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# 非線形系のロバスト適応制御器の設計

# Design of Robust Adaptive Controller for Nonlinear Systems

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## 1. Introduction

The fact is that virtually all physical plants, such as aircraft, spacecraft, automobile or robot, are nonlinear in nature, and all control systems are nonlinear to a certain extent. Thus, the topic of nonlinear control design for the output tracking, a controller which forces the plant output to track a time-varying trajectory, has attracted particular attention.

At the eighties, generalizations of pole placement and observer design techniques [1) 11)] for nonlinear systems were obtained by using differential geometric nonlinear theory which is a procedure to construct the linearizing coordinates. More recently, we were confronted with more realistic problems that were caused by various uncertainties about either plants or disturbances. Adaptive versions for nonlinear systems were announced from the mid-1980's [6) 10)] and have been recently expanded in some works [7) 4)]. On the other hand, Isidori, Khalil, Marino and Tomei have studied theories of robust versions for nonlinear systems [9) 3) 5) 8)].

However, the nonlinear control algorithms studied in the above paragraph were not available to a class of nonlinear systems affected by both constant uncertain parameters, i.e. unknown constant parameters, and unmodeled dynamics. To solve such a complex situation, in this paper, we shall present two robust adaptive nonlinear control schemes for uncertain nonlinear systems with both constant uncertain parameters and unmodeled dynamics, via indirect and direct input-output feedback linearizations, respectively. The control schemes to be developed are designed and analyzed on the following assumptions: 1) the nonlinear vector fields expressing system properties are smooth, 2) the nonlinear functions are given linearly with respect to the unknown constant parameters and the control input, 3) the full-state measurable condition is satisfied, and 4) the nonlinear systems have a well-defined relative degree.

# 2. Robust Adaptive Nonlinear Output Tracking

Let us now consider $SISO$ nonlinear systems

$$\begin{aligned} \dot{x} &= f(x,\theta) + g(x,\theta)u, \; x \in \Re^n, u \in \Re, \theta \in \Re^p \\ y &= h(x), \; y \in \Re \end{aligned} \tag{1}$$

where $x$ is the state, $u$ is the control input, $\theta$ is the unknown constant parameters, $y$ is the output, $h : \Re^n \to \Re$ is the $C^\infty$ output function, and $f$, $g$ are two $C^\infty$ vector fields with $g(x,\theta) \neq 0$. We assume that the unknown constant parameters $\theta$ are restricted to appearing linearly and the vector field $f$ is affected by the unmodeled dynamics $\Delta f$:

$$f(x,\theta) = \sum_{i=1}^{p} \theta_i f_i(x) + \Delta f(x) \tag{2}$$

with $C^\infty$ nonlinear functions $f_i$, $g_i : \Re^n \to \Re$, $i = 1, \cdots, p$, and $C^\infty$ vector field $\Delta f$. Our objective is the design of nonlinear feedback control schemes to force the output $y$ to approximately track the reference trajectory $y_m(t)$ satisfying

$$||y_m^{(i)}|| \leq \epsilon_{y_m}, \; i = 0, 1, \cdots, r \tag{3}$$

where $\epsilon_{y_m} > 0$ and $r$ is the relative degree to be introduced in a short time. For this, the control law and parameter update law must be designed on a local domain of both the state $x$ and the parameter estimates $\hat{\theta}$ of $\theta$, together with the robustness of all the closed-loop signals including $\hat{\theta}$ which may be drifted to infinity due to $\Delta f$. Let $\bar{f}(x,\theta)$ be $\sum_{i=1}^{p} \theta_i f_i(x)$ in (2) and $x_e$ be an equilibrium point of (1). Moreover, we assume that $\Delta f(x_e) = 0$ and $h(x_e) = 0$ at $x_e$.

## 2.1 Robust adaptive output tracking with indirect input-output feedback linearization

Here, we assume that the input vector field $g$ is not affected by the unknown parameters $\theta$, i.e. $g(x,\theta) = g(x)$. With this assumption, the first robust adaptive nonlinear controller is designed separately with robust adaptive law and input-output feedback linearization by state feedback. For this control scheme, let us first illustrate the robust adaptive law to identify $\theta$. The explanation can be evolved as the transformed form for (1):

$$\begin{aligned} (s+1)[x_i] &= \theta_i f_i(x) + \Delta f_i + g_i(x)u \\ &\quad + x_i, \; 1 \leq i \leq n \end{aligned} \tag{4}$$

where $\theta_i$ is the unknown constant parameter parameterized linearly with the $i-th$ nonlinear function $f_i$. In (4), we can define the following linear expression with the strictly proper $SPR$ transfer function $W(s) \equiv \frac{1}{s+1}$, the time invariant parameter vector $\bar{\theta}_i^{*T} \equiv (\theta_i \; 0 \; 1)$, the state variable vector $\omega_i \equiv (f_i(x) \; 0 \; x_i)^T$ and the piecewise continuous signal input $z_{oi} \equiv g_i(x) \; u$:

$$x_i = W(s)[\bar{\theta}_i^{*T} \omega_i + \Delta f_i + z_{oi}], \; 1 \leq i \leq n \; . \tag{5}$$

From (5), the state estimate can be constructed as

$$\hat{x}_i = W(s)[\hat{\bar{\theta}}_i^T \omega_i + z_{oi}], \; 1 \leq i \leq n \; . \tag{6}$$

Then, the estimation error $\epsilon_i \equiv \hat{x}_i - x_i$ is got from

$$\begin{aligned} \dot{e}_i &= -e_i + (\phi_i^T \omega_i - \Delta f_i) \\ \epsilon_i &= e_i, \; 1 \leq i \leq n \end{aligned} \tag{7}$$

with the parameter error $\phi_i \equiv \hat{\bar{\theta}}_i - \bar{\theta}_i^*$. However, since it is not assumed that the unmodeled dynamics $\Delta f_i$ is bounded in a compact set, we can not identify the state estimate $\hat{x}_i$ and the parameter estimate $\hat{\bar{\theta}}_i$ by only (7). Hence, to design an adaptive law which is robust with respect to $\Delta f_i$, we shall present the following normalized error dynamics with the normalizing signal $m_i$ such as $m_i^2 = 1 + ||x_i||^2$:

$$\begin{aligned} \dot{\bar{e}}_i &= -\bar{e}_i + (\phi_i^T \bar{\omega}_i - \bar{\Delta f}_i) \\ \bar{\epsilon}_i &= \bar{e}_i, \ 1 \le i \le n \end{aligned} \quad (8)$$

where $\bar{\epsilon}_i = \bar{e}_i \equiv \frac{e_i}{m_i}$, $\bar{\omega}_i \equiv \frac{\omega_i}{m_i}$, $\bar{\Delta f}_i \equiv \frac{\Delta f_i}{m_i} \in \mathcal{L}_\infty$.

