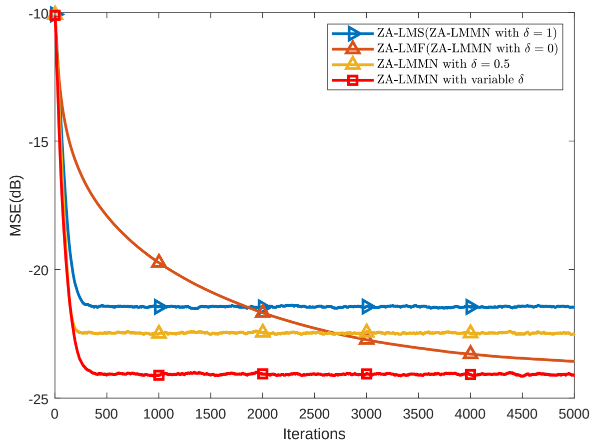

Fig. 4 The convergence curves.

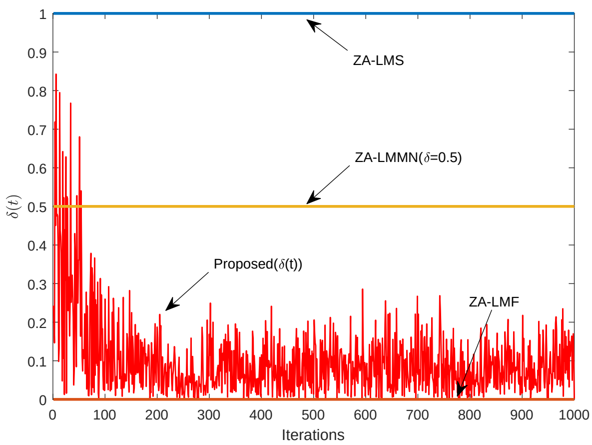

Fig. 5 The curves of $\delta(t)$.

Table 1 The parameters

Algorithm	δ in LMMN	ρ	μ	MSE(after 5000 iterations)	Iteration (MSE< 20dB)
ZA-LMS	1	5×10^{-3}	0.02	-21.4549dB	168
ZA-LMF	0	1×10^{-6}	1.8	-23.5688dB	1094
ZA-LMMN	0.5	1×10^{-5}	0.05	-22.4634dB	119
Proposed	$\delta(t)$	1×10^{-5}	0.08	-24.0671dB	115

and 0.5, respectively. The parameters $\delta(t)$ of our proposed algorithm is variable based on the residual $e(t)$. At the beginning, the parameters $\delta(t)$ of our proposed algorithm is close to 1, which biased towards the ZA-LMS algorithm. It provides a fast convergence speed. When our proposed algorithm approaches convergence, the small $\delta(t)$ which is close to 0, provides a small MSE like the ZA-LMF algorithm.

5. Conclusion

We have proposed an improved ZA-LMMN algorithm for multi-path channel estimation. Instead of a fixed δ in the ZA-LMMN algorithm, a variable $\delta(t)$ is used in our proposed algorithm. Using the variable $\delta(t)$, our proposed algorithm can achieve a fast convergence speed like the ZA-LMS algorithm at the beginning, while can achieve a small MSE like ZA-LMF algorithm at last. The simulation results verify the effectiveness of our proposed algorithm.

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