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Stability and Nonlinear Behavior of Poppet Valve Circuits

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1. Introduction

Poppet valves are extensively used in hydraulic systems as pressure regulating valves, e.g., relief valves, pilot parts of a reducing valves, check valves and cartridge valves because of the simplicity of the construction and the high pressure-sensitivity to valve displacement. It is a familiar fact to engineers in this field that the system is liable to be unstable and often suffers from noisy chatter. Since the instability of the valves is a cause of abnormal performance and a source of noise, a number of researchers have studied it for a long time.

Most such studies deal with the local behavior in the vicinity of steady states of the system. However, the system has various kinds of non-linearities, e.g., flow characteristics of valves and orifice pressure-flow characteristics, dry friction and collision of the poppet against the seat. Thus, the global behavior is sometimes very different from the local one. Although the system has a simple structure in appearance, the problem has rarely been investigated because of the difficulty of analysis due to marked non-linearity and complexity of phenomena occurring in the system.

In recent years, the situation has been drastically changed by the development of high-speed computers. That is, it is now possible to analyze the dynamics of systems with complex non-linearity by making use of numerical simulation, and the change of system responses according to variation of system parameters is easily analyzed.

This paper outlines past studies on local stability of hydraulic systems including poppet valves and then describes the course of studies on the mechanism of the instability and the global behavior of poppet valve circuits on the basis of the author's studies [17]-[21],[23].

2. Historical background

As far as I am aware, the first discussion concerning the stability of the poppet valve was the study by Lutz [1], which has been quoted in "Mechanical Vibrations" by Den Hartog [2]. Lutz has dealt with the stability of a diesel-engine fuel-injection valve which has similar construction to an inflow-type poppet valve as shown in Figure 1. He has clarified that the instability of the valve is caused by the compressibility of the working fluid.

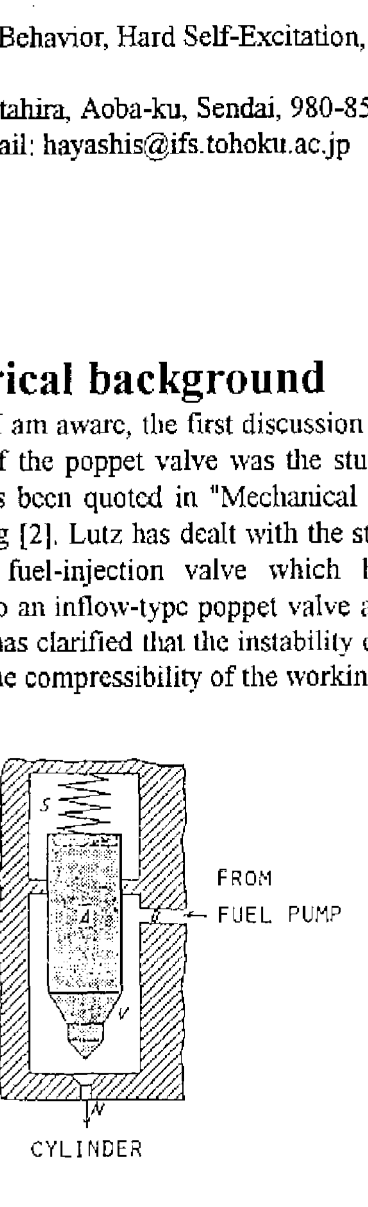


Figure 1 Diesel engine fuel injection valve

In the field of hydraulics, Backé and Rünnenburger [3] have reported a detailed study on the stability of a relief valve. They have verified the conjecture by Bickel and Acel [4] that the valve instability is not caused by the compressibility of working fluid because of smallness of valve chamber volume and short pipeline length, but rather by the interaction between the poppet motion and other components in the circuit such as pumps and actuators as shown in Figure 2.

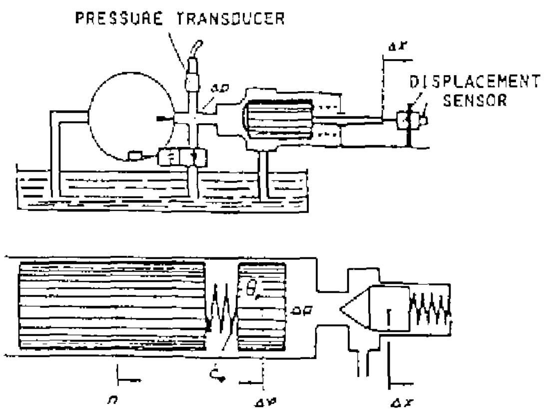


Figure 2 Poppet valve circuit by Backé

Funk [5] has reported a theoretical study on the influence of valve chamber volume and pipeline length on the stability of a poppet valve circuit shown in Figure 3, in which the poppet is connected with a supply line through an orifice and a plenum chamber. In the analysis, the pipeline was treated as a lossless and distributed-parameter element in which the compressibility of oil is taken into consideration. He has clarified the fundamental properties affecting the local stability of the system as follows : (1) a long pipeline makes the system unstable, (2) the critical frequency of stability coincides with that of the fundamental vibratory mode of the pipeline, (3) the lower the fundamental frequency is, i.e., the longer the pipeline is, the more stable the system is, (4) a large valve displacement and a large discharge coefficient of the orifice stabilizes the system. Since the conclusions are qualitative, consequently, many other studies have been attempted to complement his work.

