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非線形フィードバック制御

Polynomial Feedback Control for Nonlinear Systems

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1. INTRODUCTION

An overwhelming majority of the nonlinear problems encountered in practice are able to be described or approximated by polynomial functions with sufficiently high order, so the research on polynomial nonlinear systems deserves our attention [1]. Many researchers have considered the problem of polynomial nonlinearities by a compact description using Kronecker power, which has benefited the analysis and design greatly. Okubo and Kitamori designed a regulator in the form of infinite expansion [6], and Okubo published his stable design of regulator with a genetic algorithm for the systems, polynomial in state and linear in input [7]. Sontag et al reported the application of algebraic geometry to discrete time systems [2] and a more extensive study of observability [3]. John Baillieul studied this class of systems using methods from algebraic and differential geometry [5] and proposed a efficient condition for regulator problem [4].Unfortunately, they did not provide a feasible solution for the systems.

In this paper, we design a globally asymptotically stabilizing direct feedback control using Kronecker powers of state vector for the plants with polynomial dynamics in the state and input. A genetic algorithm is employed to find suitable gain, and algebraic geometric concept is used to simplify the design.

2. PLANT ASSUMPTIONS

Consider the nonlinear systems which dynamics are polynomials in the state and/or the control, i.e.,

$$\dot{x} = P(x, u), \tag{1}$$

where $P(\cdot,\cdot)$ is a polynomial function vector in the components of the state $x \in \Re^{n_x}$ and the control $u \in \Re^{n_u}$.

Using Kronecker power, the systems can be described as

$$\begin{aligned}\dot{x} &= B_0(x) + B_1(x)u + B_2(x)u^{[2]} \\ &\quad + \cdots + B_{N_u}(x)u^{[N_u]} \\ &= \sum_{i=0}^{N_u} B_i(x)u^{[i]}, \quad (2) \\ B_i(x) &= \sum_{j=0}^{N_x} B_{ij}(x^{[j]}), \quad (3)\end{aligned}$$

where $B_i(x) \in \Re^{n_x \times n_u^i}$, $B_{ij} \in \Re^{n_x \times (n_x^j \cdot n_u^i)}$. $x^{[j]}$ and $u^{[i]}$ denote the Kronecker power of x and u in j, i times respectively. If, for example, $n_x = 2$, $j = 2$,

$$\begin{aligned}x &= [x_1 \quad x_2]^T, \quad (4) \\ x^{[2]} &= [x_1^2 \quad x_1x_2 \quad x_1x_2 \quad x_2^2]^T. \quad (5)\end{aligned}$$

We can rewrite the above systems as

$$\begin{aligned}\dot{x} &= \sum_{i=0}^{N_u}\sum_{j=0}^{N_x} B_{ij}(x^{[j]})u^{[i]} \\ &= \sum_{i=0}^{N_u}\sum_{j=0}^{N_x} B_{ij}(x^{[j]} \otimes u^{[i]}), \quad (6)\end{aligned}$$

here, the symbol $\otimes$ denotes Kronecker product. Without loss of generality, let $B_{00} = 0$, and define a new vector $v \in \Re^{n_u}$ as $\dot{u} = v$. Combine the new vector with the original state,

$$\begin{aligned}\dot{x} &= \sum_{i=0}^{N_u}\sum_{j=0}^{N_x} B_{ij}(x^{[j]} \otimes u^{[i]}), \quad (7) \\ \dot{u} &= v, \quad (8)\end{aligned}$$

and denoting an augmented state vector as $z \in \Re^{n_z}$, $n_z = n_x + n_u$, we have

$$\begin{aligned}z &= \begin{bmatrix} x \\ u \end{bmatrix}, \quad (9) \\ x &= [I_x \quad 0]\, z = E_x z, \quad (10) \\ u &= [0 \quad I_u]\, z = E_u z, \quad (11) \\ B_{ij}(x^{[j]} \otimes u^{[i]}) &= B_{ij}(E_x^{[j]} \otimes E_u^{[i]})z^{[i+j]}, (12)\end{aligned}$$

here, $E_x^{[j]} \otimes E_u^{[i]}$ is the Kronecker product of $E_x^{[j]}$ and $E_u^{[i]}$.

Let $max[i+j] = 2N-1$, then

$$\begin{aligned}\dot{x} &= \sum_{i=0}^{N_u}\sum_{j=0}^{N_x} B_{ij}(E_x^{[j]} \otimes E_u^{[i]})z^{[i+j]} \\ &= \sum_{k=1}^{2N-1}\{\sum_{i+j=k} B_{ij}(E_x^{[j]} \otimes E_u^{[i]})\}z^{[k]}, \quad (13)\end{aligned}$$

where

$$B_{x[1,k]} = \{\sum_{i+j=k} B_{ij}(E_x^{[j]} \otimes E_u^{[i]})\}S_k. \quad (14)$$

S_k is a symmetric tensor, so $B_{x[1,k]}$ is a $[1,k]$-type covariant tensor.