We have to construct an appropriate Lyapunov type function for designing a robust adaptive law of $\dot{\hat{\theta}}_i$. Let us now consider the Lyapunov function

$$V_i(\phi_i, \bar{e}_i) = \frac{1}{2}(\bar{e}_i^2 + \phi_i^T \phi_i), \ 1 \le i \le n \ . \quad (9)$$

The following theorem introduces one way to avoid the parameter drift phenomenon and establish boundedness in the presence of $\bar{\Delta f}_i$ (or $\Delta f_i$).

**Theorem 1** *If we select a robust adaptive law defined as the differential equations*

$$\dot{\phi}_i = \dot{\hat{\theta}}_i = -\bar{\epsilon}_i \bar{\omega}_i - w_i \hat{\theta}_i, \ 1 \le i \le n \quad (10)$$

*with the leakage term $w_i(t) \geq 0$, we establish $V_i$, $\bar{e}_i$ (or $e_i$), $\phi_i$, $\bar{\epsilon}_i$ (or $\epsilon_i$) $\in \mathcal{L}_\infty$.*

*Proof:* Differentiating (9) with respect to time $t$ along the solution of (8) and using the above equation (10), we have

$$\begin{aligned} \dot{V}_i(\phi_i, \bar{e}_i) &= -\bar{\epsilon}_i^2 + \bar{\epsilon}_i \phi_i^T \bar{\omega}_i - \bar{\epsilon}_i \bar{\Delta f}_i + \phi_i^T \dot{\phi}_i \\ &= -\bar{\epsilon}_i^2 - \bar{\epsilon}_i \bar{\Delta f}_i - w_i \phi_i^T \hat{\theta}_i, \ 1 \le i \le n \\ &\le -\bar{\epsilon}_i^2 + |\bar{\epsilon}_i||\bar{\Delta f}_i| - w_i \phi_i^T \hat{\theta}_i \end{aligned} \quad (11)$$

where the leakage term $w_i(t)$ is to be chosen so that for $V_i \geq V_{0i}$ and some constant $V_{0i} > 0$, $\dot{V}_i < 0$. This property of $V_i$ implies that $V_i$, $\bar{e}_i$ (or equivalently, $e_i$), $\bar{\epsilon}_i$ (or equivalently, $\epsilon_i$), $\phi_i \in \mathcal{L}_\infty$. □

If the unknown constant parameters $\theta$ in (1) are estimated by using (10), we can see (1) as a class of nonlinear plants with the knowable parameters $\hat{\theta}$ at time $t$. However, since (1) is affected by $\Delta f$, the feedback linearization procedure must be considered with $\Delta f$. To overcome the problem, with the definition $\bar{f}(x, \hat{\theta}) \equiv \sum_{i=1}^p \hat{\theta}_i \ f_i(x)$ and Frobenius's theorem, we can define the following relative degree and the local diffeomorphism which are not dependent on $\Delta f$: in $U_\delta(\theta) \times U_\epsilon(x_e)$,

$$\begin{aligned} L_{g(x)} L^i_{\bar{f}(x,\hat{\theta})} h(x) &= 0, \ 0 \le i \le r-2 \\ L_{g(x)} L^{r-1}_{\bar{f}(x,\hat{\theta})} h(x) &\neq 0 \end{aligned} \quad (12)$$

and

$$(\xi, \eta) = (\underbrace{h(x)}_{z_1(x,\hat{\theta})}, \cdots, \underbrace{L^{r-1}_{\bar{f}(x,\hat{\theta})} h(x)}_{z_r(x,\hat{\theta})}, \eta_{r+1}, \cdots, \eta_n) \ . \quad (13)$$

**Theorem 2** *If (12) and (13) are assumed, (1) is input-output feedback linearizable into*

$$\begin{aligned} \dot{\xi} &= A_l \xi + B_l v + \Delta\phi(x, \hat{\theta}, \Delta f(x)) \\ \dot{\eta} &= q(\xi, \eta) \\ y &= \xi_1 \end{aligned} \quad (14)$$

*where $(A_l, B_l)$ is in Brunovsky controller form,*

$$\begin{aligned} \Delta\phi(x, \hat{\theta}, \Delta f) &= [\Delta\phi_1, \cdots, \Delta\phi_\gamma]^T \\ \Delta\phi_k(\xi, \hat{\theta}, \Delta f) &= L_{\Delta f} L^{k-1}_{\bar{f}(x,\hat{\theta})} h(x), \ k = 1, \cdots, r \\ u &\equiv \frac{1}{A(\xi, \eta)}[v - B(\xi, \eta)] \end{aligned} \quad (15)$$

*with*

$$A(\xi, \eta) = L_{g(x)} L^{r-1}_{\bar{f}(x,\hat{\theta})} h(x), \ B(\xi, \eta) = L^r_{\bar{f}(x,\hat{\theta})} h(x) \ .$$

*Proof:* Let us differentiate $y$ with respect to time $t$:

$$\dot{\xi}_1 = L_{\bar{f}(x,\hat{\theta})} h(x) + L_{g(x)} h(x) u + L_{\Delta f} h(x) \ .$$

Since $L_{g(x)} h(x) = 0$ in (12), we have

$$\dot{\xi}_1 = \underbrace{L_{\bar{f}(x,\hat{\theta})} h(x)}_{\xi_2(x,\hat{\theta})} + \underbrace{L_{\Delta f} h(x)}_{\Delta\phi_1(x,\hat{\theta},\Delta f)} \ .$$