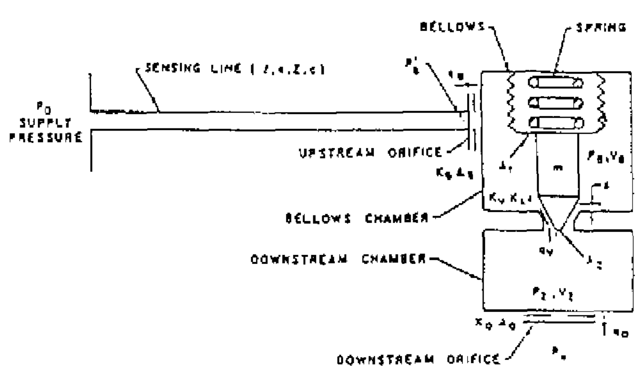


Figure 3 Poppet valve circuit by Funk

Ichikawa and Nakamura [6] have theoretically and experimentally investigated the effect of chamber volume and pipeline length on the stability of a direct-acting poppet valve circuit in detail.

Kasai [7],[8] has investigated the effect of the upstream and the downstream pipeline of the poppet valve on the stability and has derived a critical condition for stability. Furthermore, he has argued theoretically and experimentally that the circuit becomes unstable due to external periodic disturbances.

Waldling and Johnson [9], [10] have examined the effect of distributed mass dynamics of the spring suspending the poppet on the stability of the valve circuit.

Maeda [11]-[13] has indicated the occurrence of transverse self-excited vibrations due to the unsteady transverse flow force.

Concerning the instability due to internal flow of the poppet, there have been studies by McCloy and McGuigan [14]. They have pointed out that vibrations may be induced by the change of flow force accompanying the change of flow patterns on the basis of Schrenk's observation for flow patterns issuing from the poppet valve into a confined chamber[15].

Green and Woods [16] have imply five causes of instability by the flow around the poppet: (1) Instability resulting from coupled motions of the poppet with other circuit elements. (2) Instability caused by flow transition from laminar to turbulent with increasing or decreasing flow rate accompanying poppet displacement. (3) Instability caused by negative restoring flow force. (4) Instability caused by pressure difference between opening and closing of the valve (hysteresis of flow force). (5) Vibrations induced by fluctuation of supply pressure.

The instability (1) has already been stated in reference [1]-[10]. As for the instability (5), we cannot infer from the paper [16] whether the vibrations are self-excited vibrations or a kind of forced vibration. The instabilities (2),(3) and (4) are said to be unstable phenomena caused by the flow force, but the details of the phenomena are unclear, since Green and Woods have shown neither the measuring results concerning the flow force, nor the vibratory waveforms.

3. Mechanism of instability

The poppet valve circuits are classified into two typical configurations, i.e. a pilot type poppet valve circuit and a direct-acting poppet valve circuit. The outlines are shown in Figure 4(a) and (b).

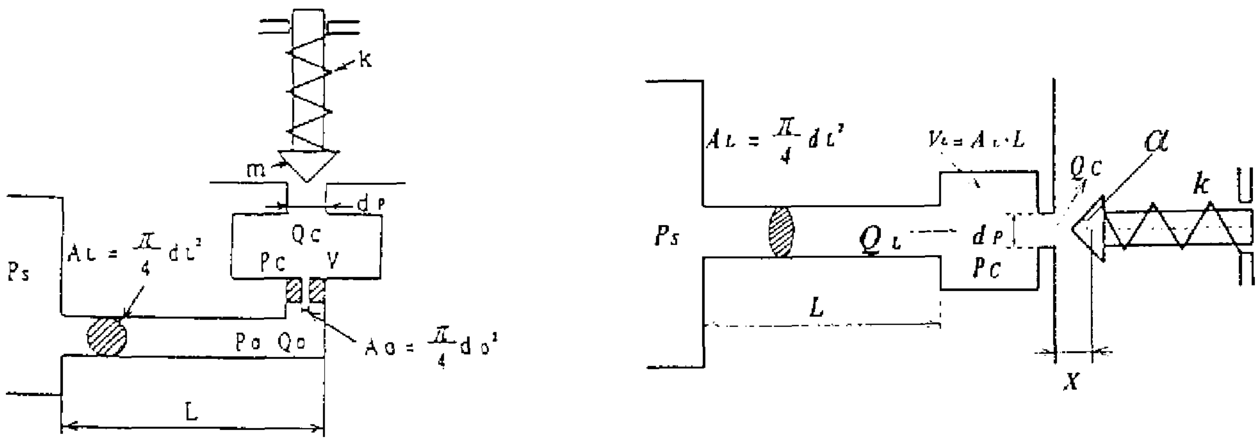
The governing equations of the circuit are given in the Appendix together with the linearized forms, the associated transfer functions and the characteristic equation. In this section, we consider the mechanism of the instability of the poppet valve circuit from the point of view of mechanical energy transferred to the poppet through the flow force [17].

