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Redundancy Resolution Based on the Singularity-Consistent Method

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1. Introduction

Kinematically redundant manipulators have been studied for more than three decades [1], and numerous redundancy resolution methods and respective control laws have been proposed so far. Such manipulators are expected to be incorporated in autonomous robots for dextrous manipulation tasks and in teleoperators for application in space and other risky environments. A few undergoing projects are based on commercial seven-DOF manipulators such as the Robotics Research K-1207 arm [2], or the Mitsubishi PA-10 arm [3], well-known through research laboratories worldwide.

Most of the research solves the underdetermined redundancy problem by making use of a generalized inverse of the Jacobian, possibly in combination with a so-called self-motion term from the null space of the Jacobian, or the augmented Jacobian method [4]. Unfortunately, these methods deteriorate due to singularities, both kinematic ones and algorithmic ones, the latter being artificially introduced by the additional constraint used to resolve the redundancy. What is still needed is a reliable approach which avoids this major drawback of the existing algorithms.

The additional constraint for redundancy resolution determines the behavior to a great extent. For example, when the constraint is formulated in terms of velocities and is a non-integrable one, the behavior lacks cyclic path tracking ability, and the motion is unpredictable. This is the case when the redundancy is solved by means of the pseudoinverse, i.e. the joint velocities are being locally minimized [5]. Similarly, when the joint torque is locally minimized, unpredictable behavior in terms of joint velocities is observed. Therefore, it is desirable to avoid such local minimization constraints, and to employ instead a constraint in terms of positions, i.e. a holonomic constraint.

In the case of a seven-DOF anthropomorphic arm such as those mentioned above, a suitable constraint is the orientation of the arm plane determined by the centers of the shoulder, elbow and wrist joints [6), 7)]. A method for representing the constraint through a so-called "arm angle" is described in [2)]. The arm-angle equation is derived in terms of joint angles only and yields a predictable arm behavior. Unfortunately, the equation introduces an algorithmic singularity.

We have proposed a method for stable control at and around kinematic singularities of nonredundant manipulators, the so-called *singularity consistent* or SC method [8)]. The method has certain advantages over the well-known damped least-squares or DLS method for singularity treatment [9), 10)]. Its most important advantage is that no directional error is introduced. Recently, the SC method was applied to the Robotics Research K-1207 arm [11)]. This arm has non-zero joint offsets, and therefore the analytical expressions obtained are quite complex. This is the reason why no closed-form inverse kinematics solution has been found so far. The SC method has been implemented in terms of differential kinematics only, and hence, some of the interesting features of the method have been lost.

In this paper we will consider an anthropomorphic seven-DOF arm with zero joint offsets and kinematic decoupling, such as the Mitsubishi PA-10 arm. This allows us to derive a closed-form solution for the null space vector of the arm, and it becomes possible to apply the SC method within an analytic framework. The paper is organized as follows. First background about the SC approach and a numerical implementation is presented. In Section 3 we derive the main result. Section 4 presents a simulation study. Finally, in Section 5 our conclusions are presented.

2. Background

2.1 The SC approach applied to a non-redundant arm

The SC approach is applied to a non-redundant arm according to the following procedure [8)].

1) Denote the end-effector path as $\boldsymbol{x}(p)$ where p is the path parameter, not necessarily time. The path parameter augments the joint variables $\boldsymbol{q} \in \Re^n$: $\bar{\boldsymbol{q}} = (\boldsymbol{q}, p)$.
2) Derive the manipulator Jacobian $\boldsymbol{J}(\boldsymbol{q}) \in \Re^{n\times n}$ and form the *column-augmented* Jacobian:

$$\boldsymbol{H}(\bar{\boldsymbol{q}}) = \left[\begin{array}{cc} \boldsymbol{J}(\boldsymbol{q}) & -\boldsymbol{S}(p) \end{array} \right].$$

   $\boldsymbol{S}(p) \in \Re^m$ denotes the unit vector along the positive direction of the end-effector's path tangent, and $d\boldsymbol{x}(p) = dp\boldsymbol{S}(p)$, $dp = \|d\boldsymbol{x}(p)\|$, see[1] . Thus, the system is represented as a closed-loop system with one degree of redundancy:

$$\boldsymbol{H}(\bar{\boldsymbol{q}})d\bar{\boldsymbol{q}} = \boldsymbol{0}. \tag{1}$$

