計測自動制御学会東北支部 第210回研究集会 (2003.7.16)
資料番号 210-1

μ計算法による多次元システムのロバスト安定性解析

A μ Approach to Robust Stability Analysis of nD Discrete-time Systems

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キーワード： 多次元システム (multidimensional systems), 安定性 (stability), 安定性余裕 (stability margin), 構造化特異値 (structured singular value),

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1. Introduction

Stability is undoubtedly the most important requirement for an multidimensional (nD) filter or system. To ensure satisfactory performance of a stable nD system, it is also often necessary to know how stable, or how far away from being unstable the system is [3]. The stability analysis problem for 2D systems, including various stability criterions and corresponding tests as well as the derivation and computation of different kinds of stability margins, has been well investigated and documented in the literature (see, e.g., [2,3,6,13,17,19]). For the nD ($n > 2$) case, however, due to the difficulties growing with the number of dimensions n, though considerable results for nD stability conditions have been obtained, only rather limited results have been reported for both nD stability test and nD stability margin computation and a lot of difficulties still remain to be challenged (see, e.g., [7–10,12,18,20,21]).

The purpose of this paper is to tackle, by utilizing μ analysis approach, the stability analysis problem for nD systems characterized by state-space models. It is shown that the problems of stability test and stability margin computation for an nD system described by Roesser model can be recast into a set of μ analysis problems in a unified way, thus can be solved effectively by using the commercially available software package [5]. In particular, the relation between the structured singular value $\mu(A)$, with A being the state matrix of an nD Roesser model, and the stability and several kinds of stability margins of the nD system characterized by A is clarified, and methods for computation of these stability margins are given.

The paper is organized as follows. Section 2 gives some preliminaries for the stability of nD systems described by Roesser model, and a short overview on the μ analysis. In Section 3, as the main results of the paper, several kinds of nD stability margins are defined in a precise way and it is show how the problems of nD stability test and stability margin computation can be accomplished in a μ analysis setting. Finally, the concluding remarks are given in Section 4.

2. Preliminaries

Roesser state-space model for an MIMO nD system [?] is given by

$$\begin{cases} \boldsymbol{x}'(i_1,\ldots,i_n) = A\boldsymbol{x}(i_1,\ldots,i_n) + B\boldsymbol{u}(i_1,\ldots,i_n) \\ \boldsymbol{y}(i_1,\ldots,i_n) = C\boldsymbol{x}(i_1,\ldots,i_n)+D\boldsymbol{u}(i_1,\ldots,i_n) \end{cases} \quad (1)$$

where $\boldsymbol{u}(i_1,\ldots,i_n) \in \mathcal{R}^r$ and $\boldsymbol{y}(i_1,\ldots,i_n) \in \mathcal{R}^l$ are the input and output vectors, respectively; $\boldsymbol{x}(i_1,\ldots,i_n) \in \mathcal{R}^m$ is the local state vector in the form

$$\boldsymbol{x}(i_1,\ldots,i_n) = \begin{bmatrix} \boldsymbol{x}_1(i_1,\ldots,i_n) \\ \boldsymbol{x}_2(i_1,\ldots,i_n) \\ \vdots \\ \boldsymbol{x}_n(i_1,\ldots,i_n) \end{bmatrix},$$

$$\boldsymbol{x}'(i_1,i_2,\ldots,i_n) = \begin{bmatrix} \boldsymbol{x}_1(i_1+1,i_2,\cdots,i_n) \\ \boldsymbol{x}_2(i_1,i_2+1,\cdots,i_n) \\ \vdots \\ \boldsymbol{x}_n(i_1,i_2,\cdots,i_n+1) \end{bmatrix}$$

with $\boldsymbol{x}_j(i_1,\ldots,i_n) \in \mathcal{R}^{m_j}$ ($j = 1,\ldots,n$, $m = \sum_{j=1}^n m_j$) being the jth (sub-)state vector of $\boldsymbol{x}(i_1,\ldots,i_n)$; and

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} \quad B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

$$C = [C_1 \; C_2 \; \cdots \; C_n]$$

with A_{jk}, B_j, C_j and D being constant matrices of suitable dimensions, particularly, $A_{jj} \in \mathcal{R}^{m_j \times m_j}$.