Let

$$\begin{aligned}A_{[1,k]} &= \begin{bmatrix} B_{x[1,k]} \\ 0 \end{bmatrix}, \quad (15) \\ B &= \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad (16)\end{aligned}$$

the systems can be unified as

$$\begin{aligned}\dot{z} &= \sum_{k=1}^{2N-1} A_{[1,k]}z^{[k]} + Bv \\ &= A_{G[1,2N-1]}G^{[2N-1]}(z) + Bv, \quad (17)\end{aligned}$$

here,

$$A_{G[1,2N-1]} = [A_{[1,1]}A_{[1,2]}\cdots A_{[1,2N-1]}], (18)$$

$$G^{[2N-1]}(z) = \begin{bmatrix} z^{[1]} \\ z^{[2]} \\ \vdots \\ z^{[2N-1]} \end{bmatrix}. \quad (19)$$

3. POLYNOMIAL FEEDBACK CONTROL

First we define a gain set $\Delta\{K_{G[1,2N-1]} \in \Re^{n_u \times \sum_{k=1}^{2N-1} n_z^k}\}$:

$$K_{G[1,2N-1]} = [K_{[1,1]}K_{[1,2]}\cdots K_{[1,2N-1]}], \quad (20)$$

where $K_{[1,k]}$ is a $[1,k]$-type covariant symmetric tensor. For the systems in (17), using the state vector's Kronecker power in $1 \sim 2N-1$ times as in (19), we choose a direct feedback control as

$$v = -K_{G[1,2N-1]}G^{[2N-1]}(z), \quad (21)$$

here, gain $K_{G[1,2N-1]} \in \Delta$. Then the closed-loop system dynamics are

$$\dot{z} = (A_{G[1,2N-1]} - BK_{G[1,2N-1]})G^{[2N-1]}(z). \quad (22)$$

Let us define a matrix set $\Omega\{\tilde{P}_{G[N,N]} \in \Re^{\sum_{k=1}^{N} n_z^k \times \sum_{k=1}^{N} n_z^k}\}$ with

$$\tilde{P}_{G[N,N]} = \begin{bmatrix} \tilde{P}_{[1,1]}^{(1,1)} & \tilde{P}_{[1,2]}^{(1,2)} & \cdots & \tilde{P}_{[1,N]}^{(1,N)} \\ \tilde{P}_{[2,1]}^{(2,1)} & \tilde{P}_{[2,2]}^{(2,2)} & \cdots & \tilde{P}_{[2,N]}^{(2,N)} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{P}_{[N,1]}^{(N,1)} & \tilde{P}_{[N,2]}^{(N,2)} & \cdots & \tilde{P}_{[N,N]}^{(N,N)} \end{bmatrix}, \quad (23)$$

where $block\tilde{P}_{[i,j]}^{(i,j)} \in \Re^{n_z^i \times n_z^j}$ is a $[i,j]$ symmetric tensor at block position (i,j). Define another matrix set $\Gamma\{P_{G[1,2N-1]} \in \Re^{n_z \times \sum_{k=1}^{2N-1} n_z^k}\}$ with

$$P_{G[1,2N-1]} = [P_{[1,1]} P_{[1,2]} \cdots P_{[1,2N-1]}], \quad (24)$$

where $P_{[1,k]}$ is a $[1,k]$ covariant symmetric tensor.

Choose a Lyapunov function candidate as

$$V(t) = \frac{1}{2} G^{T[N]}(z) \tilde{P}_{G[N,N]} G^{[N]}(z) \quad (25)$$

with $\tilde{P}_{G[N,N]} \in \Omega$. Then $V(t)$ can be rewritten as

$$V(t) = \sum_{k=1}^{2N-1} \frac{1}{k+1} z^T P_{[1,k]} z^{[k]}, \quad (26)$$

where

$$P_{[1,k]} = \begin{cases} \frac{k+1}{2} \sum_{i=1}^{k} \tilde{P}_{[1,k]}^{(i,k-i+1)} \\ \text{for } 1 \le k \le N, \\ \frac{k+1}{2} \sum_{i=k-N+1}^{N} \tilde{P}_{[1,k]}^{(i,k-i+1)} \\ \text{for } (N+1) \le k \le (2N-1). \end{cases} \quad (27)$$

i.e. there exists a projection ϕ_1:

$$\phi_1 : \Omega \to \Gamma. \quad (28)$$

The derivative of $V(t)$ in (26) is

$$\begin{aligned} \dot{V}(t) &= \sum_{k=1}^{2N-1} \dot{z}^T P_{[1,k]} z^{[k]} \\ &= \dot{z}^T P_{G[1,2N-1]} G^{[2N-1]}(z). \quad (29) \end{aligned}$$

