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安定化補償器のパラメトリゼーションに関する一考察

Parameterization of stabilizing controllers without coprime factorizability

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Abstract An alternative parameterization method of stabilizing controllers is presented by using the coordinate-free approach. The parameterization of this paper is more intuitive than previous ones. The result in this paper will not assume the existance of the coprime factorizability and not employ the Youla-Kučera parameterization.

1 Introduction

Once the existence of the doubly coprime factorization comes to be known, it is easy to parameterize all stabilizing controllers by Youla-Kučera parameterization[DLMS80, RL84, Vid85, VSF82, YJB76].

On the other hand, some class of control systems does not know whether or not a stabilizable plant in the class always has its doubly coprime factorization. The multidimensional systems with structural stability is one of such classes[Lin98, Lin99].

The objective of this paper is to present an alternative parameterization method of stabilizing conrollers *without* the coprime factorizability.

The approach we use in this paper is the coordinate-free approach[SS92, Sul94, Sul98, Mor00, Mor99a, Mor99b, Mor01c, Mor01b].

The coordinate-free approach is a factorization approach without the coprime factorizability. It is well known that the factorization approach to control systems has the advantage that it embraces, within a single framework, numerous linear systems such as continuous-time as well as discrete-time systems, lumped as well as distributed systems, one-dimensional as well as multidimensional systems, etc.[DLMS80, VSF82]. Hence the result given in this paper will be able to a number of models in addition to the multidimensional systems. In factorization approach, when problems such as feedback stabilization are studied, one can focus on the key aspects of the problem under study rather than be distracted by the special features of a particular class of linear systems. A transfer matrix of this approach is considered as the ratio of two stable causal transfer matrices. For a long time, the theory of the factorization approach had been founded on the coprime factorizability of transfer matrices, which is satisfied by transfer matrices over the principal ideal domains or thé Bézout domains. However it is known that there are models such that some stabilizable plants do not have coprime factorizations[Ana85]. Sule in [Sul94, Sul98] has presented a theory of the feedback stabilization of strictly causal plants for multi-input multi-output transfer matrices over commutative rings with some restrictions. This approach to the stabilization theory is called "coordinate-free approach" in the sense that the coprime factorizability of transfer matrices is not required. Recently, Mori and Abe in [MA01] have generalized his theory over commutative rings under the assumption that the plant is causal. They have introduced the notion of the generalized elementary factor, which is a generalization of the elementary factor introduced by Sule[Sul94], and have given the necessary and sufficient condition of the feedback stabilizability.

Since the stabilizing controllers are not unique in general, the choice of the stabilizing controllers is important for the resulting closed loop. In the classical case, that is, in the case where there exist the right-/left-coprime factorizations of the given plant, the stabilizing controllers can be parameterized by the method called "Youla-parameterization"[DLMS80, RL84, VSF82, YJB76] (also called Youla-Kučera-parameterization). However, we do not know yet whether or not there always exists right-/left-coprime factorizations of stabilizable plants of multidimensional systems[Lin98, Lin99, Lin00]. In this paper, we will give a parameterization of the stabilizing controllers *without* using the coprime factorizability of the plants.

2 Coordinate-Free Approach

First we briefly introduce the notion we use, that is, the coordinate-free approach.

Sule in [Sul94, Sul98] presented a theory of the feedback stabilization of multi-input multi-output strictly causal plants over commutative rings with some restrictions. This approach to the stabilization theory is called "coordinate-free approach"[SS92] in the sense that the coprime factorizability of transfer matrices is not required.

Let $\mathcal{R}$ denote an unspecified commutative ring. The total ring of fractions of $\mathcal{R}$ is denoted by $\mathcal{F}(\mathcal{R})$; that is, $\mathcal{F}(\mathcal{R}) = \{n/d \,|\, n, d \in \mathcal{R},\ d$: nonzerodivisor$\}$.