From the linearized equation (13) of poppet motion, the following energy equation is derived:

$$\Delta E = \int_{E_1}^{E_2} dE = -\int_{T_1}^{T_2} \delta \dot{x}^2 dt + \int_{T_1}^{T_2} f \dot{x}^2 dt , \qquad (1)$$

where E $(=m\dot{x}^2/2 + kx^2/2)$ is the mechanical energy of the poppet, $E_1=E(T_1)$, $E_2=E(T_2)$ and $T(=T_2-T_1)$ the period of a vibration.

ΔE is the incremental change of the mechanical energy



(a) Pilot type poppet valve circuit (b) Direct-acting poppet valve circuit

Figure 4 Schimatic diagrams of poppet valve circuits

for a cycle. The vibration is amplified for $\Delta E > 0$ and is attenuated for $\Delta E < 0$.

Here we assume that the valve displacement and the valve chamber pressure change sinusoidally about a steady state as

$$x = x_0 \sin \omega t$$
$$p_c = -p_0 \sin(\omega t - \phi). \tag{2}$$

From Eq. (1) and Eq. (2), we obtain the following equation.

$$\Delta E = -\pi \delta \omega x_0^2 + \pi a_p p_0 x_0 \sin \phi. \tag{3}$$

The first term representing the energy dissipation by the damping force. The second term is the work done on the poppet by the flow force during a cycle of vibration.

Since $\phi > 0$, i.e., the chamber pressure P_c inevitably delays with respect to the valve motion, thus the second term of Eq. (3) is always positive. If $\Delta E > 0$, then the total mechanical energy of the poppet increases for every cycle of the vibration and consequently the vibration is amplified. Otherwise it decays.

There are a variety of causes of the delay of the chamber pressure ϕ depending on circuit configuration. For example, if the compressibility of oil in the valve chamber is not ignored in the circuit shown in Figure 4(a), the delay of chamber pressure is given by $\phi = \tan^{-1}(T\omega) > 0$ for $L = 0$ from Eq. (17) in Appendix. Thus, the second term of Eq. (3) becomes always positive. This implies that the delay due to the compressibility of oil may be a factor of instability. The transmission delay due to a long pipeline can also be a factor of instability.

The damping force is so small that the valve is apt to vibrate excessively, unless it is actively intensified,. This is the essential reason why the poppet valve circuit is liable to become unstable. Even if the effect of the compressibility of oil can be ignored, the valve chamber pressure often delays with respect to the poppet displacement as the result of the interaction between the valve motion and other system components, e.g., pumps, actuators and other control valves. This sort of delay also causes instability.

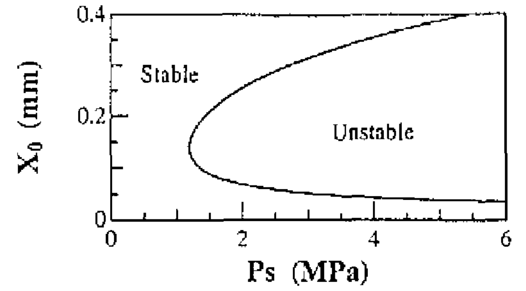
On the other hand, if the system has negative damping, i.e., $\delta < 0$, it becomes unstable. The causes of poppet valve instability are summarized as follows:

(a) delay due to compressibility of the working fluid,
(b) high-order delay due to the interaction by other components,
(c) negative damping force.

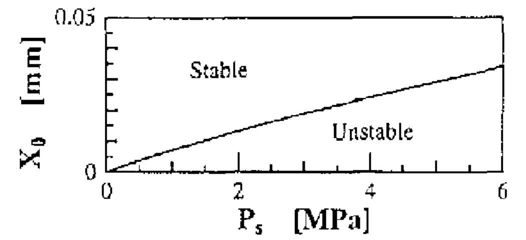
4. Local stability

Prior to describing the global nature of the system shown in Figure 4, the local stability will be mentioned briefly.