From (17), we have

$$\begin{aligned} \dot{V}(t) &= -\frac{1}{2} G^{T[2N-1]}(z) (K_{G[1,2N-1]}^T B^T \\ &\quad P_{G[1,2N-1]} + P_{G[1,2N-1]}^T B K_{G[1,2N-1]} \\ &\quad - A_{G[1,2N-1]}^T P_{G[1,2N-1]} \\ &\quad - P_{G[1,2N-1]}^T A_{G[1,2N-1]}) G^{[2N-1]}(z) \\ &= -\frac{1}{2} G^{T[2N-1]}(z) Q_{G[2N-1,2N-1]} \\ &\quad G^{[2N-1]}(z) \quad (30) \end{aligned}$$

here, just like set Ω, we define a symmetric tensor set $\Theta\{Q_{G[2N-1,2N-1]} \in \Re^{\sum_{k=1}^{2N-1} n_z^k \times \sum_{k=1}^{2N-1} n_z^k}\}$ with

$$Q_{G[2N-1,2N-1]} = \begin{bmatrix} Q_{[1,1]}^{(1,1)} & Q_{[1,2]}^{(1,2)} & \cdots & Q_{[1,2N-1]}^{(1,2N-1)} \\ Q_{[2,1]}^{(2,1)} & Q_{[2,2]}^{(2,2)} & \cdots & Q_{[2,2N-1]}^{(2,2N-1)} \\ \vdots & \vdots & \vdots & \vdots \\ Q_{[2N-1,1]}^{(2N-1,1)} & Q_{[2N-1,2]}^{(2N-1,2)} & \cdots & Q_{[2N-1,2N-1]}^{(2N-1,2N-1)} \end{bmatrix}$$

where $blockQ_{[i,j]}^{(i,j)} \in \Re^{n_z^i \times n_z^j}$ is a $[i,j]$ symmetric tensor at block position (i,j). $Q_{G[2N-1,2N-1]}$ is noted as

$$\begin{aligned} Q_{G[2N-1,2N-1]} &= K_{G[1,2N-1]}^T B^T P_{G[1,2N-1]} \\ &\quad + P_{G[1,2N-1]}^T B K_{G[1,2N-1]} \\ &\quad - A_{G[1,2N-1]}^T P_{G[1,2N-1]} \\ &\quad - P_{G[1,2N-1]}^T A_{G[1,2N-1]} \quad (31) \end{aligned}$$

where $Q_{G[2N-1,2N-1]} \in \Theta$, i.e. there exists a projection ψ_1:

$$\psi_1 : \Gamma \times \Delta \to \Theta. \quad (32)$$

So, $\tilde{P}_{G[N,N]} \in \Omega$ and $K_{G[1,2N-1]} \in \Delta$ can be projected to $Q_{G[2N-1,2N-1]} \in \Theta$ as

$$\Omega \xrightarrow{\phi_1} \Gamma \times \Delta \xrightarrow{\psi_1} \Theta. \quad (33)$$

(31) is a equation like the *Lyapunov equation* , we can call it *extended Lyapunov equation*.

If a $\tilde{P}_{G[N,N]}$ and a $K_{G[1,2N-1]}$ exist, and, for any $z \neq [0]$, satisfy

$$G^{T[N]}(z) \tilde{P}_{G[N,N]} G^{[N]}(z) \quad > \quad 0,$$

$$G^{T[2N-1]}(z)Q_{G[2N-1,2N-1]}G^{[2N-1]}(z) > 0, \quad (34)$$

(35)

noted simply as

$$\tilde{P}_{G[N,N]} > 0, \quad (36)$$

$$Q_{G[2N-1,2N-1]} > 0, \quad (37)$$

then

$$V(t) > 0 \text{ (for any } z \neq [0]), \quad (38)$$

$$\dot{V}(t) < 0 \text{ (for any } z \neq [0]), \quad (39)$$

$$V(t) = 0 \quad (z = [0] \text{ only}), \quad (40)$$

$$V(t) \to \infty \text{ (when } \| z \| \to \infty), \quad (41)$$

so, $K_{G[1,2N-1]}$ is a globally asymptotically stabilizing feedback gain from Barbasin-Krasovskii theorem (See, e.g.,[11]).