We consider that *the set of stable causal transfer functions* is a commutative ring. Throughout the paper, the set of stable causal transfer functions is denoted by $\mathcal{A}$. The total ring of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}(\mathcal{A})$ or simply $\mathcal{F}$; that is, $\mathcal{F}(\mathcal{A}) = \mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d$: nonzerodivisor$\}$. This is considered as *the set of all possible transfer functions*. The causality of transfer functions is an important physical constraint. We employ, in this paper, the definition of the causality from Vidyasagar *et al.*[VSF82, Definition 3.1].

Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$ such that $\mathcal{Z}$ includes all zerodivisors. Define the subsets $\mathcal{P}$ and $\mathcal{P}_{\rm S}$ of $\mathcal{F}(\mathcal{A})$ as follows:

$$\begin{aligned} \mathcal{P} &= \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A}\backslash\mathcal{Z}\}, \\ \mathcal{P}_{\rm S} &= \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A}\backslash\mathcal{Z}\}. \end{aligned}$$

Then every transfer function in $\mathcal{P}$ ($\mathcal{P}_{\rm S}$) is called *causal* (*strictly causal*). Analogously, if every entry of a transfer matrix is in $\mathcal{P}$ ($\mathcal{P}_{\rm S}$), the transfer matrix is called *causal* (*strictly causal*).

Matrices A and B over $\mathcal{R}$ are *right- (left-)coprime over* $\mathcal{R}$ if there exist matrices X and Y over $\mathcal{R}$ such that $XA + YB = E$ ($AX + BY = E$) holds. Further, an ordered pair (N, D) of matrices N and D over $\mathcal{R}$ is said to be a *right-coprime factorization over* $\mathcal{R}$ of P if (i) D is nonsingular over $\mathcal{R}$, (ii) $P = ND^{-1}$ over $\mathcal{F}(\mathcal{R})$, and (iii) N and D are right-coprime over $\mathcal{R}$. As the parallel notion, the *left-coprime factorization over* $\mathcal{R}$ of P is defined analogously.

Let $M_r(X)$ denote the $\mathcal{R}$-module generated by rows of a matrix X over $\mathcal{R}$. Let $X = AB^{-1} = \widetilde{B}^{-1}\widetilde{A}$ be a matrix over $\mathcal{F}(\mathcal{R})$, where A, B, $\widetilde{A}$, $\widetilde{B}$ are matrices over $\mathcal{R}$. It is known that $M_r([\,A^t \quad B^t\,]^t)$ ($M_r([\,\widetilde{A} \quad \widetilde{B}\,]^t)$) is unique up to an isomorphism with respect to any choice of fractions AB^{-1} of X ($\widetilde{B}^{-1}\widetilde{A}$ of X) (Lemma 2.1 of [MA01]). Therefore, for a matrix X over $\mathcal{R}$, we denote by $\mathcal{T}_{X,\mathcal{R}}$ and $\mathcal{W}_{X,\mathcal{R}}$ the modules $M_r([\,A^t \quad B^t\,]^t)$ and $M_r([\,\widetilde{A} \quad \widetilde{B}\,]^t)$, respectively.

The stabilization problem considered in this paper follows that of Sule in [Sul94], and Mori and Abe in [MA01], who consider the feedback system Σ [Vid85, Ch.5, Figure 5.1] as in Figure 1.

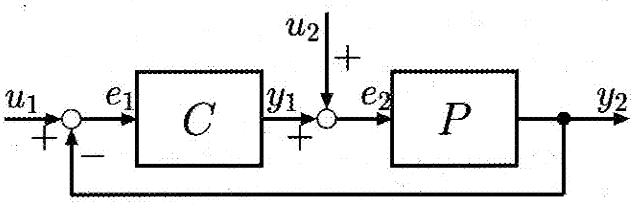

Figure 1: Feedback system Σ.

For further details the reader is referred to [MA01, Sul94, Vid85]. Throughout the paper, the plant we consider has m inputs and n outputs, and its transfer matrix, which is also called a *plant* itself simply, is denoted by P and belongs to $\mathcal{F}^{n\times m}$. We can always represent P in the form of a fraction $P = ND^{-1}$ ($P = \widetilde{D}^{-1}\widetilde{N}$), where $N \in \mathcal{A}^{n\times m}$ ($\widetilde{N} \in \mathcal{A}^{n\times m}$) and $D \in \mathcal{A}^{m\times m}$ ($\widetilde{D} \in \mathcal{A}^{n\times n}$) with nonsingular D ($\widetilde{D}$).