The transfer function matrix of the nD system (1) is given by

$$G(z) = D + CZ_n(I - AZ_n)^{-1}B \quad (2)$$

where $z = [\,z_1 \;\; z_2 \;\; \ldots \;\; z_n\,]^T \in \mathcal{C}^n$,

$$Z_n = \text{blockdiag}\{\, z_1 I_{m_1} \,,\, z_2 I_{m_2} \,,\, \ldots \,,\, z_n I_{m_n} \,\}$$

The system is then said to be (structurally) stable [12] iff the nD characteristic polynomial $d(z)$ satisfies the condition $d(z) \triangleq \det(I - AZ_n) \neq 0 \;\; \forall z \in \bar{U}^n$, where

$$\bar{U}^n = \{\, z \in \mathcal{C}^n \,:\, |z_1| \leq 1, \, |z_2| \leq 1, \, \ldots, \, |z_n| \leq 1 \,\}.$$

Now, let

$$\boldsymbol{\Delta} = \left\{ \begin{array}{l} \Delta = \text{blockdiag}\{\delta_1 I_{m_1}, \delta_2 I_{m_2}, \ldots, \delta_n I_{m_n}\} : \\ \delta_j \in \mathcal{R} \text{ for } j = 1,2,\ldots,k \text{ and} \\ \delta_j \in \mathcal{C} \text{ for } j = k+1, k+2, \ldots, n. \end{array} \right\} \quad (3)$$

Then the real/complex structured singular value of a matrix M with respect to the given structure (3), denoted by $\mu_\Delta(M)$, is defined as

$$\mu_\Delta(M) = \begin{cases} 0, \;\; \text{if } \det(I - M\Delta) \neq 0 \;\; \forall \Delta \\ (\min\{\, \bar{\sigma}(\Delta) \;:\; \det(I - M\Delta) = 0\,\})^{-1}, \\ \qquad \text{otherwise} \end{cases} \quad (4)$$

Note that the structured singular value $\mu_\Delta(M)$ is always defined and calculated with respect to a specified structure Δ. Real uncertainty blocks $\delta_j I_{m_j}$ appear only when real uncertainties, e.g. parameter uncertainties, come into the system.

The structured singular value μ has the following property [11]

$$\max_{U \in \mathcal{U}} \bar{\sigma}(MU) \leq \mu_\Delta(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}) \quad (5)$$

where

$$\mathcal{D} = \{\text{blockdiag}\{D_1, D_2, \ldots, D_n\} : D_j \in \mathcal{C}^{m_j \times m_j}, D_j = D_j^* > 0\,\}$$

$$\mathcal{U} = \{\, U \in \boldsymbol{\Delta} \;:\; UU^* = I_m \,\}$$

3. The Main Results

It is clear that every component of Z_n which plays a similar role as Δ can be either real or complex. Therefore, to solve the nD stability problem using the μ methodology, it suffices to define the complex diagonal matrices

$$\mathbf{\Delta} = \left\{ \begin{array}{r} \Delta = \text{blockdiag}\{\delta_1 I_{m_1}, \delta_2 I_{m_2}, \ldots, \delta_n I_{m_n}\} : \\ \delta_j \in \mathcal{C} \end{array} \right\} \quad (6)$$

Equipped with these notations we can now state the result on the stability of the system (1).

Lemma 1 [14,22] *The nD discrete-time system (1) is stable iff $\mu_{\mathbf{\Delta}}(A) < 1$, where $\mathbf{\Delta}$ is given in (6).*

Based on Lemma 1 and the inequality (5), a Lyapunov inequality can be readily established for testing the stability of an nD system.

Lemma 2 *The nD discrete-time system is stable if there exists a blockdiagonal positive definite matrix $X = \text{blockdiag}\{X_1, X_2, \ldots, \ldots, X_n\}$ such that*

$$A^* X A - X < 0$$

Proof. Follows directly from $\mu_{\mathbf{\Delta}}(A) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DAD^{-1})$ and on noting that $X \triangleq D^* D$ is a block diagonal positive definite matrix. ■

This result in fact coincides with the Lyapunov equation condition obtained in [1]. Though solution algorithms for 2D Lyapunov equation have been proposed in [4], no such algorithm exists for nD ($n > 2$) case to the knowledge of the authors. However, by using the μ-toolbox of MATLAB [5], one can easily find the solution X to the nD Lyapunov equation or inequation.