We can summarize the above as Theorem 1

Theorem 1 *The nonlinear systems*

$$\dot{x} = P(x,u), \quad (1)$$

where $P(\cdot,\cdot)$ is a polynomial function vector in the components of the state $x \in \Re^{n_x}$ and/or the control $u \in \Re^{n_u}$, can be augmented by $\dot{u} = v$ in $z = \begin{bmatrix} x \\ u \end{bmatrix}$, as

$$\dot{z} = A_{G[1,2N-1]}G^{[2N-1]}(z) + Bv. \quad (17)$$

If there exist a $\tilde{P}_{G[N,N]}$ and a $K_{G[1,2N-1]}$ satisfying

$$\tilde{P}_{G[N,N]} > 0, \quad (36)$$

$$Q_{G[2N-1,2N-1]} > 0, \quad (37)$$

the control $v = -K_{G[1,2N-1]}G^{[2N-1]}(z)$ is globally asymptotically stabilizing.

4. SIMPLIFICATION AND REALIZATION

In order to realize the feedback control above, we have to discard redundant information in the relationship between $K_{G[1,2N-1]}$ and $P_{G[1,2N-1]}$ as well as $Q_{G[2N-1,2N-1]}$. In algebra, the idea of factoring out extraneous or repetitive data is accomplished through the concept of a quotient set. Intuitively, the notation of a quotient is nothing more than dividing the total set of objects under consideration into those which are of interest and those that are not. The rule of separating interesting objects from uninteresting is based on the idea of two elements being equivalent. Roughly speaking, we say two objects are equivalent if they differ only in details that are not important for the problem at hand. Let us be more specific.

Correspond with the index $[\cdot]$, we introduce a index $\langle\cdot\rangle$ to denote contract expressions related to contract form of $x^{(i)}$. For example, the contract form of $x^{[2]}$ in (5) is

$$x^{\langle 2\rangle} = [\, x_1^2 \quad x_1x_2 \quad x_2^2 \,]^T, \quad (42)$$

here the entries are ordered lexicographically, and (17) becomes

$$\dot{z} = A_{G\langle 1,2N-1\rangle}G^{\langle 2N-1\rangle}(z) + Bv, \quad (43)$$

$$A_{G\langle 1,2N-1\rangle} = [A_{\langle 1,1\rangle}A_{\langle 1,2\rangle}\cdots A_{\langle 1,2N-1\rangle}] \quad (44)$$

$$G^{\langle 2N-1\rangle}(z) = \begin{bmatrix} z^{\langle 1\rangle} \\ z^{\langle 2\rangle} \\ \vdots \\ z^{\langle 2N-1\rangle} \end{bmatrix}. \quad (45)$$

Correspondingly, we denote the compacted sets $\Omega/\lambda_1\{\tilde{P}_{G\langle N,N\rangle}\}$, $\Gamma/\lambda_2\{P_{G\langle 1,2N-1\rangle}\}$, $\Delta/\lambda_3\{K_{G\langle 1,2N-1\rangle}\}$, and $\Theta/\lambda_4\{Q_{G\langle 2N-1,2N-1\rangle}\}$, the equivalence sets of Ω, Γ, Δ, Θ respectively, as

$$\Omega \overset{\lambda_1}{\to} \Omega_c,$$

$$
\begin{aligned}
\Gamma &\overset{\lambda_2}{\to} \Gamma_c, \\
\Delta &\overset{\lambda_3}{\to} \Delta_c, \\
\Theta &\overset{\lambda_4}{\to} \Theta_c. \qquad (46)
\end{aligned}
$$

here, the elements of Ω_c and Θ_c are symmetric matrices. The projections in (33) become

$$\Omega_c \overset{\phi_2}{\to} \Gamma_c \times \Delta_c \overset{\psi_2}{\to} \Theta_c \qquad (47)$$

with

$$\phi_2: \ \lambda_2 \cdot \phi_1 \cdot \lambda_1^{-1}, \qquad (48)$$

and ψ_2:

$$
\begin{aligned}
& Q_{G(2N-1,2N-1)} \\
&= K^T_{G(1,2N-1)} B^T P_{G(1,2N-1)} \\
&\quad + P^T_{G(1,2N-1)} B K_{G(1,2N-1)} \\
&\quad - A^T_{G(1,2N-1)} P_{G(1,2N-1)} \\
&\quad - P^T_{G(1,2N-1)} A_{G(1,2N-1)}. \qquad (49)
\end{aligned}
$$