For $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$, a matrix $H(P, C) \in \mathcal{F}^{(m+n)\times(m+n)}$ is defined as

$$H(P, C) = \begin{bmatrix} (E_n + PC)^{-1} & -P(E_m + CP)^{-1} \\ C(E_n + PC)^{-1} & (E_m + CP)^{-1} \end{bmatrix} \tag{1}$$

provided that $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$. This $H(P, C)$ is the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,e_1^t \quad e_2^t\,]^t$ of the feedback system Σ. If (i) $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$ and (ii) $H(P, C) \in \mathcal{A}^{(m+n)\times(m+n)}$, then we say that the plant P is *stabilizable*, P is *stabilized* by C, and C is a *stabilizing controller* of P.

Another matrix $W(P, C) \in (\mathcal{F})_{m+n}$ provided that $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$ is also defined by

$$W(P, C) = \begin{bmatrix} C(E_n + PC)^{-1} & -CP(E_m + CP)^{-1} \\ PC(E_n + PC)^{-1} & P(E_m + CP)^{-1} \end{bmatrix}, \tag{2}$$

which is the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,y_1^t \quad y_2^t\,]^t$ of the feedback system Σ. It is well known that $H(P, C)$ is over $\mathcal{A}$ if and only if $W(P, C)$ is over $\mathcal{A}$.

In the definition above, we do not mention the causality of the stabilizing controller. However, it is known that if a causal plant is stabilizable, there exists a causal stabilizing controller of the plant[MA01].

The outline of new parameterization is as follows. Let P be a causal and stabilizable plant ($P \in \mathcal{P}^{n\times m}$). Hence there exists its stabilizing controller C of $\mathcal{F}^{m\times n}$. Then $\mathrm{Diag}(P, C)$ has a doubly coprime factorization. Hence we can parameterize the stabilizing controllers of the plant $\mathrm{Diag}(P, C)$ by the Youla-Kučera parameterization. From the stabilizing controllers of the plant $\mathrm{Diag}(P, C)$, we will obtain all the stabilizing controllers of the original plant P. Thus we show in Section 4e that a stabilized closed feedback system has the doubly coprime factorizability because this will lead to give the parameterization of the stabilizing controllers. Using this, we will present the method to parameterize all stabilizing controllers in Section 5.

3 Previous Result

The parameterization of stabilizing controllers that *does not require the coprime factorizability* of the plant were originally given in [Mor99b, Mor01b].

Here we briefly outline this parameterization. Let $H(P,C)$ denote the transfer matrix of the standard feedback system defined as

$$H(P,C) = \begin{bmatrix} (E_n+PC)^{-1} & -P(E_m+CP)^{-1} \\ C(E_n+PC)^{-1} & (E_m+CP)^{-1} \end{bmatrix},$$

where P and C are plant and controller, and E_x the identity matrix of size x (m and n denotes the number of inputs and outputs, respectively, of P). We consider the set $\mathcal{H}$ of $H(P,C)$'s with all stabilizing controllers C rather than the set of all stabilizing controllers itself. We have characterized this $\mathcal{H}$ by one parameter matrix. Once having the set $\mathcal{H}$, we can easily obtain the parameterization of the stabilizing controllers.

