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Research on the Stability Control System of Double Inverted Pendulum

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1 Research significance

Inverted pendulum is the abstract model of many control problems which the barycenter is upper and the fulcrum is lower, it is unstable object. In the control process it can reflect a number of abstract and key issues effectively, like the system stability, non-linear, controllability, robustness and so on.

As a control device, with the image of visual pattern, simple structure, inverted pendulum`s component parameters and the shape of the composition is susceptible to change and it is easy to simulation; As a controlled object it is a higher order, nonlinear, multi-variable, strong coupling and unstable rapid control system.

Inverted pendulum research not only has important theoretical significance, but also has important engineering background. Walking robot joint control, the vertical degree of control in rocket launch, satellite attitude control, those are all related to the stability of the control problem upside-down objects. Therefore, the inverted pendulum control strategies can be applied to aerospace, military, robotics, industry and others` areas, to solve the balance problems , and has important application value and far-reaching social significance.

Inverted pendulum system can not to established a precise mathematical model. It can be seen approximately as a time-invariant linear system only in a small range and with a series of assumptions.

2 Establishing mathematical model

2.1

Double inverted pendulum is composed by cart, pendulums, guide and so on as fig.1

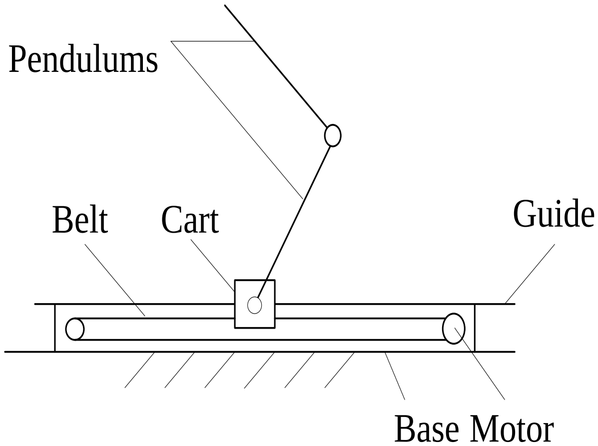

Fig.1 Configuration of double inverted pendulum

2.2

There are two ways to establish mathematical model of double inverted pendulum. One is based on Newtonian mechanics, and the other based on Lagrangian method. Decided to adopt the Lagrangian method for the following reasons:

1 The number of equations equal to the degree of freedom system

2 Analyze only the force what have known

3 Modeling process can be simplified

2.3

Before the derivation，make some assumptions：

1 The system is rigid-body

2 There is no relative sliding

3 The transfer delay of circuit system is neglected

4 Friction between the cart and the rail is direct proportion to the cart's speed, Resistance torque between the cart and pendulum 1 , pendulum 1 and pendulum 2,are direct proportion to the angular velocity.

2.4

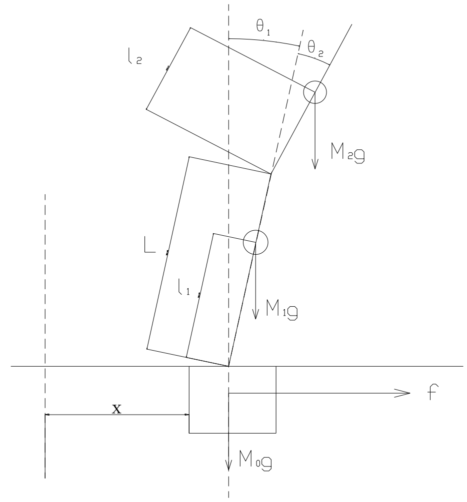

Fig.2 Force analysis

Form Fig.2, could get coordinates of center of gravity

Pendulum 1

$$\begin{cases} x_1 = x + l_1 \sin\theta_1 \\ y_1 = l_1 \cos\theta_1 \end{cases} \tag{1}$$

Pendulum 2

$$\begin{cases} x_2 = x + L_1 sin\theta_1 + l_2 sin(\theta_1+\theta_2) \\ y_2 = L_1 cos\theta_1 + l_2 cos(\theta_1+\theta_2) \end{cases} \tag{2}$$

2.5

Lagrange equation

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i$$

$$L(q,\dot{q}) = T(q,\dot{q}) - V(q,\dot{q}) \tag{3}$$

T is total kinetic energy, V is total potential energy and D is total energy loss.