There even exist redundant information in set Ω_c and set Θ_c. Letting the entries which are far from the diagonals equal zero in matrix $\tilde{P}_{G(N,N)}$ and $Q_{G(2N-1,2N-1)}$ as possible as we can, we can construct symmetric matrices with only independent entries remained, denoted as $\tilde{P}_{G(\langle N,N\rangle)}$ and $Q_{G(\langle 2N-1,2N-1\rangle)}$ respectively. Set $\Omega_c/\lambda\{\tilde{P}_{G(\langle N,N\rangle)}\}$ and $\Theta_c/\lambda\{Q_{G(\langle 2N-1,2N-1\rangle)}\}$ are the equivalence set of Ω_c and Θ_c. We denote them as Ω_s and Θ_s, i.e.

$$
\begin{aligned}
\Omega_c &\overset{\lambda}{\to} \Omega_s, \\
\Theta_c &\overset{\lambda}{\to} \Theta_s. \qquad (50)
\end{aligned}
$$

So, our task has been changed equivalently to find a $\tilde{P}_{G(\langle N,N\rangle)}$ and a $K_{G(1,2N-1)}$ to satisfy

$$\tilde{P}_{G(\langle N,N\rangle)} > 0, \ Q_{G(\langle 2N-1,2N-1\rangle)} > 0 \qquad (51)$$

$$nonumber \quad \Rightarrow \ V(t) > 0, \ \dot{V}(t) < 0. \qquad (52)$$

The following equivalence relations will help to understand the above transformations explicitly,

$$
\begin{aligned}
& G^{T[N]}(z) \tilde{P}_{G[N,N]} G^{[N]}(z) \\
&= G^{T(N)}(z) \tilde{P}_{G(N,N)} G^{(N)}(z) \\
&= G^{T(N)}(z) \tilde{P}_{G(\langle N,N\rangle)} G^{(N)}(z) \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
& G^{T[2N-1]}(z) Q_{G[2N-1,2N-1]} G^{[2N-1]}(z) \\
&= G^{T(2N-1)}(z) Q_{G(2N-1,2N-1)} G^{(2N-1)}(z) \\
&= G^{T(2N-1)}(z) Q_{G(\langle 2N-1,2N-1\rangle)} G^{(2N-1)}(z) \qquad (54)
\end{aligned}
$$

and Fig. 1 indicates the commutation diagrammatically.

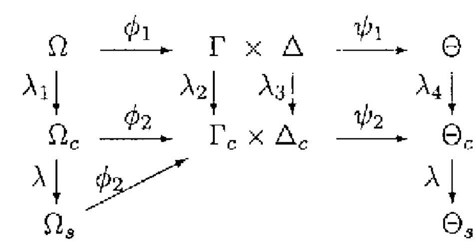

Fig. 1 Commutative diagram of P and Q

Corollary 1 *If all eigenvalues of $\tilde{P}_{G(\langle N,N\rangle)}$ and $Q_{G(\langle 2N-1,2N-1\rangle)}$ are positive, the control*

$$v = -K_{G(1,2N-1)} G^{(2N-1)}(z)$$

is globally asymptotically stabilizing.

It is hardly feasible to obtain a $K_{G(1,2N-1)}$ from given $\tilde{P}_{G(\langle N,N\rangle)}$ and $Q_{G(\langle 2N-1,2N-1\rangle)}$ computationally, so a genetic algorithm is employed, using Corollary 1, to search $K_{G(1,2N-1)}$, which is the efficient condition (NOT necessary!) for (52).

5. GENETIC ALGORITHM

By recursively building the positive definite leading principal minors of a symmetric matrix, we can map the strictly positive definite matrix from a series of numbers. For the symmetric matrix $P_{an} \in \Re^{n \times n}$, we can divided it as,

$$P_{an} = \begin{bmatrix} P_{an-1} & P_{bn-1} \\ P^T_{bn-1} & p_{cn} \end{bmatrix} \qquad (55)$$

with P_{an-1} a matrix in $(n-1) \times (n-1)$, P_{bn-1} a $(n-1)$-vector, and p_{cn} a scalar. We have

$$|P_{an}| = |P_{an-1}|(p_{cn} - P^T_{bn-1} P^{-1}_{an-1} P_{bn-1}). \qquad (56)$$