In the second step with the knowable value $\hat{\theta}$ at time $t$,

$$\dot{\xi}_2 = \underbrace{L^2_{\bar{f}(x,\hat{\theta})}h(x)}_{\xi_3(x,\hat{\theta})} + L_{g(x)}L_{\bar{f}(x,\hat{\theta})}h(x)u + \underbrace{L_{\Delta f}L_{\bar{f}(x,\hat{\theta})}h(x)}_{\Delta\phi_2(x,\hat{\theta},\Delta f)} .$$

Since $L_{g(x)}L_{\bar{f}(x,\hat{\theta})}h(x)$ is again zero for all $x$ in $U_\delta(\theta) \times U_\epsilon(x_e)$, we shall differentiate again and again until the integer $r$:

$$\dot{\xi}_r = L^r_{\bar{f}(x,\hat{\theta})}h(x) + L_{g(x)}L^{r-1}_{\bar{f}(x,\hat{\theta})}h(x)u + L_{\Delta f}L^{r-1}_{\bar{f}(x,\hat{\theta})}h(x) .$$

With the above definitions $A$, $B$ and $\Delta\phi_r$, the normal dynamics (14) can be got. □

**Remark 1** *With the assumption $\Delta f(x_e) = 0$ and the knowable value $\hat{\theta}$ at time $t$, it is obvious that $\Delta\phi(x_e, \hat{\theta}, \Delta f(x_e)) = 0$ and $||\Delta\phi(x, \hat{\theta}, \Delta f)|| \leq \kappa||\xi||$ for $\kappa > 0$.*

Using the pole placement control input

$$v = y_m^{(r)} + \alpha_{r-1}(y_m^{(r-1)} - \xi_r) + \cdots + \alpha_0(y_m - \xi_1) \quad (16)$$

with Hurwitz polynomial coefficients $\alpha_i$, $i = 0, \cdots, r-1$, the robust output tracking can be approximately achieved. If we define the tracking error $e_y$ by

$$e_{y_i} = y^{(i-1)} - y_m^{(i-1)}, \; i = 1, \cdots, r, \quad (17)$$

the feedback control laws (15) and (16) lead to the closed-loop system

$$\begin{aligned} \dot{e}_y &= \bar{A}e_y + \Delta\phi(x, \hat{\theta}, \Delta f) \\ \dot{\eta} &= q(e_y + \bar{y}_m, \eta) \end{aligned} \quad (18)$$

where $\bar{A}$ is an exponentially stable $r \times r$ matrix in the bottom companion form [2, 9], and $\bar{y}_m = (y_m, \dot{y}_m, \cdots, y_m^{(r-1)})^T$ with the boundedness $||\bar{y}_m|| \leq \epsilon_{y_m}$ in (3).

**Remark 2** *Since the above dynamics can be derived from the full state variables $x$ and the desired trajectory vector $\bar{y}_m$, it is natural that the feedback control laws (15) and (16) become a state-feedback control scheme.* •

Assuming that the zero dynamics $q(0, \eta)$ in (14) is locally exponentially minimum phase, the following theorem indicates that the feedback control laws (15) and (16) make the tracking error $y(t) - y_m(t)$ bounded.

**Theorem 3** *If the nonlinear system (14) has the relative degree $r$, its zero dynamics is locally asymptotically stable and the boundary condition (3) of $y_m$ is satisfied, the feedback control laws (15) and (16) lead to*

$$\lim_{t\to\infty} ||y(t) - y_m(t)|| < \epsilon_{e_y} \quad (19)$$

*and there exists a positive number $\epsilon_\eta$ such that*

$$||\eta(t)|| \leq \epsilon_\eta . \quad (20)$$

*Proof:* To show that $e_y$ remains bounded, let us now consider the following Lyapunov function for (18):

$$V(e_y, \eta) = e_y^T P e_y + \mu V_2(\eta) \quad (21)$$

where $\mu > 0$ is a constant to be determined and $P > 0$ is such that $\bar{A}^T P + P\bar{A} = -I$. The time derivative of $V$ along the trajectories of (18) is equivalent to

$$\begin{aligned} \dot{V} &= 2e_y^T P \dot{e}_y + \mu\frac{\partial V_2}{\partial \eta}\dot{\eta} \\ &= -||e_y||^2 + 2e_y^T P\Delta\phi + \mu\frac{\partial V_2}{\partial \eta}[q(0,\eta) \\ &\quad + q(e_y + \bar{y}_m, \eta) - q(0, \eta)] \\ &\leq -||e||^2 - \mu k_3||\eta||^2 \\ &\quad + \mu k_4 l_q||\eta||(||e|| + \epsilon_{y_m}) . \end{aligned} \quad (22)$$

Here, it is natural that due to the local exponential stability of the zero dynamics, the converse Lyapunov theorem [9] implies

$$\begin{aligned} k_1\|\eta\|^2 &\leq V_2(\eta) \leq k_2\|\eta\|^2 \\ \frac{\partial V_2}{\partial \eta}q(0,\eta) &\leq -k_3\|\eta\|^2 \\ \|\frac{\partial V_2}{\partial \eta}\| &\leq k_4\|\eta\| \end{aligned} \tag{23}$$

for some positive constants $k_1$, $k_2$, $k_3$, $k_4$, and $q$ is a locally Lipschitz vector function, i.e.,

$$\|q(e_y + \bar{y}_m, \eta) - q(0,\eta)\| \leq l_q(\|e_y\| + \epsilon_{y_m}) \tag{24}$$

with a Lipschitz constant $l_q$. Hence, using that $\|\Delta\phi\| \leq \kappa\|x\|$ and $\|P\| \leq \lambda_{max}$, we have

$$\begin{aligned} \dot{V} &\leq -\|e_y\|^2 + \kappa\|e_y\|\|P\|\|x\| - \mu k_3\|\eta\|^2 \\ &\quad +\mu k_4 l_q\|\eta\|(\|e_y\| + \epsilon_{y_m}) \\ &\leq -\|e_y\|^2 + \kappa\lambda_{max}\|e_y\|\|x\| - \mu k_3\|\eta\|^2 \\ &\quad +\mu k_4 l_q\|\eta\|(\|e_y\| + \epsilon_{y_m}) . \end{aligned} \tag{25}$$