We consider now the stability margins of A. For a stable A, the *individual stability margin* σ_j, $j = 1, 2, \ldots, n$, of the jth dimension, i.e. with respect to the indeterminant z_j is defined as

$$\sigma_j = \sup \left\{ \begin{array}{l} s_j \ : \ d(\hat{z}) \neq 0 \\ \forall\ |\hat{z}_k| \leq 1, k = 1, \ldots, n, k \neq j \\ \text{and } |\hat{z}_j| \leq 1 + s_j \end{array} \right\} \quad (7)$$

It is clear that for a stable system σ_j is a positive number. In the similar way, we can define the *joint stability margin* $\sigma_{j_1, j_2, \ldots, j_q}$, $\{j_1, j_2, \cdots, j_q\} \subset \{1, 2, \ldots, n\}$, of the indeterminants z_{j_k}, $k = 1, 2, \ldots, q$

$$\sigma_{j_1, j_2, \ldots, j_q} = \sup \left\{ \begin{array}{l} s_{j_1, j_2, \ldots, j_q} \ : \ d(\hat{z}) \neq 0 \\ \forall\ |\hat{z}_k| \leq 1, k \notin \{j_1, j_2, \cdots, j_q\}, \\ \text{and } |\hat{z}_k| \leq 1 + s_{j_1, j_2, \ldots, j_q}, \\ k \in \{j_1, j_2, \cdots, j_q\} \end{array} \right\} \quad (8)$$

Note that, by the definition, $\sigma_{j_1, j_2, \ldots, j_q}$ is independent of the order of $j_1, j_2, \cdots, j_q$. The joint stability margin $\sigma_{1,2,\ldots,n}$ of all the indeterminants will be called the *total stability margin* of the system and is denoted by σ for notational simplicity. Obviously,

$$\sigma = \sup \{ s : d(\hat{z}) \neq 0\ \forall |\hat{z}_j| \leq 1 + s, j = 1, 2, \ldots, n \} \quad (9)$$

The purpose of the rest of this section is to give a small μ test for finding out (exactly) the stability margins. We begin with the calculation of σ_j and then come to the general joint stability margins $\sigma_{j_1, j_2, \ldots, j_q}$ and the total stability margin of the system.

It is clear that σ_j can be equally represented as

$$\sigma_j = \sup \left\{ \begin{array}{l} s_j \ : \ d(\hat{z}) \neq 0 \\ \forall\ |\hat{z}_k| \leq 1, k = 1, \ldots, n, k \neq j \\ \text{and } |(1 + s_j)^{-1} \hat{z}_j| \leq 1 \end{array} \right\} \quad (10)$$

To determine σ_j, we introduce the selector E_j which is the jth block column of the identity matrix having the same block dimensions as the matrix A. It is clear that premultiplying A by E_j^*, the transpose of E_j, yields the jth block row of A

$$E_j^* A = [A_{j1}\ A_{j2}\ \ldots\ A_{jn}] \triangleq A_{r,j}$$

and similarly, post-multiplying A by E_j yields the jth block column of A denoted $A_{c,i}$, that is why we call E_j a selector. Also, premultiplying E_j^* by E_j yields the matrix

$$E_j E_j^* \triangleq S_{r,j} = \begin{bmatrix} 0 \\ E_j^* \\ 0 \end{bmatrix}, \text{ and } S_{r,j}A = \begin{bmatrix} 0 \\ A_{r,j} \\ 0 \end{bmatrix}, \tag{11}$$

where the zeros above E_j^* and $A_{r,j}$ stand for $(\sum_{k=1}^{j-1} m_k) \times m$ zero matrices, whereas those below E_j^* and $A_{r,j}$ stand for $(\sum_{k=j+1}^{n} m_k) \times m$ zero matrices. Similarly,

$$E_i E_i^* \triangleq S_{c,i} = [0 \;\; E_i \;\; 0], \text{ and } AS_{c,i} = [0 \;\; A_{c,i} \;\; 0] \tag{12}$$