Figure 5 shows stability maps calculated from the characteristic equation (15); they are plotted using supply pressure P_s and valve displacement X. Figure 5(a) shows the map for a pilot type circuit having no pipeline L=0. The system is stable for the small valve displacement, whereas it becomes unstable for the relatively large valve



(a) Pilot type poppet valve circuit



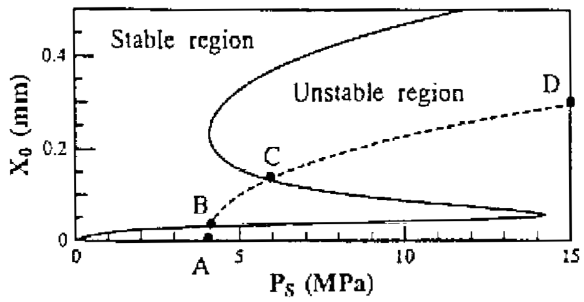
(b) Direct-acting poppet valv circuit

Figure 5 Stability map of poppet valve circuits

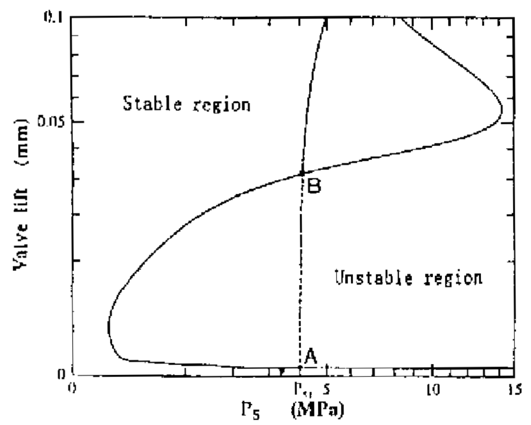
displacement. In this case, the unstable vibration has the poppet valve frequency f_p indicated in Eq.(4). On the other hand, for a direct-acting poppet valve circuit $L \neq 0$, the small valve displacement region becomes unstable as seen in Figure 5(b) and the unstable vibration has the pipeline frequency f_L.

$$f_p = \sqrt{k_x/m}\Big/2\pi, \quad f_L = \sqrt{\beta A_L/\rho V_c L}\Big/2\pi \tag{4}$$

Figure 6(a) shows the stability of a pilot type poppet valve circuit having the pipeline with viscous resistance and the partly enlarged map is shown in Figure 6(b).



(a) Stability map



(b) Partly enlarged map

Figure 6 Stability of a pilot type poppet valve circuit

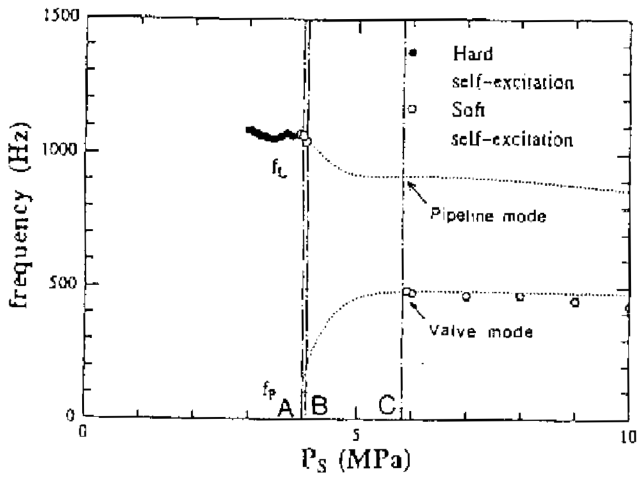


Figure 7 Natural frequencis of pilot type circuit

As seen in Figure 6, the instability region consists of the valve mode instability and the pipeline mode instability. As increasing P_s along the static valve displacement curve shown by a dashed line, the system becomes unstable in the interval AB and the interval CD ; the former corresponds to the pipeline mode and the latter to the valve mode as seen in Figure 7 which shows the linear frequencies calculated from the characteristic equation (15). Each curve coincides with the respective natural frequencies of Eq. (4) at the valve cracking point $P_s = P_{si}$.

The symbols ○ and ● in Figure 7 indicate the self-excited frequencies of the pipeline and the valve mode calculated by numerical simulation respectively. From the energy transfer equation (3), it is inferred that the valve mode instability is caused by the high pressure sensitivity of the pilot type valve $|\partial P_c/\partial X|_0$ as seen in Figure 8 and Eq. (5), which is derived substituting Eq. (17) in Appendix into Eq. (3).

$$\Delta E = \pi x_0^2 \left[-\delta\omega + a_p \left| \frac{\partial P_c}{\partial X} \right| \left(\frac{T\omega}{1+(T\omega)^2} \right) \right] \tag{5}$$

On the other hand, it is reasoned that the pipeline mode instability is owing to the amplification of p_0 by the resonance of the pipeline mode.