From (3) and (17), it is given that

$$\begin{aligned} \|x\| &\leq l_x(\|\xi\| + \|\eta\|) \\ &\leq l_x(\|e_y\| + \epsilon_{y_m} + \|\eta\|) . \end{aligned} \tag{26}$$

Then,

$$\begin{aligned} \dot{V} &\leq -\|e_y\|^2 + \kappa\lambda_{max}l_x\|e_y\|(\|e_y\| + \epsilon_{y_m} + \|\eta\|) \\ &\quad -\mu k_3\|\eta\|^2 + \mu k_4 l_q\|\eta\|(\|e_y\| + \epsilon_{y_m}) \\ &\leq -(\frac{1}{2}\|e_y\| - \kappa\lambda_{max}l_x\|\eta\|)^2 + (\kappa\lambda_{max}l_x)^2\|\eta\|^2 \\ &\quad -(\frac{1}{2}\|e_y\| - \kappa\lambda_{max}l_x\epsilon_{y_m})^2 + (\kappa\lambda_{max}l_x)^2\epsilon_{y_m}^2 \\ &\quad -\mu k_3(\frac{1}{2}\|\eta\|^2 - \frac{k_4}{k_3}l_q\epsilon_{y_m})^2 + \frac{\mu}{k_3}(k_4 l_q\epsilon_{y_m})^2 \\ &\quad -(\frac{1}{2} - \kappa\lambda_{max}l_x)\|e_y\|^2 - \frac{3}{4}\mu k_3\|\eta\|^2 \\ &\leq -(\frac{1}{2} - \kappa\lambda_{max}l_x)\|e_y\|^2 \\ &\quad -(\frac{3}{4}\mu k_3 - (\kappa\lambda_{max}l_x)^2)\|\eta\|^2 + [(\kappa\lambda_{max}l_x)^2 \\ &\quad +\frac{\mu}{k_3}(k_4 l_q)^2]\epsilon_{y_m}^2 . \end{aligned} \tag{27}$$

Defining that $\lambda_{max} \leq \frac{1}{4\kappa l_x}$ and $\mu \geq \frac{4(\kappa\lambda_{max}l_x)^2}{k_3}$,

$$\begin{aligned} \dot{V} &\leq -\frac{1}{4}\|e_y\|^2 - \frac{1}{2}\mu k_3\|\eta\|^2 + [(\kappa\lambda_{max}l_x)^2 \\ &\quad +\frac{\mu}{k_3}(k_4 l_q)^2]\epsilon_{y_m}^2 . \end{aligned} \tag{28}$$

It is obtained that $\|e_y\|$ and $\|\eta\|$ are bounded since $\dot{V} \leq 0$ whenever $\|e_y\|$ or $\|\eta\|$ is large. □

The robust adaptive control scheme with indirect feedback linearization dealt above can be simply illustrated with the block diagram in Figure 1.

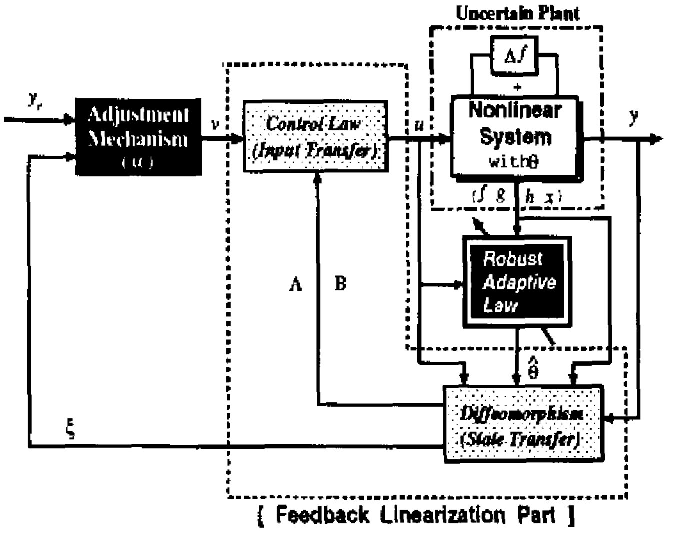



Fig. 1 Robust adaptive output tracking for indirect feedback linearization

## 2.2 Robust adaptive output tracking with direct input-output feedback linearization

When the input vector field $g$ is dependent on the unknown constant parameters $\theta$, i.e. $g(x,\theta) = \sum_{i=1}^{p}\theta_i g_i(x)$, $\theta$ can not be estimated indirectly as the previous method. Thus, to solve the estimation problem, we here replace the state variables $\xi$ with their estimates $\hat{\xi}$ by replacing the unknown constant parameters $\theta$ appearing in $\xi$ by their estimates $\hat{\theta}$. Using $\hat{\theta}$, $\hat{\xi}$ and Frobenius's theorem, let us define the following relative degree and the local diffeomorphism for (1) in $U_\delta(\theta) \times U_\epsilon(x_e)$:

$$L_{g(x,\hat{\theta})}L^i_{\hat{f}(x,\hat{\theta})}h(x) = 0, \; 0 \leq i \leq r-2$$

$$L_{g(x,\hat{\theta})} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) \neq 0 \tag{29}$$

and

$$(\hat{\xi}, \eta) = (\underbrace{\hat{\xi}_i = L_{\bar{f}(x,\hat{\theta})}^{i-1} h(x)}_{\hat{z}_i(x,\hat{\theta})},\ i = 1, \cdots, r, \quad \eta_{r+1}, \cdots, \eta_n), \tag{30}$$

respectively. Assuming $\Delta f(x_e) = 0$ and defining $\bar{f}(x, \hat{\theta}) \equiv \sum_{i=1}^{p} \hat{\theta}_i\ f_i(x)$, the normal dynamics can be summarized by the following theorem.