Furthermore, we introduce the matrices

$$A_{\beta,r} \triangleq \begin{bmatrix} \beta_1 A_{11} & \beta_1 A_{12} & \dots & \beta_1 A_{1n} \\ \beta_2 A_{21} & \beta_2 A_{22} & \dots & \beta_2 A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_n A_{n1} & \beta_n A_{n2} & \dots & \beta_n A_{nn} \end{bmatrix} \tag{13}$$

$$A_{\beta,c} \triangleq \begin{bmatrix} \beta_1 A_{11} & \beta_2 A_{12} & \dots & \beta_n A_{1n} \\ \beta_1 A_{21} & \beta_2 A_{22} & \dots & \beta_n A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1 A_{n1} & \beta_2 A_{n2} & \dots & \beta_n A_{nn} \end{bmatrix} \tag{14}$$

then, from $\det(I - UV) = \det(I - VU)$ we get

$$\begin{aligned} d_\beta(z) &\triangleq \det(I - A_{\beta,c}Z_n) = \det(I - AZ_{n,\beta}) \\ &= \det(I - Z_{n,\beta}A) = \det(I - Z_n A_{\beta,r}) \\ &= d(z_\beta) \end{aligned} \tag{15}$$

where

$$\begin{aligned} &z_\beta \triangleq [\beta_1 z_1 \; \beta_2 z_2 \; \dots \; \beta_n z_n]^T \\ &Z_{n,\beta} = \text{blockdiag}\{\beta_1 z_1 I_{m_1}, \beta_2 z_2 I_{m_2}, \dots \beta_n z_n I_{m_n}\} \end{aligned} \tag{16}$$

Hence, $[\,\hat{z}_1 \; \hat{z}_2 \; \dots \; \hat{z}_n\,]^T$ is a root of $d(z)$ iff

$$[\,z_1 \; z_2 \; \dots \; z_n\,]^T = [\,\beta_1^{-1}\hat{z}_1 \; \beta_2^{-1}\hat{z}_2 \; \dots \; \beta_n^{-1}\hat{z}_n\,]^T$$

is a root of $d_\beta(z)$. $d_\beta(z)$ will have smaller and smaller roots by increasing $\beta_j > 1$. Hence, to determine the stability margin σ_j, we fix all $\beta_k = 1$ $k = 1, 2, \dots, j-1, j+1, \dots, n$ but increase β_j from 1 to a certain positive number $\beta_{j,0}$ to be specified below. Since $[\,\hat{z}_1 \; \hat{z}_2 \; \dots \; \hat{z}_n\,]^T$ is a root of $d(z)$ iff

$$[\,z_1 \; z_2 \; \dots \; z_n\,]^T = [\,\hat{z}_1 \; \dots \; \hat{z}_{j-1} \; \beta_j^{-1}\hat{z}_j \; \hat{z}_{j+1} \; \dots \; \hat{z}_n\,]^T$$

is a root of $d_{\beta_j}(z) = \det(I - A_{\beta_j,r}Z_n)$ where

$$A_{\beta_j,r} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{j-1,1} & A_{j-1,2} & \dots & A_{j-1,n} \\ \beta_j A_{j1} & \beta_j A_{j2} & \dots & \beta_j A_{jn} \\ A_{j+1,1} & A_{j+1,2} & \dots & A_{j+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix}, \tag{17}$$

from equation (10) we readily realize that $\sigma_j = \beta_{j,0} - 1$ where $\beta_{j,0}$ is the minimum positive number of $\beta_j > 1$ such that

$$\min\{\max\{|z_k|\} \; : \; \det(I - A_{\beta_{j,0},r}Z_n) = 0\} = 1 \tag{18}$$

The following theorem characterizes the stability margin σ_j in terms of the structured singular value of a certain matrix $M_j(w)$.