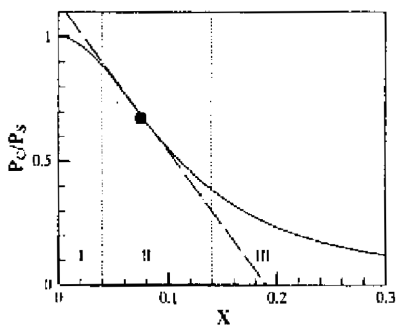


Figure 8 Static characteristics of pilot poppet valve

5.Nonlinear behavior

The poppet valve circuits shown in Figure 4 frequently exhibit strange behaviors, i.e., disagreement of theoretical results with experimental ones and occurrence of chaotic vibrations which can be unexpected from the local nature of the system. However, no study has been performed to clarify the phenomena. The resent development of numerical simulation exhibits the power to analyze such a complex system. Here the author will describe several global behaviors of the poppet valve circuit on the basis of his studies [17]-[21], [23].

5.1 Hard self-excited vibration

Figure 9 shows a result of numerical simulation carried out to examine the influence of disturbances on the system behavior. The response for the velocity disturbance smaller than a critical value converges to a stable steady state. On the other hand, the response for the larger disturbance grows into a self-excited vibration.

This sort of self-excited vibrations, that occur at stable steady states due to large disturbances, is called "hard self-excitation" [17], which corresponds to a situation that a stable limit cycle encircles an unstable limit cycle, in which a stable equilibrium point is included [22]. By taking this phenomenon into consideration, the self-excited region is considerably widened in comparison with the locally unstable region as shown in Figure 10.

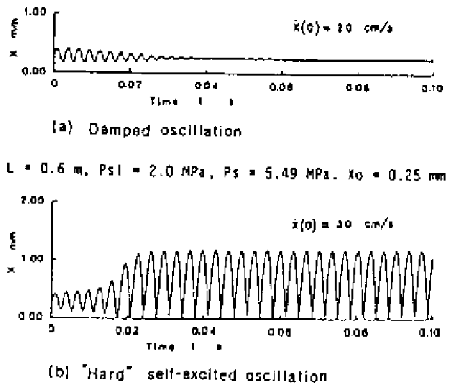


Figure 9 Responses for velocity disturbances

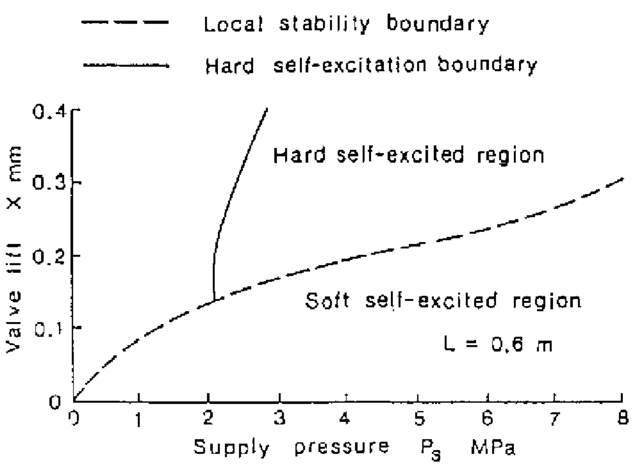


Figure 10 Self-excited region

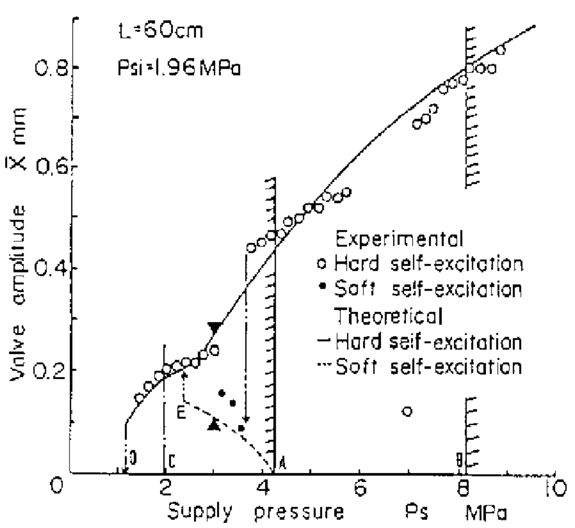


Figure 11 Amplitude of self-excited vibration

The most noticeable characteristics of the hard self-excitation is that once the self-excited vibration occurs, the vibration does not stop until a far lower supply pressure than the cracking pressure is reached. Figure 11 shows the change of amplitude of vibrations with supply pressure P_s, where the cracking pressure P_{si} (=2.0 MPa) is indicated by a chain line and the hatched areas indicate locally unstable regions. As seen in Figure 11, the self-excited vibration occurs also in the stable region and continues until P_s=1.1 MPa.

It has been pointed out that the self-excited vibration of the valve mode is caused by asymmetry of the static characteristics, P_c vs. X [19].

The symbols ● in Figure 7 shows another type of "hard self-excitation" of the pipeline mode, which occurs due to large disturbances beyond a critical value even for a supply pressure lower than the cracking pressure P_{si}, although the poppet stably rests on the valve seat.

