

## 受動4脚走行ロボットの歩容解析

### Passive Quadruped Gait Searching

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## 1. Introduction

Since McGeer's pioneering work of gravity driven biped walker, more and more researchers take passive balance paradigm as a control scheme for legged locomotion. They believe that legged machines should be designed with structures that are naturally stable, not requiring feedback of body posture. Inspired from cyclic leg swinging motion of animals, Tompson and Raibert showed that a hip string configured monopod robot can hop without any inputs, provided with appropriately chosen initial conditions<sup>1)</sup>. Francois and Samson, Hyon and Emura respectively derived controllers which enable the one-legged robot to converge to passive running gaits<sup>2) 3)</sup>. As for quadruped running, Poulakakis et al. found that under a massless leg assumption, there exists some energy efficient gaits within a leg compliance profile, which stabilize themselves.<sup>4)</sup>

This paper is a direct extension of the paper<sup>3)</sup>. Based on a more realistic model than previous studies, we aim to find passive quadruped running gait which contributes to energy-efficient running controller design. To simplify analysis, the running model we use is confined in sagittal plane. The physical legs are modeled as single back and front virtual leg respectively. However, this simplified model is still involved with quite complex nonlinear dynamics that prevents analytical treatment. Therefore, a numerical gait searching method is applied in this paper.

## 2. Planar Quadruped Model

Figure 1 shows our planar quadruped model. Hip joint provides leg rotation. Legs are connected to torso through springs. Each leg includes upper and lower sections connected via linear spring. To summarize, leg has two DOF: a rotational one, a linear

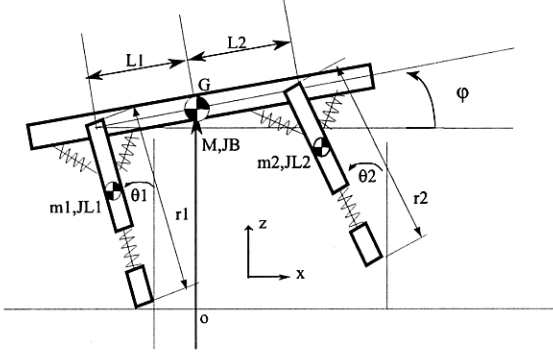


Fig. 1 Robot model

one. The hip springs provide hip compliance.

The running of the robot has four different dynamics corresponding to each phase ( $f_1$ : **double legs flight**,  $f_2$ : **back leg stance phase**,  $f_3$ : **double legs stance**,  $f_4$ : **front leg stance**). A sequential combination of the phases results in the locomotion of the robot. With Lagrange method, we built up mathematical model of each phase. As there are no inputs to the robot, the models can be described as nonlinear autonomous system:

$$\dot{\mathbf{x}} = \mathbf{f}_i(\mathbf{x}) \quad (i = 1, \dots, 4), \quad (1)$$

where  $\mathbf{x} = [\theta_1, \theta_2, \phi, \dot{x}, \dot{z}, r_1, r_2, \theta_1, \theta_2, \phi, x, z, r_1, r_2]$ . As we can see from Fig.1, the model of robot can be completely described by generalized coordinates  $q = [\theta_1, \theta_2, \phi, x, z, r_1, r_2] \in R^7$ . However, the DOF of the phases is 5,4,3,4 respectively. Therefore the variables which are not chosen as state variable in Lagrange equation should be calculated using the constraints subjected to the corresponding phase.

At landing, the foot of robot hits the ground, which is treated as the transition between two successive phases. This process is dealt with using an inelastic impact model. As an assumption, massless toes are considered in our model. With the method of *Lagrange multipliers* we derive the gen-

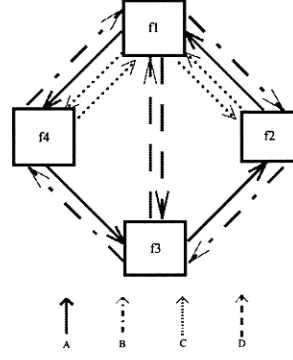


Fig. 2 Four examples of quadruped running gaits

eralized components of the impulse moment. Combining this result with constraints subjected to state after touching down, we can obtain the following impulse equation:

$$\mathbf{x}_+ = \mathbf{h}_i(\mathbf{x}_-) \quad (j = 1, \dots, 4), \quad (2)$$

For the details, please see the paper <sup>5)</sup>.

### 3. Passive Gait

#### 3.1 Poincaré's Method

In our case a periodic running represents a cyclic orbit. An important conceptual tool for understanding the periodic orbit of non-linear system is the *Poincaré map*. It replaces an  $n^{th}$  order continuous-time autonomous system by an  $(n - 1)^{th}$  order discrete-time system. In order to define a Poincaré's map for a legged system a reference point in the cyclic motion must be selected and then the dynamics equations must be integrated starting from the point until the next cycle. Therefore a periodic running can be treated as root of function:

$$H(\mathbf{x}_0) = F(\mathbf{x}_0) - \mathbf{x}_0, \quad (3)$$

where the function  $F$  maps the initial point to its corresponding point after one cycle. To solve the

root, we use *Newton-Raphson* method:

$$d\mathbf{x}_0 = -\left(\frac{\partial H(\mathbf{x}_0)}{\partial \mathbf{x}_0}\right)^{-1} H(\mathbf{x}_0) \quad (4)$$

$$\mathbf{x}_0(n+1) = Kd\mathbf{x}_0 + X_0(n) \quad (5)$$

### 3.2 Implementation of Algorithm

Quadruped running forms as a mix of continuous-time dynamics, switching actions, and jump phenomena. Its corresponding hybrid system can be described as:

$$\dot{\mathbf{x}} = f_i(\mathbf{x}, \mathbf{y}) \quad (i = 1, \dots, 4) \quad (6)$$

$$0 = g(\mathbf{x}, \mathbf{y}) \quad (7)$$

$$\mathbf{x}_+ = h_j(\mathbf{x}_-, \mathbf{y}_-) \quad (j = 1, \dots, 4) \quad (8)$$

where,  $\mathbf{y}$  is trigger variable which triggers the transition of different phases under the condition defined by function  $g$ . Specifically it represents the distance of toe from the ground. The value of zero indicates a touching down of foot, which triggers the phase transition. The transition process is defined by function  $h_j$ , which deals with both smoothed changing of state variables and stepped ones due to, for example, leg touching down.  $\mathbf{x}_-$ ,  $\mathbf{y}_-$  refer the value of  $\mathbf{x}, \mathbf{y}$  just prior to the phase transition and  $\mathbf{x}_+$  refers to the value just after the phase transition. Based on this model, the jacobian matrix used in (4) can be calculated as followed <sup>6)</sup>:

1) For continuous-time phases:

$$\frac{d\mathbf{x}\mathbf{x}_0}{dt} = \frac{\partial f_i}{\partial \mathbf{x}} \cdot \mathbf{x}\mathbf{x}_0 \quad (9)$$

2) At the transition of two successive phases:

$$\mathbf{x}\mathbf{x}_{0+} = \mathbf{x}\mathbf{x}_{0-} - (f_+ - f_-)\tau\mathbf{x}_0 \quad (10)$$

where

$$\tau\mathbf{x}_0 = -\frac{\mathbf{1}_k((g_y^-)^{-1}g_x^-)|_{\tau^-}\mathbf{x}\mathbf{x}_0(\tau^-)}{\mathbf{1}_k((g_y^-)^{-1}g_x^-)|_{\tau^-}f^-} \quad (11)$$

$$\mathbf{x}\mathbf{x}_0(t_0) = I \quad , \quad (12)$$

$f_-, f_+$  take values at the moment just before and after phases transition respectively,  $\mathbf{1}_k = [0 \dots \overset{k}{1} \dots 0]$ , and  $t_0$  corresponds to the time of start point.

## 4. Result

Using the above algorithm, we found two kinds of gaits shown in Fig. 3 and Fig. 4, both of which include the four available phases. They differ with the order in which system propagate throughout the phases. In gait A, the front leg will hit the ground first after a **double legs flight** phase, while the back leg will hits first as for gait B. As respected, the passive trajectories we found lose no energy due to the sudden impact of leg's touching down (Fig. 5).

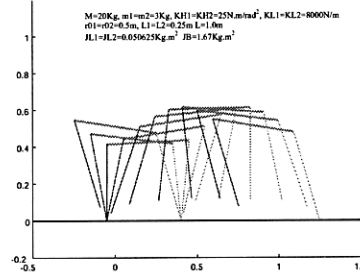


Fig. 3 Gait A

Another finding is that all the periodic trajectories are unstable, which is different from Poulakakis's result. Although the motion starting from the fixed point can continue for some steps (13 steps as maximum), it falls finally. Extensive investigations of eigenvalues corresponding to the trajectories revealed that they all have eigenvalues whose magnitude is bigger than unity. As Fig. 7 shows, during first several cycles, system exhibits roughly smooth curves. The uncontinuous change of phase transition is not obvious. As the system have no inputs, lost energy can not be compensated. With the system proceeding forward, it deviates from the

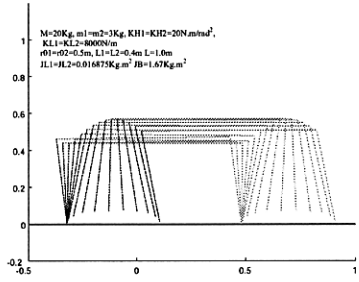


Fig. 4 Gait B

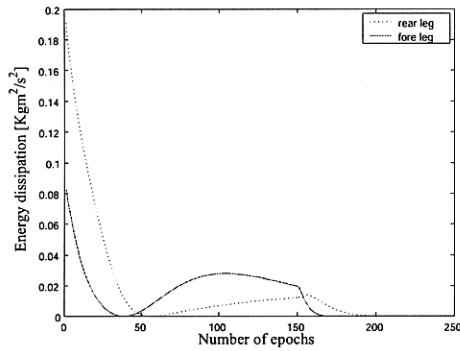


Fig. 5 Energy dissipation

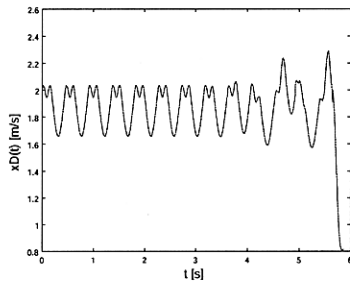


Fig. 6 Horizontal velocity plot of Gait 1

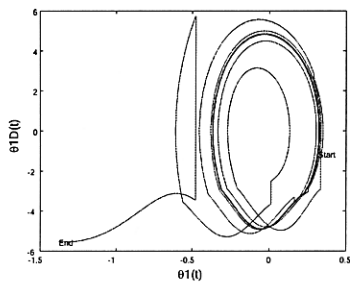


Fig. 7 Phase plot  $\theta_1 - \dot{\theta}_1$  of Gait 1

passive running gait more. We also found that all the passive running gaits have symmetricity. This observation agree with symmetricity result of Poulakakis<sup>4)</sup>. Another interesting observation is that it seems that different kinds of gaits do not exist under one profile. This fact disagree with the fact found from legged animals.

To find various running gaits, more complex model will be needed. Nevertheless, we believe the results of the paper can be utilized to find efficient running controller, which is left for future work.

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