3) Find the solution to the last equation as

$$d\bar{\boldsymbol{q}} = b\bar{\boldsymbol{f}}(\bar{\boldsymbol{q}}) \tag{2}$$

   where b is any small scalar, and $\bar{\boldsymbol{f}}(\bar{\boldsymbol{q}}) \in \ker\boldsymbol{H}(\bar{\boldsymbol{q}}) \subset \Re^{n+1}$.

The last equation represents an autonomous dynamical system, parameterized by the vector $\boldsymbol{S}(p)$. It is important to note that the vector field $\bar{\boldsymbol{f}}(\bar{\boldsymbol{q}})$ should *not* be derived as a function of the pseudoinverse $\boldsymbol{H}^+$, e.g. as a vector from the null-space projector $(\boldsymbol{I} - \boldsymbol{H}^+\boldsymbol{H})$. Instead, we derive each entry as $\bar{f}_i = (-1)^{i+1}\det\boldsymbol{H}_i,\ i = 1, ..., n+1$, where

[1] dp can be assumed to be dimensionless end-effector "speed." In such case, the units of the elements of $\boldsymbol{S}(p)$ match those of $d\boldsymbol{x}(p)$.

$\boldsymbol{H}_i$ stands for $\boldsymbol{H}$ with its i-th column removed [13], [14]. The advantage of this derivation is that it yields an inherently stable behavior around critical points when any of the determinants approaches zero. This includes the case of a kinematic singularity, i.e. $\det \boldsymbol{H}_{n+1} \to 0$.

2.2 The SC approach applied to a seven-DOF arm in terms of differential kinematics

Theoretically, when applying the SC method to a n-DOF kinematically redundant arm, the following two steps are envisioned. First, augument the m-dimensional task-space constraint (i.e. the end-effector motion constraint) by $n-m$ additional constraints. This technique is well-known as the "augumented task-space" approach [4]. Second, proceed with the SC method as if the system is a non-redundant one. The algorithmic singularities due to the additional constraints will be treated thereby as kinematic singularities. It turns out, however, that from a practical viewpoint the solution is not as straigtforward because of the complexity of the system after the augumentation. Even with just a single degeree of redundancy it might be impossible to derive a closed-form solution and to apply the SC method to its full extent. Therefore, in [11] a numerical approach has been proposed. The arm angle was employed as an additional constraint.

3. The SC approach applied to a seven-DOF arm with kinematic decoupling property

Let us consider an antropomorphic seven-DOF arm with zero joint offsets and kinematic decoupling property, such that the three-DOF spherical wrist can be treated independently from the four-DOF positioning arm. The latter is displayed in Figure 1. We note that the Mitsubishi PA-10 arm has such a structure. We will focus from now on the positioning arm, since it contains the redundancy.

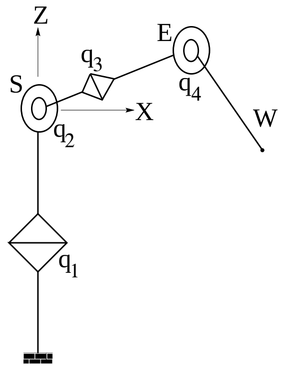

Fig. 1 The four-DOF positioning arm.

The direct kinematics of the manipulator arm is solved based on the Denavit and Hartenberg notation presented in [15]. Upon differentation, we obtain the Jacobian as follows:

$$j_{11}(\boldsymbol{q}) = d_3 S_1 S_2 + d_5(C_4 S_2 + (C_2 C_3 S_1 + C_1 S_3) S_4)$$
$$j_{12}(\boldsymbol{q}) = -d_3 C_1 C_2 + d_5(C_1 C_3 S_2 S_4 - C_1 C_2 C_4)$$
$$j_{13}(\boldsymbol{q}) = d_5(C_3 S_1 + C_1 C_2 S_3) S_4$$
$$j_{14}(\boldsymbol{q}) = d_5(C_4(S_1 S_3 - C_1 C_2 C_3) + C_1 S_2 S_4)$$
$$j_{21}(\boldsymbol{q}) =$$
$$-d_3 C_1 S_2 + d_5((S_1 S_3 - C_1 C_2 C_3) S_4 - C_1 C_4 S_2)$$
$$j_{22}(\boldsymbol{q}) = -d_3 C_2 S_1 - d_5(C_2 C_4 S_1 - C_3 S_1 S_2 S_4)$$
$$j_{23}(\boldsymbol{q}) = -d_5(C_1 C_3 - C_2 S_1 S_3) S_4$$
$$j_{24}(\boldsymbol{q}) = -d_5(C_4(C_2 C_3 S_1 + C_1 S_3) - S_1 S_2 S_4)$$
$$j_{31}(\boldsymbol{q}) = 0$$
$$j_{32}(\boldsymbol{q}) = -d_3 S_2 - d_5(C_4 S_2 + C_2 C_3 S_4)$$
$$j_{33}(\boldsymbol{q}) = d_5 S_2 S_3 S_4$$
$$j_{34}(\boldsymbol{q}) = -d_5(C_3 C_4 S_2 + C_2 S_4)$$

where d_3 and d_5 denote link lengths, C_i and S_i stand for $\cos q_i$ and $\sin q_i$, respectively.

There are two types of kinematic singularities: (1) extended/folded-arm singularity ($q_4 = 0, \pm\pi$), and (2) shoulder singularity, when the end-point is on the Z axis.

Denote by $\boldsymbol{v} \in \Re^3$ the (desired) wrist center point velocity, and by $\hat{\boldsymbol{v}}$ the respective unit vector. This vector will be used to parameterize the dynamical system as explained in Subsection 2.1. We adjoint $-\hat{\boldsymbol{v}}$ to the Jacobian, to obtain the 3×5 column-augmented Jacobian. The solution of the respective homogeneous equation (cf. Eq. 1) is obtained as

$$d\bar{\boldsymbol{q}} = b_{sm}\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}}) + b_{ep}\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}}) \tag{3}$$

where b_{sm} and b_{ep} are arbitrary scalars and $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ and $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ are vector fields from the kernel of $\boldsymbol{H}(\bar{\boldsymbol{q}})$. b_{sm} and b_{ep} are considered as design parameters which determine the speed along the two vector fields. The vector fields we obtained in analytical form using Mathematica. $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ has the following components:

$$\begin{aligned}
\bar{f}_{sm1}(\bar{\boldsymbol{q}}) &= -d_5C_3S_4^2/c\\
\bar{f}_{sm2}(\bar{\boldsymbol{q}}) &= d_5S_2S_3S_4^2/c\\
\bar{f}_{sm3}(\bar{\boldsymbol{q}}) &= S_4\\
\bar{f}_{sm4}(\bar{\boldsymbol{q}}) &= 0\\
\bar{f}_{sm5}(\bar{\boldsymbol{q}}) &= 0
\end{aligned}$$

where $c = (d_3 + d_5C_4)S_2 + d_5C_2C_3S_4$. $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ determines the self-motion of the manipulator since $\bar{f}_{sm5}(\bar{\boldsymbol{q}})$ is identically zero. As expected, the joint four velocity is also always zero during self-motion of the arm. Further, it is apparent that whenever the arm is at the extended/folded arm singularity, no self-motion will be performed since S_4 and hence the entire self-motion vector field vanishes. This matches the physical condition and shows the appropriateness of the expression.

Next, let us consider the c term. This term vanishes at the shoulder singularity. Since it appears in the denominator of the first two components, we can expect problems around the shoulder singularity, if self-motion via $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ is attempted. On the other hand, note that self-motion at the shoulder singularity means arm-plane rotation around the vertical Z axis, i.e. joint one rotation. Therefore, we do not use the above vector field in the neighborhood of the shoulder singularity.