Figure 12 shows the responses for slightly different initial velocities under the operating condition indicated by the symbol ▼ in Figure 6(b). It is conjectured to be caused by a shift of the equivalent static state into the unstable region in Figure 6(b) ; the center of the poppet's positional vibration locates at some distance from the valve seat due to the collision of the poppet against the seat.

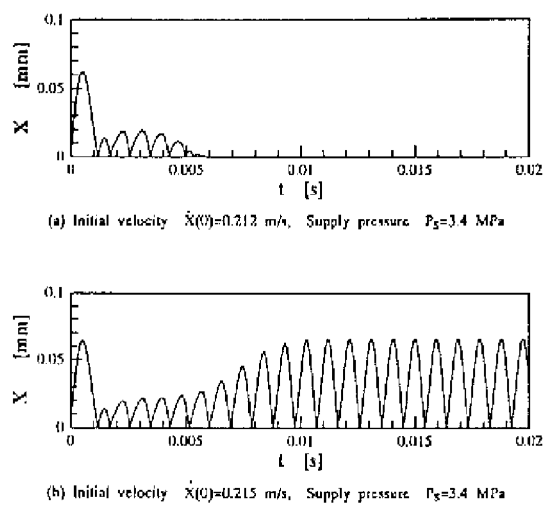


Figure 12 Responses for a supply pressure under cracking pressure

5.2 Chaotic vibrations

Chaotic vibrations are sometimes observed in the poppet valve circuits. This section describes the chaotic vibrations induced in the circuit [19], [20],[22].

5.2.1 Feigenbaum chaos

Figure 12 is an amplitude diagram on which all the maximum points of vibratory waveforms of poppet displacement are plotted against supply pressure P_s. The dashed curve represents the static valve lift. The points A and B correspond to those of Figure 6(b) ; they indicate the critical points of stability. Other static points except the interval AB in the figure are locally stable as described previously.

The self-excited vibration undergoes a series of period-doubling bifurcations with increasing P_s leading to chaotic vibrations. Figure 13 shows the waveform, the power spectrum and the phase plane trajectory of a chaos occurring in this region. With further increase in P_s, a period-3 window appears. At higher P_s still the vibration enters the second chaotic regime. The chaotic vibration disappears around P_s=3.85 MPa. The vibration goes through period-2 and period-3 and returns to period-1 vibration around the cracking pressure P_s=4.0 MPa.

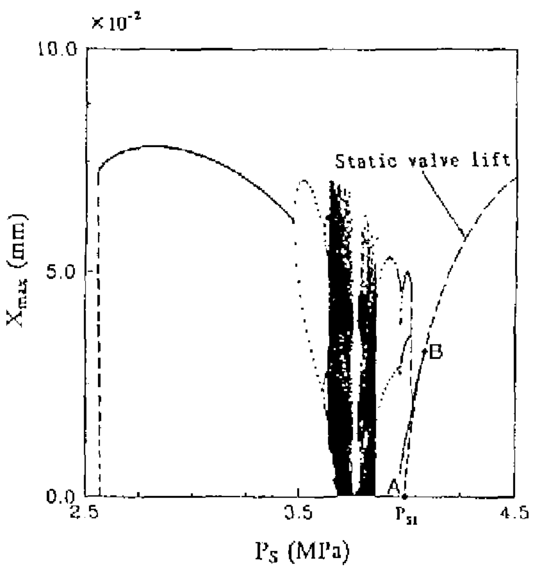


Figure 13 Bifurcation diagram

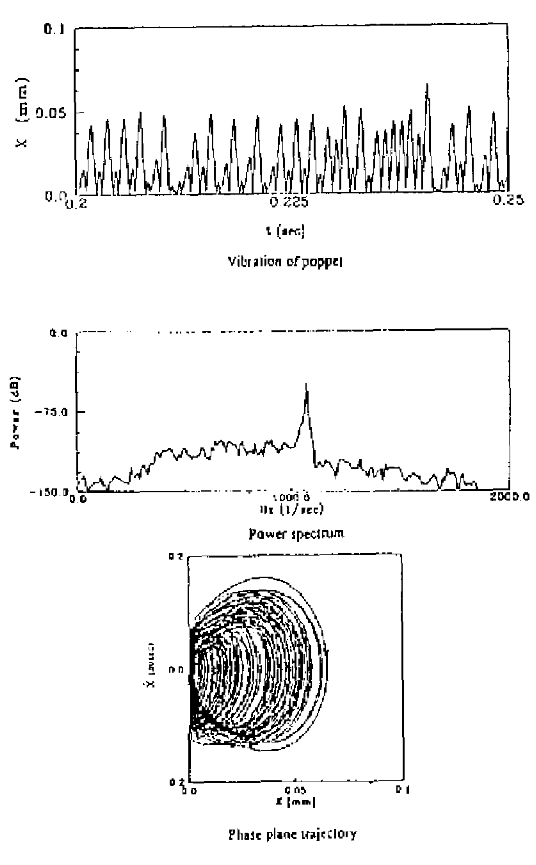


Figure 14 Chaotic vibration

It can be proved that the period-doubling bifurcation sequence in Figure 13 obeys the typical Feigenbaum's route [24], and Feigenbaum number 4.6692016···can be derived from the period-doubling bifurcation diagram.