Let $|P_{an-1}|(p_{cn}-P_{bn-1}^TP_{an-1}^{-1}P_{bn-1})=\alpha_{nn}^2+\beta_n>0$, and $P_{bn-1}^T=[\alpha_{n1}\quad \alpha_{n2}\quad \cdots \quad \alpha_{nn-1}]$, a randomly generated n-vector $\alpha_n\in\Re^{n\times 1}$,

$$\alpha_n^T=[\alpha_{n1}\quad \alpha_{n2}\quad \cdots \quad \alpha_{nn-1}\quad \alpha_{nn}], \tag{57}$$

and a positive real number $\beta_n>0$. Then

$$p_{cn} = \frac{\alpha_{nn}^2+\beta_n}{|P_{an-1}|}+P_{bn-1}^TP_{an-1}^{-1}P_{bn-1}. \tag{58}$$

So, we can build a symmetric matrix P_{an} with $|P_{an}|=\alpha_{nn}^2+\beta_n>0$ from a symmetric matrix P_{an-1} with $|P_{an-1}|>0$, and a randomly generated vector $\alpha_n^T=[\alpha_{n1}\quad \alpha_{n2}\quad \cdots \quad \alpha_{nn-1}\quad \alpha_{nn}]$ as well as a scalar $\beta_n>0$.That means we can build a symmetric matrix with positive definite leading principal minors, say, strictly positive symmetric matrix P, from a randomly generated vector $\alpha_P^T=[\alpha_{11}\quad \alpha_{21}\quad \alpha_{22}\quad \cdots \quad \alpha_{nn}]$ as well as n-vector $\beta=[\beta_1\quad \beta_2\quad \cdots \quad \beta_n]$ which entries are positive real numbers, i.e.

$$\omega:\ \alpha_P\rightarrow P. \tag{59}$$

Specifically,

$$\omega:\ \alpha_{\tilde{P}}\rightarrow \tilde{P}_{G(\langle N,N\rangle)} \tag{60}$$

with $\alpha_{\tilde{P}}\in\Re^{N_{\tilde{P}}\times 1}$.

$$\begin{aligned} N_{\tilde{P}} &= \sum_{k=2}^{2N} {}_{n_z}H_k \\ &= {}_{n_z+1}H_{2N}-(n_z+1). \end{aligned} \tag{61}$$

Also we can easily divide a free vector $\alpha_K\in\Re^{N_K\times 1}$,

$$\begin{aligned} N_K &= n_z\times\sum_{k=1}^{2N-1} {}_{n_z}H_k \\ &= n_z\times({}_{n_z+1}H_{2N-1}-1), \end{aligned} \tag{62}$$

to form the rows of $K_{G\langle 1,2N-1\rangle}$ in order, say form $K_{G\langle 1,2N-1\rangle}$.

$$\mu:\ \alpha_K\rightarrow K_{G\langle 1,2N-1\rangle}. \tag{63}$$

Coding $\alpha_G=[\alpha_{\tilde{P}}\quad \alpha_K]$, we get a gene, noted as α_{GC}.

The evaluation function for the GA is selected as

$$\begin{aligned} f_Q &= min\{D_1,D_2,\cdots,D_{N_Q}\}, &(64)\\ N_Q &= \sum_{k=1}^{2N-1} {}_{n_z}H_k={}_{n_z+1}H_{2N-1}-1, &(65)\\ f_Q &\rightarrow max, &(66)\end{aligned}$$

where D_i are the leading principal minors of matrix $Q_{G(\langle 2N-1,2N-1\rangle)}$. By the operations of crossover, mutation, elite selection etc., we search evolutionarily for a proper gene which maps a $\tilde{P}_{G(\langle N,N\rangle)}$ with all positive eigenvalues and a $K_{G\langle 1,2N-1\rangle}$ to a $Q_{G(\langle 2N-1,2N-1\rangle)}$ with all positive eigenvalues.

The procedure is as follows:

Procedure

step1 Generate α_{GC} randomly;

step2 Compute $\alpha_G=[\alpha_{\tilde{P}}\ \alpha_K]$ from α_{GC};

step3 Compute $P_{G\langle 1,2N-1\rangle}$ from $\alpha_{\tilde{P}}$:

$$\begin{aligned} \omega: &\quad \alpha_{\tilde{P}}\rightarrow \tilde{P}_{G(\langle N,N\rangle)},\\ \phi_2: &\quad \tilde{P}_{G(\langle N,N\rangle)}\rightarrow P_{G\langle 1,2N-1\rangle}; \end{aligned}$$

step4 Compute $K_{G\langle 1,2N-1\rangle}$ from α_K;

step5 Compute $Q_{G(\langle 2N-1,2N-1\rangle)}$;

step6 Evaluation, and judgement of stop or not; If $f_Q>0$, the $K_{G\langle 1,2N-1\rangle}$ is what we want to construct a controller in (21) that makes the systems (17) globally asymptotically stable;

step7 GA operation;

step8 Go to step2.