When $q_i = x$, $Q_i = f$, from (3) could get

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial V}{\partial x} + \frac{\partial D}{\partial \dot{x}} = f \tag{4}$$

When $q_i = \theta_1$, $Q_i = 0$, from (3) could get

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) - \frac{\partial T}{\partial \theta_1} + \frac{\partial V}{\partial \theta_1} + \frac{\partial D}{\partial \dot{\theta}_1} = 0 \tag{5}$$

When $q_i = \theta_2$, $Q_i = 0$, from (3) could get

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) - \frac{\partial T}{\partial \theta_2} + \frac{\partial V}{\partial \theta_2} + \frac{\partial D}{\partial \dot{\theta}_2} = 0 \tag{6}$$

By calculating, get

$$(M_0 + M_1 + M_2)\ddot{x} + (M_1 l_1 C_1 + M_2 L C_1 + M_2 l_2 C_{12})\ddot{\theta}_1 + (-M_1 l_1 S_1 - M_2 L S_1 - M_2 l_2 S_{12})\dot{\theta}_1{}^2 + M_2 l_2 C_{12}\ddot{\theta}_2 - 2M_2 l_2 S_{12}\dot{\theta}_1\dot{\theta}_2 - M_2 l_2 S_{12}\dot{\theta}_2{}^2 + R_0\dot{x} = f$$

$$\left(J_1 + M_1 l_1{}^2 + J_2 + M_2 L^2 + M_2 l_2{}^2 + 2M_2 L l_2 C_2\right)\ddot{\theta}_1 + (-2M_2 L l_2 S_2)\dot{\theta}_1\dot{\theta}_2 + (M_1 l_1 C_1 + M_2 L C_1 + M_2 l_2 C_{12})\ddot{x} + \left(J_2 + M_2 l_2{}^2 + M_2 L l_2 C_2\right)\ddot{\theta}_2 + (-M_2 L l_2 S_2)\dot{\theta}_2{}^2 - M_1 g l_1 S_1 - M_2 g(L S_1 + l_2 S_{12}) + R_1\dot{\theta}_1 = 0$$

$$(M_2 l_2 C_{12})\ddot{x} + \left(J_2 + M_2 l_2{}^2 + M_2 L l_2 C_2\right)\ddot{\theta}_1 + \left(J_2 + M_2 l_2{}^2\right)\ddot{\theta}_2 + (M_2 L l_2 S_2)\dot{\theta}_1{}^2 - M_2 g l_2 S_{12} + R_2\dot{\theta}_2 = 0 \tag{7}$$

Make (7) to matrix form, could get

$$M(\theta_1,\theta_2)\begin{pmatrix}\ddot{x}\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + N\left(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2\right)\begin{pmatrix}\dot{x}\\ \dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} = G(u,\theta_1,\theta_2) \tag{8}$$

2.6 Linearization

Double inverted pendulum is a multi-variable, complex nonlinear system. In order to facilitate control strategy, linear the model near the balance point.

Set the balance point

$$\theta_1 = \theta_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0 \tag{9}$$

So, could get

$$S_1 \approx \theta_1 \qquad S_2 \approx \theta_2$$

$$C_1 \approx 1 \qquad C_2 \approx 1 \tag{10}$$

Then get the new equations

$$M(0,0)\begin{pmatrix}\ddot{x}\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + N(0,0,0,0)\begin{pmatrix}\dot{x}\\ \dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} = P(0,0)\begin{pmatrix}x\\ \theta_1\\ \theta_2\end{pmatrix} + \begin{pmatrix}1\\ 0\\ 0\end{pmatrix} f \tag{11}$$

$$M(0,0) = \begin{pmatrix} M_0 + M_1 + M_2 & M_1 l_1 + M_2 L + M_2 l_2 & M_2 l_2 \\ M_1 l_1 + M_2 L + M_2 l_2 & J_1 + M_1 l_1{}^2 + J_2 + M_2 L^2 + M_2 l_2{}^2 + 2M_2 L l_2 & J_2 + M_2 l_2{}^2 + M_2 L l_2 \\ M_2 l_2 & J_2 + M_2 l_2{}^2 + M_2 L l_2 & J_2 + M_2 l_2{}^2 \end{pmatrix}$$

$$N(0,0,0,0) = \begin{pmatrix} R_0 & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & R_2 \end{pmatrix}$$

$$P(0,0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_1 g l_1 + M_2 g L + l_2 & l_2 \\ 0 & M_2 g l_2 & M_2 g l_2 \end{pmatrix} \tag{12}$$