5.2.2 Chaos in direct-acting circuit

The stability of the direct-acting poppet valve is considerably degraded by the direct interaction between the motion of the poppet valve and the oil column in the pipeline and various kinds of chaotic vibrations are observed in the circuit [22].

Figure 16 shows the natural frequencies of the system and the frequencies of self-excited vibrations calculated by numerical simulation. The curves $[f_1]$, $[f_2]$, $[f_3]$·· represent the pipeline mode frequencies which are the imaginary parts of the characteristic roots. The curve $[f_0]$ corresponds to the valve mode vibration, which is always stable regardless of pipeline length. The stability of the system is indicated by two horizontal broken lines, between which every mode becomes unstable [23].

In the system, several modes become unstable at the same time for the same operating condition. With increasing pipeline length, the number of unstable vibratory modes increases. However, not all the unstable modes are necessarily excited, but only a few modes selectively occur. The vibrations occurring in the system are classified and indicated by symbols ○-◎ in Figure 16.

It is crucial to note that chaotic vibrations shown by the symbols △ and ϕ occur in the region indicated by A, B, B' and C, although the mathematical model used here is deterministic and does not include any statistical factors. These chaotic vibrations obtained by numerical simulation have been confirmed to be valid qualitatively by an experiment. [21].

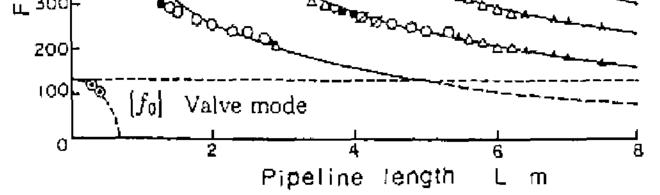


Figure 16 Modal frequencies direct-acting poppet valve circuit and frequencies of self-excited vibrations
(○=period-one, ●=period-two, □=period-three, ○=period-four, ▽=period-five, ■=period-six, ○=period-eight, ▲=almost periodic, △=chaotic, ϕ=intermittent chaos, ◎=damped vibration. Period-n means 1/n sub-harmonics with the frequency f/n)

In the region A, only a mode $[f_1]$ is unstable and the self-excited vibration transforms into Feigenbaum-type chaotic vibration through period-doubling bifurcation with increasing L as a parameter. This chaotic vibration is similar to that indicated in Figures 13-15.

In the region B, the three modes $[f_1]$, $[f_2]$ and $[f_3]$ are unstable and the almost periodic vibration composed of two unstable modes $[f_1]$ and $[f_2]$ changes into Lorenz-type chaotic vibration. In region B', the same sort of chaotic vibration as in the region B occurs.

In the region C, a vibration with beating waveform occurs. The power spectrum indicates side bands on both sides of peaks of the mode $[f_2]$ and its higher modes. The vibration is regarded as an intermittent chaos.

Of various self-excited vibrations, simple periodic vibrations produce sound close to pure tones, but almost periodic and chaotic vibrations produce unpleasant noisy tones. Thus, prevention of these vibrations is important also from the viewpoint of noise reduction.

6. Conclusions

Studies on the local instability of poppet valve circuits have been reviewed and the global behavior has been described on the basis of the author's studies by numerical simulation. The global behavior of the poppet valve circuits is analyzed and occurrence of hard self-excited vibrations and various chaotic vibrations are indicated. Since the poppet valve motion is affected by various non-linearities, complexity of the dynamic behavior is greater than was at first expected.

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Appendix

Governing equations

The governing equations for the circuits shown in Figure 4 are derived as follows:

(a) Pilot type poppet valve circuit

The equation of motion of the poppet is

$$m\frac{d^2X}{dt^2}+\delta\frac{dX}{dt}+k(X+X_I)=F \tag{6}$$

where the flow force acting on the poppet is expressed as

$$F=A_pP_c\left(1-4c_p\frac{X}{d_p}\sin 2\alpha\right). \tag{7}$$

The relation between the velocities before and after a collision of the poppet against the valve seat is

$$\dot{X}(t^+)=-e\dot{X}(t^-), \tag{8}$$

where e is the coefficient of restitution for the poppet to the valve seat and t^- and t^+ are times just before and after collision, respectively.