6. NUMERICAL EXAMPLE

For the systems of $n_x=2$, $n_u=1$, and $N=2$,

$$\dot{x} = [B_{01}\quad B_{02}\quad B_{03}]\begin{bmatrix}x^{[1]}\\ x^{[2]}\\ x^{[3]}\end{bmatrix}+\Big[B_{10}u^{[1]}$$

$$
\begin{aligned}
&+ B_{11}x^{[1]} \otimes u^{[1]} + B_{12}x^{[2]} \otimes u^{[1]}\Big] + \Big[B_{20}u^{[2]} \\
&+ B_{21}x^{[1]} \otimes u^{[2]}\Big] + \Big[B_{30}u^{[3]}\Big] \qquad (67)
\end{aligned}
$$

$$z = \begin{bmatrix} x \\ u \end{bmatrix} \qquad (68)$$

$$x = [I_x \quad 0]\, z = E_x z \qquad (69)$$

$$u = [0 \quad I_u]\, z = E_u z \qquad (70)$$

$$
\begin{aligned}
\dot{z} &= [B_{01}E_x + B_{10}E_u]\, z^{[1]} \\
&+ [B_{02}E_x \otimes E_x + B_{11}E_x \otimes E_u \\
&+ B_{20}E_u \otimes E_u]\, z^{[2]} + [B_{03}E_x \otimes E_x \otimes E_x \\
&+ B_{12}E_x \otimes E_x \otimes E_u + B_{21}E_x \otimes E_u \otimes E_u \\
&+ B_{30}E_u \otimes E_u \otimes E_u]\, z^{[3]} \qquad (71)
\end{aligned}
$$

Numerically,

$$
\begin{aligned}
\dot{x}_1 &= -2x_1 + 0.2x_2 + 0.12x_1^2 - 0.15x_1x_2 \\
&+ 0.2x_2^2 - 3x_1^3 + 0.6x_1^2x_2 - 0.4x_1x_2^2 \\
&+ 0.2x_2^3 + u + 0.1x_1u + 0.2x_2u \\
&- 0.1x_1^2u + 0.6x_1x_2u + 0.7x_2^2u \\
&+ u^2 + 2x_1u^2 + x_2u^2 + 0.2u^3 \qquad (72)
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_2 &= x_1 - 3.6x_2 - 0.25x_1^2 - 0.6x_1x_2 \\
&+ 0.3x_2^2 - 0.4x_1^3 - 0.51x_1^2x_2 - 0.21x_1x_2^2 \\
&- 5x_2^3 + u - 0.1x_1u + 0.5x_2u \\
&+ 0.3x_1^2u - x_1x_2u + x_2^2u - u^2 \\
&- 0.5x_1u^2 + 0.5u^3 \qquad (73)
\end{aligned}
$$

i.e.

$$B_{01} = \begin{bmatrix} -2 & 0.2 \\ 1 & -3.6 \end{bmatrix}, \qquad (74)$$

$$B_{02} = \begin{bmatrix} 0.12 & -0.075 & -0.075 & 0.2 \\ -0.25 & -0.3 & -0.3 & 0.30 \end{bmatrix}, \qquad (75)$$

$$B_{03} = \begin{bmatrix} -3 & 0.2 & 0.2 & -0.13 & 0.2 & -0.13 & -0.13 & 0.2, \\ -0.4 & -0.17 & -0.17 & -0.07 & -0.17 & -0.07 & -0.07 & -5 \end{bmatrix}, \qquad (76)$$

$$B_{10} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad (77)$$

$$B_{11} = \begin{bmatrix} 0.1 & 0.2 \\ -0.1 & 0.5 \end{bmatrix}, \qquad (78)$$

$$B_{12} = \begin{bmatrix} -0.1 & 0.3 & 0.3 & 0.7 \\ 0.3 & -0.5 & -0.5 & 1 \end{bmatrix}, \qquad (79)$$

$$B_{20} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad (80)$$

$$B_{21} = \begin{bmatrix} 2 & 1 \\ -0.5 & 0 \end{bmatrix}, \qquad (81)$$

$$B_{30} = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}. \qquad (82)$$

Select $\beta_i > 0$ $(i = 1 \sim 9)$, say vector β as

$$\beta = [\; 0.1 \quad 1 \quad 5 \quad 15 \quad 50 \quad 200 \quad 2000 \quad 10000 \quad 40000 \;] \qquad (83)$$