Theorem 1 *The single stability margin σ_j of the system (2) is given by*

$$\sigma_j = \inf\{\, w > 0 \; : \; \mu_{\Delta_j}(M_j(w)) \geq 1\,\}$$

where

$$\begin{aligned} M_j(w) &= \begin{bmatrix} A & E_j \\ wE_j^*A & 0 \end{bmatrix} \\ \mathbf{\Delta}_j &= \{\Delta_j = \text{blockdiag}\{\, Z_n, \delta_j I_{n_j}\,\}, \, \delta_j \in \mathcal{R}\} \end{aligned} \tag{19}$$

Proof. For notational simplicity, we drop w and denote $M_j(w)$ simply by M_j. Using the Schur formula of the determinant we can easily show that for $\Delta_j \in \mathbf{\Delta}_j$

$$\begin{aligned} &\det(I - M_j\Delta_j) \\ &= \det\left(\begin{bmatrix} I - AZ_n & -E_j\delta_j \\ -wE_j^*AZ_n & I \end{bmatrix}\right) \\ &= \det(I - (A + E_j(\delta_j w)E_j^*A)Z_n) \end{aligned}$$

From the definition of $\mu_{\Delta_j}(M_j)$ it follows that

$$\mu_{\Delta_j}(M_j) < 1, \iff \mu_{\Delta_j}^{-1}(M_j) > 1$$
$$\iff \det(I - (A + E_j\delta_j' E_j^* A)Z_n) \neq 0$$
$$\forall\ |z_k| \leq 1 \text{ and } -w \leq \delta_j' \leq w$$

where $\delta_j' = \delta_j w$. From (11) and (17) we have

$$A + E_j\delta_j' E_j^* A = A_{\beta_j',r}$$

where $\beta_j' = 1 + \delta_j'$ with $-w \leq \delta_j' \leq w$.

To complete the proof we show in the following that $A_{\beta_j,r}$ is stable for all $1 \leq \beta_j \leq \beta_{j,0}$ iff it is stable for all $\beta_j' = 1 + \delta_j'$ with $-(\beta_{j,0} - 1) \leq \delta_j' \leq (\beta_{j,0} - 1)$. Indeed, Since $[1\ ,\ \beta_{j,0}] \subset [2 - \beta_{j,0}\ ,\ \beta_{j,0}]$ $\forall \beta_{j,0} \geq 1$, the "if" part is obvious. To show the "only if" part, we make first the claim that A is stable iff $A_{\beta,r}$ and $A_{\beta,c}$ are stable for all

$$\beta = [\pm 1 \quad \pm 1 \quad \ldots \quad \pm 1]^T.$$

The second claim we make is that A is stable iff $A_{\beta,r}$ and $A_{\beta,c}$ are stable for all β with $\|\beta\|_\infty \leq 1$. We consider now the "only if" part. It is clear that for every $\beta_{j,0} > 1$ there exists a $\delta_j' > 0$ such that $\beta_j = 1 + \delta_j'$. Hence, we show only that the stability of $A_{\beta_j,r}$ with $\beta_j = 1 + \delta_j' > 1$ implies also the stability of $A_{\beta_j',r}$ with $\beta_j' = 1 - \delta_j'$. Indeed, when $0 < \delta_j' \leq 1$, $\beta_j' = 1 - \delta_j' \in [0\ ,1)$. Hence, $|\beta_j'| < 1$ and the condition of the second claim is satisfied. Stability of A thus implies that $A_{\beta_j',r}$ is stable. When $1 < \delta_j' \leq 2$, $\beta_j' = 1 - \delta_j' \in [-1\ ,0)$. In this case the first and second claims ensure that $A_{\beta_j',r}$ is stable as long as A is stable. For every $\delta_j' > 2$, there exists uniquely a positive $\hat{\delta}_j = \delta_j' - 2$ such that $-\beta_j' = \delta_j' - 1 = \hat{\delta}_j + 1 \triangleq \hat{\beta}_j$. Hence, stability of $A_{\beta_j',r}$ is equivalent to that of $A_{\hat{\beta}_j,r}$. Since $\hat{\delta}_j < |\delta_j'| = \delta_j'$, the later is implied by the stability of $A_{\beta_j,r}$.