The continuity equation of the valve chamber is

$$\frac{V_c}{\beta}\frac{dP_c}{dt}=Q_o-Q_c. \tag{9}$$

The equation of motion of the fluid column in the pipeline by a lumped parameter approximation is

$$\frac{\rho L}{A_L}\frac{dQ_o}{dt}=P_S-P_o. \tag{10}$$

The flow characteristic of the poppet valve is

$$Q_c=c_p\pi d_pX\sin\alpha\sqrt{\frac{2}{\rho}P_c}, \tag{11}$$

putting P_c=0, when P_c<0.

The flow characteristic of the orifice is

$$Q_o=c_oA_o\sqrt{\frac{2}{\rho}|P_o-P_c|}\,\mathrm{sgn}(P_o-P_c). \tag{12}$$

(b) Direct-acting poppet valve circuit

Putting the discharge coefficient of the orifice $c_o\to\infty$ in Eq. (12) and $V_c=A_LL$ in Eq. (9), we obtain the governing equations for a direct-acting poppet valve circuit.

Linearized equations and transfer functions

We obtain the following linearized equations from Eqs. (6)-(12) at a certain steady state.

$$\begin{aligned} &m\ddot{x}+\delta\dot{x}+k_xx=a_pp_c\\ &C\dot{p}_c=q_o-q_c,\quad I\dot{q}_o=-p_o\\ &q_c=c_xx+c_cp_c,\quad q_o=c_v(p_o-p_c)\end{aligned} \tag{13}$$

where $C(=V_c/\beta)$ is the valve chamber capacitance, I $(=(\rho L)/A_L)$ the pipeline inertance, $c_x=(\partial Q_c/\partial X)_0$, $c_c=(\partial Q_c/\partial P_c)_0$, $c_v=(\partial Q_o/\partial (P_o-P_c))_0$, $k_x=k-(\partial F/\partial X)_0$, $a_p=[\partial F/\partial P_c]_0$ and the subscript 0 means a steady state.

From Eq. (13), the following transfer functions are derived.

$$\frac{X(s)}{P_c(s)}=\frac{a_p}{m}\frac{1}{s_2+2\varsigma_p\omega_ps+\omega_p^2} \tag{14}$$

$$\frac{P_c(s)}{X(s)}=-\frac{c_x}{c_vCI}\frac{(Ic_vs+1)}{(s^2+2\varsigma_L\omega_Ls+\omega_L^2)}$$

The characteristic equation of the system is

$$s^4+A_3s^3+A_2s^2+A_1s+A_0=0 \tag{15}$$

where

$$\begin{aligned} &A_0=\omega_L^2\omega_p^2+\omega_L^2|(\partial P_c/\partial X)_0|a_p/m,\\ &A_1=2\varsigma_L\omega_L\omega_p^2+2\varsigma_p\omega_p\omega_L^2+a_pc_x/(mC),\\ &A_2=\omega_L^2+\omega_p^2+(2\varsigma_p\omega_p)(2\varsigma_L\omega_L),\\ &A_3=2\varsigma_L\omega_L+2\varsigma_p\omega_p,\\ &\omega_p^2=k_x/m,\quad \omega_L^2=(1+c_p/c_v)/(CI)\\ &2\varsigma_p\omega_p=\delta/m,\quad 2\varsigma_L\omega_L=c_x/C+1/(Ic_v)\\ &|(\partial P_c/\partial X)_0|=c_x/(c_v+c_p)\end{aligned} \tag{16}$$

For L=0, or I=0, the P_c/X in Eq. (14) is reduced to

$$\frac{P_c(s)}{X(s)}=-\frac{K}{Ts+1}, \tag{17}$$

where $K=c_x/(c_v+c_p)=|\partial P_c/\partial X|_0$ and $T=C/(c_v+c_p)$.

Nomenclature

- A_L : cross-sectional area of pipeline
- A_o : opening area of orifice
- A_p : cross-sectional area of valve seat
- c_o : discharge coefficient of orifice
- c_p : discharge coefficient of poppet valve
- d_p : diameter of poppet valve seat
- F : axial flow force acting on poppet
- f : perturbation of flow force
- f_L : frequency of pipeline mode
- f_p : frequency of poppet valve mode
- k : stiffness of poppet spring
- L : length of pipeline
- m : poppet mass
- P_c : pressure in valve chamber
- P_o : upstream pressure of orifice
- P_s : supply pressure
- p_c : perturbation of chamber pressure
- Q_c : flow rate through poppet valve
- Q_o : flow rate through orifice
- q_c : perturbation of valve flow
- q_o : perturbation of orifice flow
- V_c : volume of valve plenum chamber
- X : valve lift
- X_0 : static valve lift
- x : perturbation of valve lift
- X_I : initially compressed length of suspension spring
- α : half angle of poppet
- β : bulk modulus of working fluid (oil)
- δ : viscous damping coefficient
- ρ : density of working fluid (oil)