The other vector field $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ has some lengthy expressions for three of its components, which we will omit here. The exceptions are the third component which is identically zero, and the fifth component which is S_4 and hence, non-zero, except at the extended/folded arm kinematic singularity. The latter shows that $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ contributes to the end-point (or wrist center-point) motion, and at the extended/folded arm singularity the end-point will be instantaneously at rest (the arm is in the state of instantaneous self-motion) thus complying with the physical motion constraint at this configuration. This demonstrates again the appropriateness of the expressions we derived for the Jacobian null space.

The third component of $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ being identically zero is a somewhat surprising result. Note that the q_3 angle determines the orientation of the arm plane, and whenever this joint is immobilized, the orientation will be maintained. Thus, we obtained a nice decoupling property: with $b_{sm} = 0$, $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ will drive the wrist center-point in the desired direction of motion without changing the arm-plane angle; with $b_{ep} = 0$, $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ will drive the self-motion of the arm. Hence, effective arm motion control will be possible by proper blending of b_{sm} and b_{ep}.

There is one exception, when the arm is at the shoulder singularity. It is intuitively clear that joint three motion has no contribution to self-motion in this case. As already mentioned above, self-motion would imply joint one rotation only, which can be achieved via the $\bar{\boldsymbol{f}}_{ep}(\bar{\boldsymbol{q}})$ vector field in combination with a proper command direction, i.e. $\hat{\boldsymbol{v}}$ being out of the arm plane. The vector field must be modified thereby[2] in order to avoid the influence of the term c apperaring as denominator in the first two components.

We would like to emphasize that no explicit redundancy resolution criterion is used. Nevertheless, we obtained effective arm-plane angle control. Of course, no additional constraint means also no algorithmic singularities. This is a significant advantage of the method compared with other redundancy resolution schemes.

4. A simulation study

We will demonstrate the performance of the method by means of two simulations. The link lengths are $d_3 = d_5 = 1$ m. The desired end-point motion is along a horizontal straight-line, with $\hat{\boldsymbol{v}} = (1, 0, 0)$. The b's are constant, i.e. natural motion is performed and hence the initial and final velocity is non-zero [8)]. Note that natural motion is quite suitable for analysis since it satisfies the specific end-point motion constraint at a singularity.

In the first simulation, the initial joint angles are $(0, 0, 0, -90)$ deg. This yields initial wrist-center location at $(1, 0, 1)$ m. $b_{ep} = 1$, while b_{sm} is set to zero. This means that the arm-angle will be maintained (no self-motion). The result is shown in Figure 2. From the end-point position graph it is apparent that after approx. 1 s the arm moves smoothly through the extended-arm singularity at the workspace boundary. Motion is reversed and continues along the straight-line to cross the shoulder singularity (when $x = 0$). Later on, the arm is once again fully extended and moves through the extended-arm singularity on the opposite side. Motion continues in a cyclic manner.

In the second simulation we set $b_{ep} = 1$ and $b_{sm} = 1$ which yields end-point motion and self-motion with equal weight. Also, we change the initial joint angles as $(0, 0, 60, -90)$ deg in order to avoid the shoulder singularity. As already noted, self-motion (via the $\bar{\boldsymbol{f}}_{sm}(\bar{\boldsymbol{q}})$ vector field) at this singularity is physically impossible, so no attempt should be made to approach it. The result is shown in Figure 3. It becomes apparent that the extended-arm singularity is reached and crossed (end-point motion is reversed) without any problem. This clearly shows that the arm-plane motion (self-motion) can be superimposed even at the extended/folded arm singularity without deteriorating the smoothness.

5. Conclusion

We have presented a new and efficient method for redundancy resolution for a seven-DOF arm with zero joint-offsets and kinematic decoupling, envisioning the Mitsubishi PA-10 arm. The method is efficient in the sense that (1) no additional singularities are introduced and (2) motion does not deteriorate around kinematic singularities. The effective redundancy resolution scheme is based on arm-plane rotation control which is an intuitive and practically valuable scheme. In a future work it would be straightforward to derive a feedback con-

[2] e.g. by choosing b_{ep} to be in proportion to c.