**Theorem 4** *If there exists a region $U_\delta(\theta) \times U_\epsilon(x_e)$ satisfying (29) and (30), (1) is input-output feedback linearizable into*

$$\begin{aligned} \dot{\hat{\xi}} &= A_l \hat{\xi} + B_l v + W\Phi + M\dot{\hat{\theta}} + \Delta\phi(x, \hat{\theta}, \Delta f(x)) \\ \dot{\eta} &= q(\hat{\xi}, \eta) \\ y &= \hat{\xi}_1 \end{aligned} \tag{31}$$

*where $(A_l, B_l)$ is in Brunovsky controller form, $\Phi = \theta - \hat{\theta}$, $M = \frac{\partial \hat{\xi}}{\partial \hat{\theta}}$,*

$$\begin{aligned} W &= [w_1, \cdots, w_r]^T \\ w_i(x, \hat{\theta})\Phi &= \sum_{j=1}^{p} (\theta_j - \hat{\theta}_j)[L_{f_j(x)} L_{\bar{f}(x,\hat{\theta})}^{i-1} h(x)], \\ & \quad i = 1, \cdots, r-1 \\ w_r(x, u, \hat{\theta})\Phi &= \sum_{j=1}^{p} (\theta_j - \hat{\theta}_j)[L_{f_j(x)} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) \\ & \quad + u L_{g_j(x)} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x)] \\ \Delta\phi(x, \hat{\theta}, \Delta f) &= [\Delta\phi_1, \cdots, \Delta\phi_r]^T \\ \Delta\phi_k(x, \hat{\theta}, \Delta f) &= L_{\Delta f} L_{\bar{f}(x,\hat{\theta})}^{k-1} h(x),\ k = 1, \cdots, r \\ u &\equiv \frac{1}{\hat{A}(\hat{\xi}, \eta)}[v - \hat{B}(\hat{\xi}, \eta)] \end{aligned} \tag{32}$$

*with*

$$\begin{aligned} \hat{A}(\hat{\xi}, \eta) &= L_{g(x,\hat{\theta})} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) \\ \hat{B}(\hat{\xi}, \eta) &= L_{\bar{f}(x,\hat{\theta})}^{r} h(x) . \end{aligned}$$

*Proof:* Rewriting (1) into

$$\begin{aligned} \dot{x} &= f(x, \theta) + g(x, \theta)u + f(x, \hat{\theta}) \\ & \quad + g(x, \hat{\theta})u - f(x, \hat{\theta}) - g(x, \hat{\theta})u \end{aligned}$$

and using the assumption (29), the following result is obtained by differentiating $y$ with respect to time $t$:

$$\begin{aligned} \dot{\hat{\xi}}_1 &= \underbrace{L_{\bar{f}(x,\hat{\theta})} h(x)}_{\hat{\xi}_2} + \sum_{j=1}^{p} (\theta_j - \hat{\theta}_j)[L_{f_j(x)} h(x)] \\ & \quad + \underbrace{L_{\Delta f(x)} h(x)}_{\Delta\phi_1} . \end{aligned} \tag{33}$$

In the next step, since $\hat{\theta}(t)$ is the time function, $\dot{\hat{\xi}}_2$ is dependent on not only the unmodeled dynamics $\Delta f$ but also the time function $\hat{\theta}$:

$$\begin{aligned} \dot{\hat{\xi}}_2 &= \underbrace{L_{\bar{f}(x,\hat{\theta})}^2 h(x)}_{\hat{\xi}_3} + \sum_{j=1}^{p} (\theta_j - \hat{\theta}_j)[L_{f_j(x)} L_{\bar{f}(x,\hat{\theta})} \\ & \quad h(x)] + \frac{\partial \hat{\xi}_2}{\partial \hat{\theta}} \dot{\hat{\theta}} + \underbrace{L_{\Delta f(x)} L_{\bar{f}(x,\hat{\theta})} h(x)}_{\Delta\phi_2} . \end{aligned} \tag{34}$$

After continuing this differentiation operation up to $r$, we can obtain from $L_{g(x,\hat{\theta})} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) \neq 0$ in (29) that

$$\begin{aligned} \dot{\hat{\xi}}_r &= L_{\bar{f}(x,\hat{\theta})}^{r} h(x) + L_{g(x,\hat{\theta})} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) u \\ & \quad + \sum_{j=1}^{p} (\theta_j - \hat{\theta}_j)[L_{f_j(x)} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x) \\ & \quad + u L_{g_j(x)} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x)] + \frac{\partial \hat{\xi}_r}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ & \quad + \underbrace{L_{\Delta f(x)} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x)}_{\Delta\phi_r} . \end{aligned} \tag{35}$$

With the definitions $\hat{A} = L_{g(x,\hat{\theta})} L_{\bar{f}(x,\hat{\theta})}^{r-1} h(x)$, $\hat{B} = L_{\bar{f}(x,\hat{\theta})}^{r} h(x)$ and the input transformation (32), we can obtain the normal dynamics (31). □

**Remark 3** *It is noted that unlike (14) affected by only $\Delta f$, (31) is dependent on both $\hat{\theta}$ and $\Delta f$. Since $\Delta f$ does not affect the determination of (30), it is natural that the zero dynamics $q(0, \eta)$ in (31) do not depend on only $\hat{\theta}$ but also $\Delta f$. If there exist two positive constants $\kappa_1$ and $\kappa_2$ such that $||x|| \leq \kappa_1 ||\hat{\xi}||$ and $||\Delta\phi(x, \hat{\theta}, \Delta f)|| \leq \kappa_2 ||x||$ for all $\hat{\theta} \in U_\delta(\theta)$,*

$$||\Delta\phi(x, \hat{\theta}, \Delta f)|| \leq \kappa_2 ||x|| \leq \kappa_1 \kappa_2 ||\hat{\xi}|| . \tag{36}$$

Since $\hat{\xi}$ is the unobservable variables with $\hat{\theta}$, let us now estimate $\hat{\xi}$ by the state observer dynamics with $\bar{\xi}(0) = \hat{\xi}(0)$:

$$\dot{\hat{\xi}} = A_l\hat{\xi} + B_l v + M\dot{\hat{\theta}} + \bar{A}(\bar{\xi} - \hat{\xi}) \tag{37}$$

where $\bar{A}$ is the bottom companion matrix with the coefficients $\alpha_i$, $i = 0, \cdots, r-1$ chosen above.