This set $\mathcal{H}$ and all stabilizing controllers are obtained as in the following way. Let H_0 be $H(P,C_0)$, where C_0 is a stabilizing controller of P. Let $\Omega(Q)$ be a matrix defined as follows:

$$\Omega(Q) = (H_0 - \begin{bmatrix} E_n & O_{n\times m} \\ O_{m\times n} & O_{m\times m} \end{bmatrix}) \qquad (3)$$
$$Q(H_0 - \begin{bmatrix} O_{n\times n} & O_{n\times m} \\ O_{m\times n} & E_m \end{bmatrix}) + H_0$$

with a stable causal and square matrix Q of size $m+n$, where $O_{x\times y}$ denotes zero matrix of size $x \times y$. Then we have the identity

$$\mathcal{H} = \{\Omega(Q) \mid Q \text{ is stable causal and } \Omega(Q) \text{ is nonsingular}\} \qquad (4)$$

(Theorems 4.2 and 4.3 of [Mor01b]). Hence any stabilizing controller has the following form:

$$-[\,O_{m\times n} \quad E_m\,]\,\Omega(Q)^{-1} \begin{bmatrix} E_n \\ O_{m\times n} \end{bmatrix}, \qquad (5)$$

provided that $\Omega(Q)$ is nonsingular. This is a parameterization of stabilizing controllers by parameter matrix Q *without the coprime factorizability of the plant.*

4 Doubly Coprime Factorizability of the Stabilized Closed Feedback System

We state here the key results of the new parameterization. Recall first the following result[MA01].

Proposition 4.1 (Proposition 4.1 of [MA01]) *Suppose that P and C are matrices over $\mathcal{F}$. Suppose further that* $\det(E_n + PC)$ *is a unit of $\mathcal{F}$. Then $\mathcal{T}_{H(P,C),\mathcal{A}} \simeq \mathcal{T}_{P,\mathcal{A}} \oplus \mathcal{T}_{C,\mathcal{A}}$ holds.*

If C is a stabilizing controller of P, then the matrix $H(P,C)$ is over $\mathcal{A}$ so that $\mathcal{T}_{H(P,C),\mathcal{A}}$ is free. Thus by Proposition 4.1, $\mathcal{T}_{P,\mathcal{A}} \oplus \mathcal{T}_{C,\mathcal{A}}$ is also free. This leads that the plant $\mathrm{Diag}(P,C)$ has a right coprime factorization over $\mathcal{A}$. From this observation, we give the following proposition.

Proposition 4.2 *Suppose that C_0 is a stabilizing controller of the plant P. Then $P_1 :=$ $\mathrm{Diag}(C_0,P)$ has both right- and left-coprime factorizations over $\mathcal{A}$.*

To prove this proposition, we use the following lemma.

Lemma 4.1 *Suppose that C_0 is a stabilizing controller of the plant P. Then*

$$C_1 := \begin{bmatrix} O & E_n \\ -E_m & O \end{bmatrix}$$

is a stabilizing controller of $P_1 :=$ $\mathrm{Diag}(C_0,P)$.

Proof. One can check straightforwardly that $H(P_1,C_1)$ is over $\mathcal{A}$. □

Proof of Proposition 4.2. Let C_1 be as in Lemma 4.1. Let

$$N_1 := \widetilde{N}_1 \qquad (6)$$
$$:= \begin{bmatrix} C_0(E_n+PC_0)^{-1} & -C_0P(E_m+C_0P)^{-1} \\ PC_0(E_n+PC_0)^{-1} & P(E_m+C_0P)^{-1} \end{bmatrix},$$
$$D_1 := \begin{bmatrix} (E_n+PC_0)^{-1} & -P(E_m+C_0P)^{-1} \\ C_0(E_n+PC_0)^{-1} & (E_m+C_0P)^{-1} \end{bmatrix}, \qquad (7)$$
$$\widetilde{D}_1 := \begin{bmatrix} (E_m+C_0P)^{-1} & -C_0(E_n+PC_0)^{-1} \\ P(E_m+C_0P)^{-1} & (E_n+PC_0)^{-1} \end{bmatrix}, \qquad (8)$$
$$Y_1 := \widetilde{Y}_1 := \begin{bmatrix} O & E_n \\ -E_m & O \end{bmatrix}, \qquad (9)$$
$$X_1 := \widetilde{X}_1 := \begin{bmatrix} E_n & O \\ O & E_m \end{bmatrix}. \qquad (10)$$

Then $P_1 = N_1 D_1^{-1} = \widetilde{D}_1^{-1}\widetilde{N}_1$ hold. Further $\widetilde{Y}_1 N_1 + \widetilde{X}_1 D_1 = E_{m+n}$, $\widetilde{N}_1 Y_1 + \widetilde{D}_1 X_1 = E_{m+n}$ hold. Hence (N_1, D_1) and $(\widetilde{D}_1, \widetilde{N}_1)$ are right- and left-coprime factorizations of P_1, respectively. □

As a derivative of Proposition 4.2, we have the following proposition, the proof of which is omitted.