Because *M(0,0)* is symmetric matrix, and it is positive definite matrix in practice. By calculating, could get

$$\begin{pmatrix}\ddot{x}\\ \ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} = -M(0,0)^{-1}N(0,0,0,0)\begin{pmatrix}\dot{x}\\ \dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} + M(0,0)^{-1}P(0,0)\begin{pmatrix}x\\ \theta_1\\ \theta_2\end{pmatrix} + M(0,0)^{-1}\begin{pmatrix}1\\ 0\\ 0\end{pmatrix} f \tag{13}$$

Make

$$X = [x \quad \theta_1 \quad \theta_2 \quad \dot{x} \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{14}$$

Then could get double inverted pendulum linear equation of state

$$\dot{X} = AX + Bf$$

$$Y = CX \tag{15}$$

$$A = \begin{pmatrix} 0_{3\times3} & I_{3\times3} \\ A_{21} & A_{22} \end{pmatrix}$$

$$A_{21} = M(0,0)^{-1}P(0,0)$$
$$A_{22} = -M(0,0)^{-1}N(0,0,0,0)$$

$$B = \begin{pmatrix} 0_{3\times1} \\ B_2 \end{pmatrix}$$

$$B_2 = M(0,0)^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{16}$$

Substitution of the data:

$M_0 = 0.42kg$ $\quad$ $M_1 = 0.4kg$

$M_2 = 0.2kg$ $\quad$ $l_1 = 0.3m$

$L = 0.6m$ $\quad$ $l_2 = 0.2m$

$g = 9.8m/s^2$ $\quad$ $R_0 = 1.87$

$R_1 = 0.0054$ $\quad$ $R_2 = 0.00368$

$J_1 = \frac{m_1L^2}{3} = 0.048$ $\quad$ $J_2 = \frac{4m_2{l_2}^2}{3} = 0.0107$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -4.527 & 0.9241 & -2.876 & 0.01252 & -0.007366 \\ 0 & 20.05 & -6.954 & 4.334 & -0.06201 & 0.07831 \\ 0 & -15.14 & 34.91 & -3.743 & 0.1149 & -0.3604 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.538 \\ -2.318 \\ 2.002 \end{pmatrix} \tag{17}$$

3 Performance analysis of double inverted pendulum system

3.1 Stability Analysis

The characteristic equation of double inverted pendulum system {A, B, C} is

$$\det\{\lambda I - A\} = 0 \tag{18}$$

By calculating, could get the Characteristic Roots

{7.82328, 5.40889, -5.03301, 2.72428, -2.62504, 1} (19)

Base on the Lyapunov Law, the non-linear control system can be analyzed by the linearization system. As showed there are positive characteristic roots, so there are four poles in the right half of S plane. Therefore the system is unstable.

3.2 Controllability and Observability

Controllability Criterion:

N-order linear time-invariant continuous system, $\dot{X} = AX + Bf$, can be completely controllable if and only if the Controllability matrix of the control system

$$S_c = [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B\,] \tag{20}$$

is full-rank, or $rank(S_C) = n$.

Observability Criterion:

N-order linear time-invariant continuous system, $\begin{matrix} \dot{X} = AX + Bf \\ Y = CX + Df \end{matrix}$, can be completely observable if and only if the observabilit matrix of the control system

$$S_0 = [C \;\; CA \;\; CA^2 \;\; \cdots \;\; CA^{n-1}\,]^T \tag{21}$$

is full-rank, or $rank(S_o) = n$

By calculating, $rank(S_C) = 6$, and $rank(S_o) = 6$. So the double inverted pendulum linear control system is controllable and observable.

4 Future work

As the double inverted pendulum system is controllable and observable, there are many ways to achieve stability control.

PID Control

State Feedback Control

Fuzzy Control

Self Adaptive Control

Neural Networks Control

In the study, intend to adopt fuzzy control method. So, how to design the fuzzy controller is the key of this part.

References

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Appendix

X : Is 0 when cart be in the equilibrium position.

To right is +

θ_1*:* Angle between Vertical axis and pendulum 1

Counterclockwise is +

θ_2: Angle between pendulum 1 and pendulum 2

Counterclockwise is +

M_0: Mass of cart

M_1: Mass of pendulum 1

M_2: Mass of pendulum 2

L: Length of pendulum 1's axis to pendulum 2'axis

l_1: Length of pendulum 1's Center of Gravity to its rotation axis

l_2: Length of pendulum 2's Center of Gravity to its rotation axis

J_1: Moment of Inertia of pendulum 1

J_2: Moment of Inertia of pendulum 2

f: Force on cart by DC torque motor

R_0: Co-efficient of friction for cart

R_1: Co-efficient of friction for pivot 1

R_2: Co-efficient of friction for pivot 2