This implies that we can get the error dynamics

$$\dot{s} = \bar{A}s - W\Phi - \Delta\phi \tag{38}$$

in which $s \equiv \bar{\xi} - \hat{\xi}$. However, from the boundary condition in (36), the stability of (38) can not be previously decided with the exception of $\Delta\phi$. Thus, we shall present the following normalized dynamics for (38) with the normalizing signal $m^2 = 1 + ||\hat{\xi}||^2$ of $\hat{\xi}$:

$$\dot{\bar{s}} = \bar{A}\bar{s} - \Phi\bar{W} - \bar{\Delta\phi} \tag{39}$$

where $\bar{s} \equiv \frac{s}{m}$, $\bar{W} \equiv \frac{W}{m}$, $\bar{\Delta\phi} \equiv \frac{\Delta\phi}{m}$. To design a robust adaptive law satisfying the stability of the closed-loop system with respect to $\bar{\Delta\phi}$, the following Lyapunov function is here used:

$$V(\bar{s}, \Phi) = \frac{\bar{s}^T P \bar{s}}{2} + \frac{\Phi^T \Omega^{-1} \Phi}{2}. \tag{40}$$

**Theorem 5** *If the robust adaptive law*

$$\dot{\hat{\theta}} = -\Omega\bar{W}^T P\bar{s} - \Omega L\hat{\theta} \tag{41}$$

*where $P = P^T > 0$ with $||P|| \leq \lambda_{max}$, without loss of generality, to the Lyapunov equation $\bar{A}^T\ P + P\ \bar{A} = -I$, $\Omega = \Omega^T > 0$ is the adaptive gain matrix of dimension p, and L is the leakage matrix which is constructed by the diagonal scalar signals $w_i(t) = \nu_i||\bar{s}||$ with the design constants $\nu_i > 0$, $i = 1, \cdots, p$, is used for (39), all the states in (38) and (39) become approximately stable.*

*Proof:* Using the leakage matrix $L$ with

$$w_i(t) = \nu_i||\bar{s}||,\ i = 1, \cdots, p$$

where $\nu_i > 0$, the time derivative of $V(\bar{s}, \Phi)$ along the trajectory of (39) is given by

$$\begin{aligned}
\dot{V} &= \bar{s}^T P \dot{\bar{s}} + \Phi^T \Omega^{-1} \dot{\Phi} \\
&= -\frac{1}{2}\bar{s}^T\bar{s} - \bar{s}^T P\bar{W}\Phi - \bar{s}^T P\bar{\Delta\phi} + \Phi^T\Omega^{-1}\dot{\Phi}. \\
&\leq -\frac{1}{2}||\bar{s}||^2 + ||\bar{s}||\lambda_{max}||\bar{\Delta\phi}|| + ||\nu||||\bar{s}||\Phi^T\hat{\theta} \\
&\leq -\frac{1}{2}||\bar{s}||(||\bar{s}|| - 2\lambda_{max}||\bar{\Delta\phi}|| + ||\nu||\Phi^T\Phi \\
&\quad -||\nu||\theta^T\theta)
\end{aligned}$$

where the last inequality is obtained by using $\Phi^T\hat{\theta} \leq -\frac{1}{2}\Phi^T\Phi + \frac{1}{2}\theta^T\theta$. This implies that for $V \geq V_0 \equiv \frac{1}{||\nu||}(||\nu||\theta^T\theta + 2\lambda_{max}||\bar{\Delta\phi}||)$, $\dot{V} \leq 0$, i.e., $\Phi$, $\hat{\theta}$, $\bar{s}$ (or $s$) $\in \mathcal{L}_\infty$. □

**Remark 4** *The basic concept behind this choice of $w_i(t)$ is that since in the ideal case $\Delta f = 0$, $\bar{s}$ is guaranteed to converge to zero, the leakage term will go to zero with $\bar{s}$. Therefore, the ideal properties of the robust adaptive law (41) when $\Delta f = 0$ will not be affected by the leakage.*

Then, the robust output tracking can be derived by using

$$v = y_m^{(r)} + \alpha_{r-1}(y_m^{(r-1)} - \bar{\xi}_r) + \cdots + \alpha_0(y_m - \bar{\xi}_1)$$

by assuming that the zero dynamics $q(0, \eta)$ is locally exponentially stable. The robust adaptive output tracking, i.e., $e_1 \to B_\epsilon(0)$, is analyzed as follows. Letting $e_i = \hat{\xi}_i - y_m^{(i-1)}$, $i = 1, \cdots, \gamma$, we have the following tracking error dynamics

$$\dot{e} = \bar{A}e + W\Phi + M\dot{\hat{\theta}} + \Delta\phi\ . \tag{42}$$

Defining the total error $r \equiv e + s$ or equivalently, $r_i = \bar{\xi}_i - y_m^{(i-1)}$, $i = 1, \cdots, \gamma$, from (39) and (40),

$$\begin{aligned}
\dot{r} &= \bar{A}r + M\dot{\hat{\theta}} \\
&= \bar{A}r - M\Omega\bar{W}_2^T P\bar{s} - M\Omega L\hat{\theta} 
\end{aligned} \tag{43}$$

with $\bar{s} \in \mathcal{L}_\infty$. It seems that (42) is a linear time-varying filter with the bounded input $\bar{s}$, the small perturbation $\hat{\Psi}$, and the internal dynamics $\dot{\eta} = q(\hat{\xi}, \eta)$. We define $e$ as the output of the following asymptotically stable linear filter with stable internal dynamics

$$
\begin{aligned}
\dot{r} &= \bar{A}r - M\Omega\bar{W}_2^T P\tilde{s} - M\Omega L\hat{\theta} \\
\dot{\eta} &= q(\hat{\xi}, \eta) \\
e &= r - s \,.
\end{aligned} \tag{44}
$$

Finally, the stability of the closed-loop system (44) is found by considering the following Lyapunov function for the differential equations (39) and (43):

$$V(r, \eta) = \frac{r^T \bar{P} r}{2} + \mu v_2(\eta) \tag{45}$$

where $\mu > 0$ is a constant to be determined, $\bar{P} > 0$ is such that $\bar{A}^T\bar{P} + \bar{P}\bar{A} = -I$, and $v_2$ is a Lyapunov function for the zero dynamics $\dot{\eta} = q(0, \eta)$. The rest of this output tracking analysis is similar to the proof of Theorem 3.