Proposition 4.3 *A plant $P \in \mathcal{P}^{n\times m}$ is stabilizable if and only if there exists a transfer function $F \in \mathcal{F}^{m'\times n'}$ with $m' \le m$ and $n' \le n$ such that plant* $\mathrm{Diag}(F,P)$ *has both right- and left-coprime factorizations over $\mathcal{A}$.*

5 New Parameterization of Stabilizing Controllers

Now we give a parameterization of stabilizing controllers. Let P be a plant and C_0 a stabilizing controller. Consider a new plant $P_1 :=$ $\mathrm{Diag}(P,C_0)$ as shown in (a) of Figure 2. By Proposition 4.2, P_1 both right- and left-coprime factorizations over $\mathcal{A}$. We still use symbols in the equations (6) to (10). Then all stabilizing controllers, say C_1, of the plant P_1 is parameterized with parameter matrix $R, S \in (\mathcal{A})^{m+n}$ as follows:

$$
\begin{aligned}
C_1 &= (\widetilde{X}_1 - R\widetilde{N}_1)^{-1}(\widetilde{Y}_1 + R\widetilde{D}_1) && (11)\\
&= (Y_1 + N_1 S)(X_1 - D_1 S)^{-1}. && (12)
\end{aligned}
$$

Then the block diagram of the feedback system consisting of P_1 and C_1 is given in (b) of Figure 2. As in the figure, C_1 is decomposed as follows:

$$\begin{bmatrix} C_{111} & C_{112} \\ C_{121} & C_{122} \end{bmatrix} := C_1.$$

Consider now the following matrix

$$\begin{bmatrix} O_{m\times n} & E_m & O_{m\times m} & O_{m\times n} \\ O_{n\times n} & O_{n\times m} & O_{n\times m} & E_n \end{bmatrix} \times W(P_1, C_1) \begin{bmatrix} O_{m\times n} & O_{m\times m} \\ E_n & O_{n\times m} \\ O_{n\times n} & O_{n\times m} \\ O_{m\times n} & E_m \end{bmatrix}. \quad (13)$$

Note that this matrix is the transfer matrix from $[u_{12} \;\; u_{22}]^t$ to $[y_{12} \;\; y_{22}]^t$ in (c) of Figure 2. Let

$$C_{\text{New}} = C_{122} - C_{121}(E_m + C_0 C_{111})^{-1} C_0 C_{112}.$$

Using this C_{New}, (c) of Figure 2 can be rewritten as (d) of the figure. This is a feedback system of P and C_{New}. Hence C_{New} is a new stabilizing controller of the plant P. In addition, the matrix of (13) can be written as $W(P, C_{\text{New}})$.

Recall here that C_{New} depends on either the parameter matrix R in (11) or S in (12). Without loss of the generality, we use the parameter matrix R hereafter. Using R, we further calculate (13). Then we have

$$
\begin{aligned}
(13) &= W(P, C_{\text{New}}) \\
&= W(P, C_0) \\
&\quad + (W(P, C_0) + \begin{bmatrix} O & E_m \\ O & O \end{bmatrix}) \\
&\quad \times R(-W(P, C_0) + \begin{bmatrix} O & O \\ E_n & O \end{bmatrix}). \quad (14)
\end{aligned}
$$

We now have the main result of this paper.