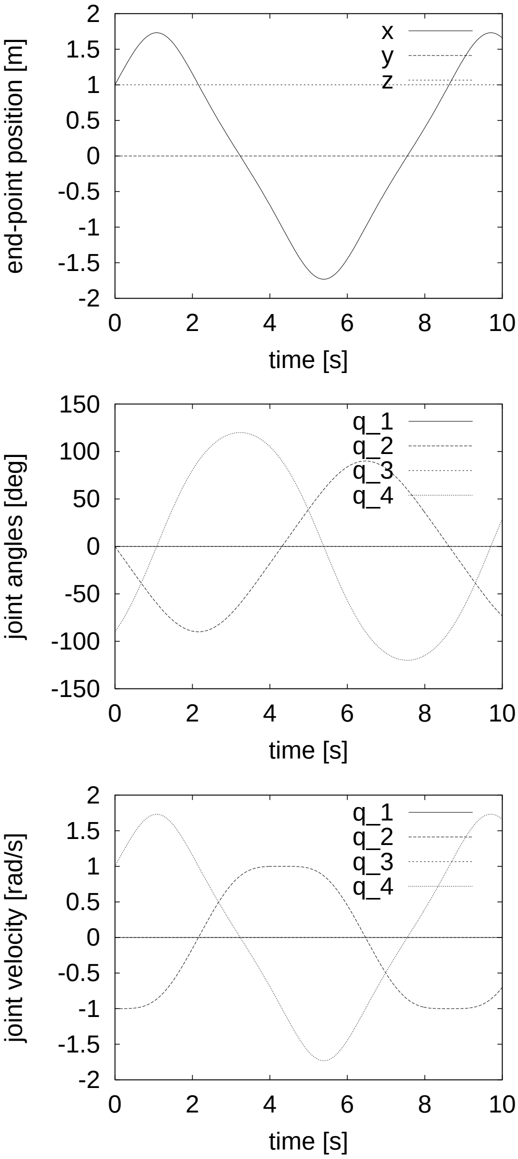

Fig. 2 Straight-line motion with constant arm-angle.

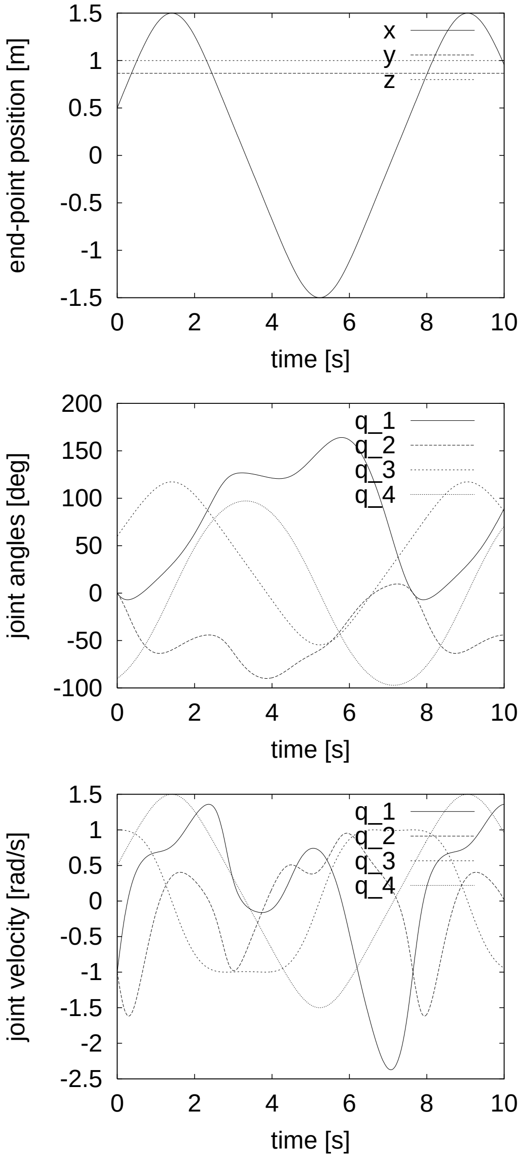

Fig. 3 Straight-line motion with varying arm-angle.

troller based on the scheme, following a procedure similar to the one we proposed earlier for a non-redundant arm.

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