For 31-vector $\alpha_{\tilde{P}}$ and 19-vector α_K, we code α_G as $16bit \times 50 = 800bit$ of number 20. Set elite number 2, mutation $6bit$ in $800bit$ of number 5 and a_{ij} range: $-3.2767 \sim 3.2767$. The evolution process is shown in Fig. 2. After generation 2185, we find

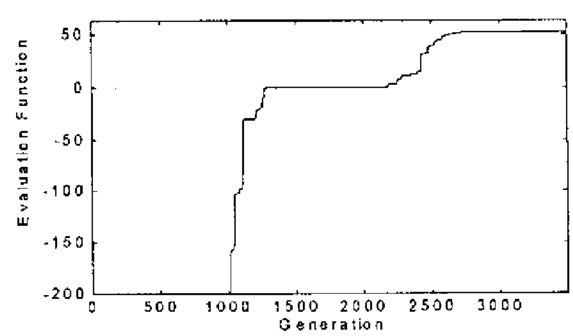

Fig. 2 Search process of GA

$\tilde{P}_{G(\langle N,N \rangle)}$, $K_{G(1,2N-1)}$ to make $Q_{G(\langle 2N-1,2N-1 \rangle)}$ strictly positive definite. At generation 3500, we have

$$
\tilde{P}_{G(\langle 2,2 \rangle)} = \left[\begin{array}{ccc|c}
9.3556 & 0.9359 & 1.0866 & -0.0141 \\
0.9359 & 0.3941 & 0.2692 & 0.6423 \\
1.0866 & 0.2692 & 2.9639 & -0.4765 \\
\hline
-0.0141 & 0.6423 & -0.4765 & 3.5746 \\
0 & 0.1579 & 0.4169 & 0.3894 \\
0 & 0 & 0.6558 & -0.5764 \\
0 & 0.1681 & 0.2883 & 0 \\
0 & 0 & 0.4443 & 0 \\
0 & 0 & 0.2439 & 0
\end{array}\right.
$$

$$
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0.1579 & 0 & 0.1681 & 0 \\
0.4169 & 0.6558 & 0.2883 & 0.4443 \\
\hline
0.3894 & -0.5764 & 0 & 0 \\
3.4543 & 0.3085 & 0.2258 & 0 \\
0.3085 & 4.3473 & 0.0413 & 1.2491 \\
0.2258 & 0.0413 & 9.8990 & -0.1904 \\
0 & 1.2491 & -0.1904 & 5.4216 \\
0 & 0.8032 & 0 & -0.2205
\end{array}
$$

$$
\left.\begin{array}{c}
0 \\
0 \\
0.2439 \\
\hline
0 \\
0 \\
0.8032 \\
0 \\
-0.2205 \\
4.2133
\end{array}\right] \tag{84}
$$

$$
\begin{aligned}
K_{G\langle 1,3\rangle} = & \left[\ 3.0685 \quad -0.4742 \quad 2.3685 \ \vdots\ -1.5744 \right. \\
& -1.3194 \quad -1.1279 \quad -3.1729 \quad -1.8758 \\
& 1.7020 \ |\ 0.1662 \quad 2.8137 \quad 2.4222 \\
& -0.3733 \quad -1.1492 \quad -1.2693 \quad -1.2928 \\
& \left. 2.6138 \quad 2.4023 \quad 3.1722\ \right]
\end{aligned} \tag{85}
$$

Using the gain matrix in (85), the feedback control system response is simulated in Fig. 3 with initial state

$$
z(0) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}. \tag{86}
$$

The simulation indicates that the state converge to

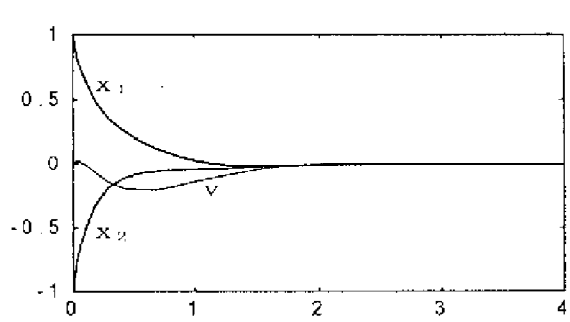

Fig. 3 The response of the system

zero asymptotically stably.

7. CONCLUSION

This paper provided a general method to design globally asymptotically stable polynomial feedback control for nonlinear systems. It is convenient to search gain matrix by a generalized GA algorithm. The method, of course, suits for bilinear systems and multilinear systems even for systems with rational functions.

The further research should include how to get the gain matrix analytically or judge the existence of $K_{G\langle 1,2N-1\rangle}$ and $\tilde{P}_{G\langle\langle N,N\rangle\rangle}$ definitely.

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