Theorem 5.1 *Suppose that C_0 is a stabilizing controller of the plant P. Let*

$$
\begin{aligned}
\begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} &:= (W(P, C_0) + \begin{bmatrix} O & E_m \\ O & O \end{bmatrix}) \quad (15)\\
&\quad \times R(-W(P, C_0) + \begin{bmatrix} O & O \\ E_n & O \end{bmatrix}),
\end{aligned}
$$

where $W_{11} \in \mathcal{A}^{m\times n}$, $W_{12} \in (\mathcal{A})_m$, $W_{21} \in (\mathcal{A})_n$, $W_{22} \in \mathcal{A}^{n\times m}$ and R is a parameter matrix $\in (\mathcal{A})^{m+n}$. Then all stabilizing controller of the plant P is given by the following matrix

$$W_{11}(E_n - W_{21})^{-1} \quad (16)$$

with $(E_n - W_{21})$ nonsingular, where R in (15) is a parameter matrix.

Proof. From the previous discussion of this paper, we have known that the transfer matrix of (16) is a stabilizing controller. Thus we will show only that any stabilizing controller of the plant can have the form of (16).

Due to the space limitation, only the outline of the remaining proof is shown. The method of the proof is analogous to the proof of Theorem 3.3 of [MA01]. In the case where $\mathcal{A}$ is a local ring, there always exists a doubly coprime factorization of any stabilizable plant. Thus we use local-global principle[Kun85]. Let $\mathcal{A}_\mathfrak{p}$ denote a local ring of $\mathcal{A}$ at a prime ideal $\mathfrak{p}$. For each local ring $\mathcal{A}_\mathfrak{p}$, (14) still holds. Patching them, we see that (14) holds over $\mathcal{A}$. From this, we obtain a stabilizing controller of the form of (16). □

We now have a new parameterization of the stabilizing controllers. It can be checked that the new parameterization we have obtained is equivalent to the previous result given in Section 3. This is done by the translation of the matrices of (1) and (2), but the details are omitted.

6 Related Works

Recently the author in [Mor01a] has given the parameterization method of all stabilizing controllers which requires *only one* of right-/left-coprime factorizations. The relationship between the results of this paper and [Mor01a] should be investigated. The author considers that Proposition 4.3 may have some relation to the result of [Mor01a].

Also the author now investigates the method to reduce the number of parameters of the parameterization of all stabilizing controllers. This result will report in another materials.