The previous analysis shows that

$$\mu_{\Delta_j}(M_j) \geq 1$$
$$\iff \det(I - A_{\beta_j,r} Z_n) = 0$$

for some $z \in \mathcal{C}^n$ with $\|z\|_\infty \leq 1$ and $\delta_j' = w$, and the infimum of all such w is σ_j. This completes the proof. ∎

To calculate $\sigma_{j_1,j_2,\ldots,j_q}$ we use the matrix

$$E_{j_1,j_2,\ldots,j_q} \triangleq [E_{j_1} \quad E_{j_2} \quad \ldots \quad E_{j_q}] \qquad (20)$$

Theorem 2 *The joint stability margin $\sigma_{j_1,j_2,\ldots,j_q}$ of the system (2) is given by*

$$\sigma_{j_1,j_2,\ldots,j_q} = \inf \left\{ \begin{array}{l} w > 0\ : \\ \mu_{\Delta_{j_1,j_2,\ldots,j_q}}(M_{j_1,j_2,\ldots,j_q}(w)) \geq 1 \end{array} \right\}$$

where

$$M_{j_1,j_2,\ldots,j_q}(w) = \begin{bmatrix} A & E_{j_1,j_2,\ldots,j_q} \\ wE^*_{j_1,j_2,\ldots,j_q}A & 0_{m_{j_1,j_2,\ldots,j_q}} \end{bmatrix}$$
$$\Delta_{j_1,j_2,\ldots,j_q} \triangleq \operatorname{diag}\left\{Z_n, \delta I_{m_{j_1}}, \delta I_{m_{j_2}}, \ldots, \delta I_{m_{j_q}}\right\}$$

with $m_{j_1,j_2,\ldots,j_q} = \sum_{k\in\{j_1,j_2,\ldots,j_q\}} m_k$ and $\delta \in \mathcal{R}$.

The proof for Theorem 2 is similar to that for Theorem 1 and is omitted.

Theorem 3 *The total stability margin of the system (2) is given by*

$$\sigma = \frac{1}{\mu_\Delta(A)} - 1$$

where Δ is given in (6).

Proof. In this case, $q = n$, and we can take $j_k = k$, $k = 1, 2, \ldots, n$. Thus $E_{j_1,j_2,\ldots,j_n} = I_m$ and

$$M_{j_1,j_2,\ldots,j_n}(w) = \begin{bmatrix} A & I_m \\ wA & 0 \end{bmatrix} \triangleq M(w)$$
$$\Delta_{j_1,j_2,\ldots,j_n} = \operatorname{diag}\{\delta I_{m_1}, \delta I_{m_2}, \ldots, \delta I_{m_n}\} = \delta I_m$$

$$A + wE_{j_1,j_2,\ldots,j_n}\Delta_{j_1,j_2,\ldots,j_n}E^*_{j_1,j_2,\ldots,j_n}A = (1+\delta')A$$

where $-w \leq \delta' \leq w$. From

$$\mu_\Delta(\alpha A) = |\alpha|\mu_\Delta(A)$$

we readily see that the minimum w making $\mu_\Delta[(1+\delta')A] = 1$ is given by $(1+w)\mu_\Delta(A) = 1$. Hence $w = \frac{1}{\mu_\Delta(A)} - 1$. ■

Note that Lemma 1 is a special case of Theorem 3. For positive total stability margin, i.e. $\sigma > 0$, $\mu_\Delta(A) < 1$ so the system (1) is stable. The value of σ characterizes the robustness of the stability. On the other hand, negative value of σ means that $\mu_\Delta(A) > 1$ and the system is unstable. The value σ indicates how far away for the system to be stable. Therefore, we see that the μ-analysis approach provides more insights to the nD stability analysis than just giving a simple stability test.

4. Concluding Remarks

The stability analysis problem for nD systems characterized by state-space models has been investigated by using μ analysis approach. It has been shown that the problems of stability test and stability margin computation for an nD system described by Roesser model can be formulized into a set of μ analysis problems in a unified way, thus can be solved effectively by applying the available μ-toolbox of MATLAB [5].

It is noted, however, that only the lower and upper bounds for μ can be obtained by using the μ-toolbox. The upper bound of μ provides a lower bound for the stability margins and the corresponding minimizer D in 5 can be used to construct a diagonal Lyapunov function. The lower bound of μ provides an upper bound for the stability margins. Numerical examples show that in certain circumstances the lower and upper bounds of μ can be equal. We shall address this issue in a full version of this paper later.

Although in this paper only the stability problems of systems described in Roesser model have been discussed, the same problems of nD systems described by other kinds of state-space models can be also investigated by transforming the models into corresponding Roesser models.

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