The robust adaptive control scheme discussed in this subsection can be also illustrated with the following block diagram.

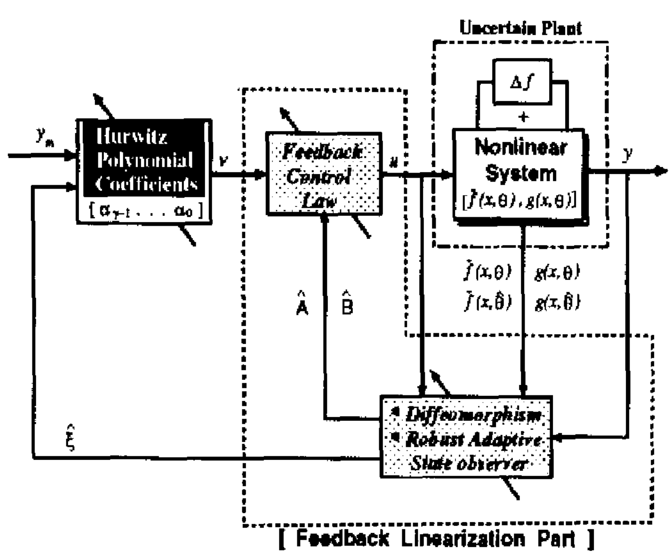
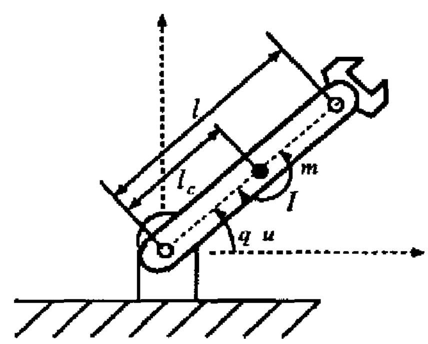



Fig. 2 Robust adaptive output tracking for direct feedback linearization

# 3. Simulations and Results

To show performance of two nonlinear feedback control schemes presented, we shall consider the single-link rigid robot rotating on a vertical plane




- $I$ : the inertia of link
- $m$ : the mass of link
- $l$ : the length of link
- $g_{gc}$ : the gravity constant
- $u$ : the torque input

Fig. 3 Single-link rigid robot

modeled by

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \theta\cos(x_1) + \frac{1}{I}u + \Delta f \\
y &= x_1 = q
\end{aligned} \tag{46}
$$

where $\theta \equiv -\frac{m g_{gc} l}{2I}$ is the unknown constant parameter given by the true value 1 and $\Delta f$ is the unmodeled dynamics caused by the parameter oscillation $\Delta f = 0.3\cos(300t)\cos(x_1)$. The objective of control is to make the output $y$ track the desired position

$$y_m(t) = \frac{\pi}{4}\sin(t - \frac{\pi}{2}) + \frac{\pi}{4} \tag{47}$$

which is specified by the motion planning system.

## 3.1 Robust adaptive control (1)

Let us now start with the first robust adaptive nonlinear control scheme. The following terms indicate the state estimator, the parameter estimator and the Lyapunov function for (46), respectively,

$$\dot{\hat{x}}_2 = -(\hat{x}_2 - x_2) + \hat{\theta}\cos(x_1) + \frac{1}{I}u \quad (48)$$

$$\dot{\hat{\theta}} = -e_{x_2}\cos(x_1) - \gamma||e_{x_2}||\hat{\theta} \quad (49)$$

$$V(e_{x_2}, \phi) = \frac{1}{2}(e_{x_2}^2 + \phi^2) \quad (50)$$

where $e_{x_2} \equiv \hat{x}_2 - x_2$, $\gamma = 0.3$, $\phi \equiv \hat{\theta} - \theta$. According to the design procedures in Subsection 2.1, we can have the following normal dynamics for (46):

$$\begin{aligned}\dot{\xi}_1 &= \underbrace{x_2}_{\xi_2 = z_2(x,\hat{\theta})} \\ \dot{\xi}_2 &= \underbrace{(\hat{\theta}\cos(x_1))}_{B(x,\hat{\theta})} + \underbrace{(\frac{1}{I})}_{A(x,\hat{\theta})} u + \underbrace{\Delta f}_{\Delta\phi} \\ y &= \underbrace{x_1}_{\xi_1 = z_1(x,\hat{\theta})} \quad (51) \\ u &= \frac{1}{A(x,\hat{\theta})}(v - B(x,\hat{\theta})) .\end{aligned}$$

Then, using the pole placement control input $v = \ddot{y}_m + 2(\dot{y}_m - \xi_2) + 2(y_m - \xi_1)$, the output tracking may be approximately established. The parameter estimate $\hat{\theta}(t)$ converges to a bounded range in Figure 4, and Figure 5 indicates the desired result of the output tracking.