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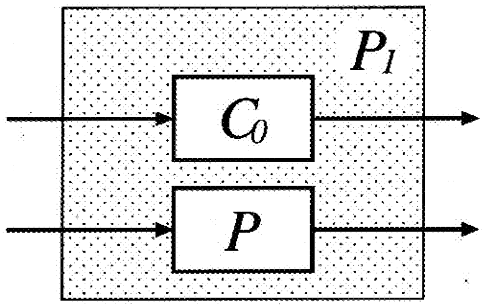
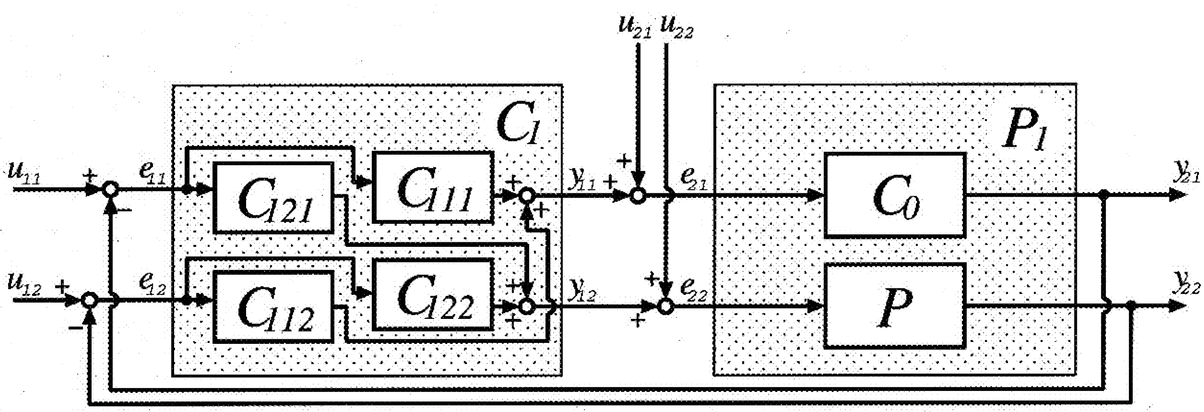
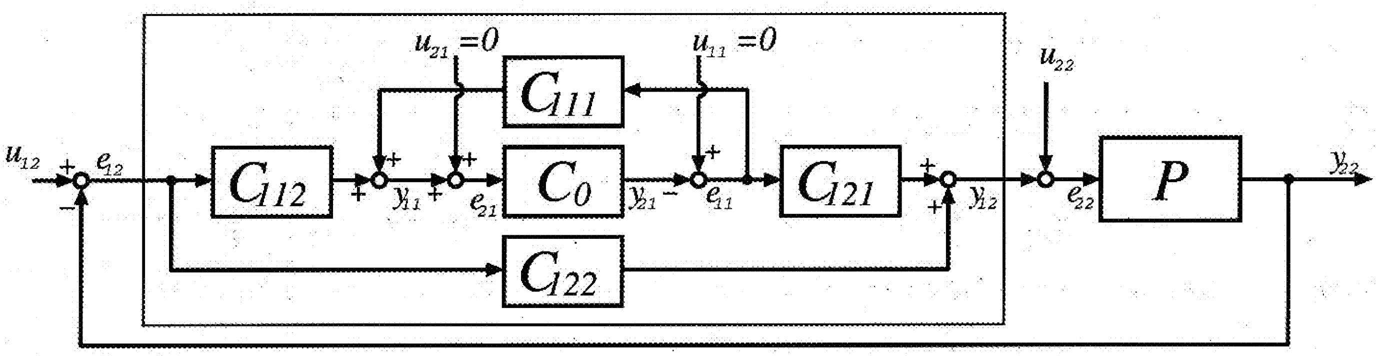


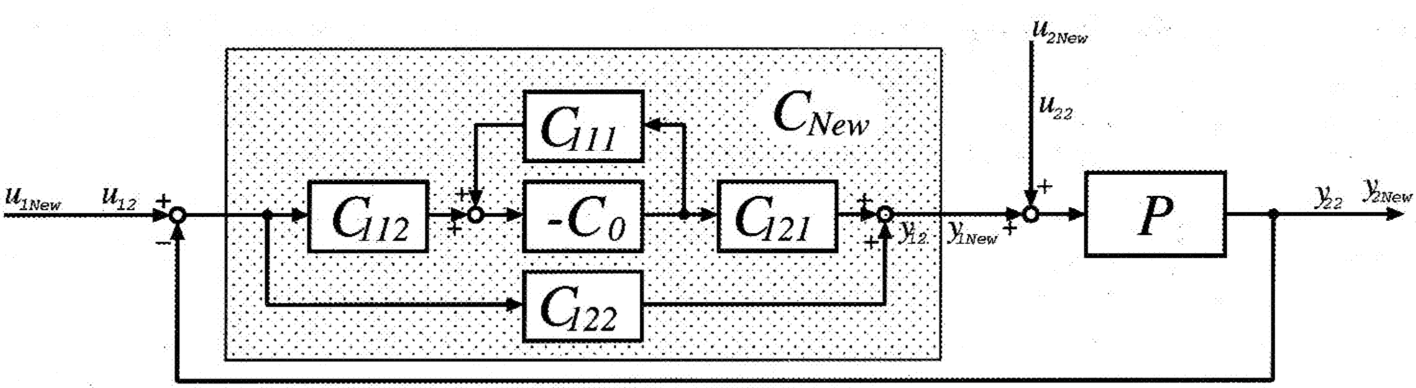
(a) New plant $P_1 := \mathrm{Diag}(P, C_0)$.



(b) New plant P_1 and its stabilizing controller C_1.



(c) Relocating the components of P_1 and C_1.



(d) Original plant P and its newly obtained stabilizing controller C_{New}.

Figure 2: Construction of a stabilizing controller of the plant.