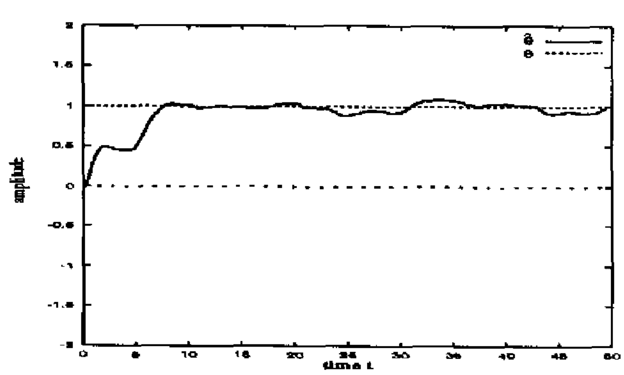


Fig. 4 System responses (1) - $\hat{\theta}(t)$, $\theta$

## 3.2 Robust adaptive control (2)

We shall now use the second control method. Since $\theta$ can not be separately estimated, the con-

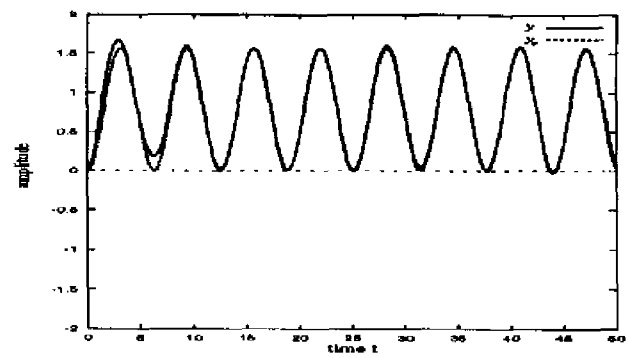


Fig. 5 System responses (2) - $y(t)$, $y_m(t)$

troller may be derived from the normal dynamics

$$\begin{aligned}\dot{\hat{\xi}}_1 &= \underbrace{x_2}_{\hat{\xi}_2 = \hat{z}_2(x,\hat{\theta})} \qquad y = \underbrace{x_1}_{\hat{\xi}_1 = \hat{z}_1(x,\hat{\theta})} \quad (52) \\ \dot{\hat{\xi}}_2 &= \underbrace{(\hat{\theta}\cos(x_1))}_{\hat{B}(x,\hat{\theta})} + \underbrace{(\frac{1}{I})}_{\hat{A}(x,\hat{\theta})} u + (\theta - \hat{\theta})\cos(x_1) + \underbrace{\Delta f}_{\Delta\phi} \\ u &= \frac{1}{\hat{A}(x,\hat{\theta})}(v - \hat{B}(x,\hat{\theta})) .\end{aligned}$$

It is clear that the normal form (52) has the unobservable state variables $\hat{\xi}_1$ and $\hat{\xi}_2$. Thus, defining $\bar{\xi}$ as the state estimate and cancelling $\Delta\phi$ in (52), the following state observer is considered to estimate the unobservable dynamics (52):

$$\begin{aligned}\dot{\bar{\xi}}_1 &= \hat{\xi}_2 + (\bar{\xi}_2 - \hat{\xi}_2) = \bar{\xi}_2 \\ \dot{\bar{\xi}}_2 &= v + [-2(\bar{\xi}_1 - \hat{\xi}_1) - 2(\bar{\xi}_2 - \hat{\xi}_2)] \quad (53)\end{aligned}$$

with the robust adaptive law

$$\dot{\hat{\theta}} = -s_2\cos(x_1) - \gamma||s_2||\hat{\theta} \quad (54)$$

where $s_2 \equiv \bar{\xi}_2 - \hat{\xi}_2$, $\gamma = 0.8$. Since $\Delta\phi = \Delta f \in \mathcal{L}_\infty$, the normalizing signal is not considered here. The output tracking is finally achieved by using $v = \ddot{y}_m + 2(\dot{y}_m - \bar{\xi}_2) + 2(y_m - \bar{\xi}_1)$. Then, the time responses of (54) and the output tracking are shown in Figure 6 and Figure 7, respectively.

# 4. Conclusions and Prospects

In this paper, we introduced two robust adaptive nonlinear control schemes for *SISO* nonlinear

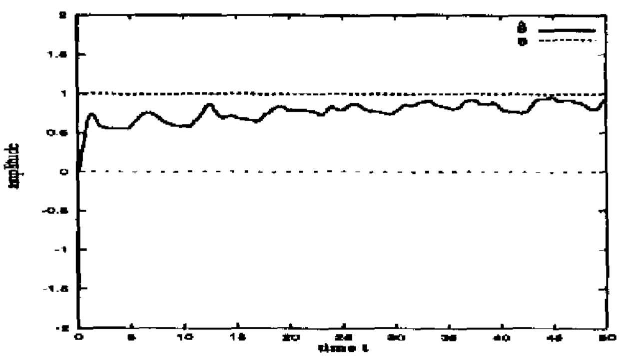


Fig. 6 System responses (1) - $\hat{\theta}(t)$, $\theta$

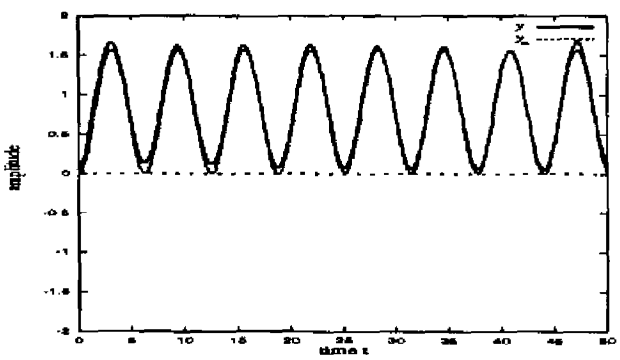


Fig. 7 System responses (2) - $y(t)$, $y_m(t)$

systems in the presence of both unknown constant parameters and unmodeled dynamics. Then, the main contributions of this paper could be summarized as shown below. First, it was noted that all the robust adaptive laws developed in this paper did not lead to any overparameterization for parameter identification. Second, it was shown that the systematic designs for robust adaptive laws were similar to those for uncertain linear systems. Lastly, it was also important to note that these nonlinear control methods led to the robustness and approximate output tracking of all the closed-loop system with robust adaptive laws. Because the control schemes developed were designed on some restrictive assumptions such as the full-state measurable condition, the local domain of both state variables and parameter estimate, and the single-input, single-output nonlinear systems, we need to study some prospective works like output feedback control, global domain control, and multi-input, multi-output control based on the control schemes developed in this paper.

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