計測自動制御学会東北支部 第 253 回研究集会 (2009.11.20)
資料番号 253-6

前置補償器を用いた安定化補償器のパラメトリゼーション
Parametrization of Stabilizing Controllers with Precomensators

○ 森 和好 (会津大コンピュータ理工学部)
Kazuyoshi MORI, School of Computer Science and Engineering
The University of Aizu, Aizu-Wakamatsu 965-8580, JAPAN

Keywords: Youla parameterization, Coprime Factorization, Parameterization of Stabilizing Controllers.

連絡先： 〒965-8580 福島県会津若松市一箕町鶴賀会津大学 コンピュータ理工学部
Tel: 0242-37-2615, Fax: 0242-37-2747, Email: Kazuyoshi.MORI@IEEE.ORG

Abstract

So far, the author developed the parametrization of all strictly causal stabilizing controllers in the framework of the factorization approach[1]. The objective of this paper is to extend the previous results, the parametrization of all strictly causal stabilizing controllers with some conditions. Our approach is the factorization approach, so that the result can be applied to numerous linear system models.

1 Introduction

Since stabilizing controllers of a plant are not generally unique, the choice of the stabilizing controllers is important for the resulting closed-loop. In the classical case, that is, in the case where the given plant admits coprime factorizations, the stabilizing controllers can be parametrized by the so-called Youla-Kučera-parametrization [2, 3, 4, 5, 6, 7]. However, this parametrization may include stabilizing controllers which are not causal and, in the case of a discrete-time system, may result in a closed-loop that does not contain even one-step delay, which is not physically realizable. There are models such that some stabilizable plants do not admit coprime factorizations [8]. A parametrization that can be applied even to stabilizable plants that do not admit doubly coprime factorizations is given by Quadrat in [9, 10] and the present author in [11, 12], which may also include stabilizing controllers that are not causal and closed-loop systems that are causal but not strictly causal.

For some models, such as the discrete-time system model, the closed-loop system must be strictly causal in order to be physically realizable. Kučera in [13, pp. 283–287] and [14, p. 178] discussed the methods to obtain strictly causal stability controllers of discrete-time systems. In addition, Lin in [15] presented the method to obtain all strictly causal stabilizing controllers of two-dimensional discrete-time systems, which can be also applied to discrete-time systems.

So far, the author developed the parametrization of all strictly causal stabilizing controllers in the framework of the factorization approach [16, 1]. The objective of this paper is to extend the previous results. Since the factorization approach has been used, the result can be applied to numerous linear system models.

2 Preliminaries

We employ the factorization approach [2, 4, 5, 6] and the symbols used in [11, 12, 17]. The reader is referred to Appendix A of [5] for algebraic preliminaries if necessary.

Denote by $\mathcal{A}$ an integral domain that is the set of all stable causal transfer functions. The total field of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d \neq 0\}$. This $\mathcal{F}$ is considered to be the set of all possible transfer functions. Let $\mathcal{Z}$ be a finitely generated prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$. Furthermore, let $\mathcal{P} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A} - \mathcal{Z}\}$ and $\mathcal{P}_{\rm s} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A} - \mathcal{Z}\}$. A transfer function is said to be *causal* (*strictly causal*) if and only if it is in $\mathcal{P}$ ($\mathcal{P}_{\rm s}$)[1]. Analogously, a transfer matrix is said to be *causal* (*strictly causal*) if and only if every entry in the matrix is in $\mathcal{P}$ ($\mathcal{P}_{\rm s}$). Note that $\mathcal{Z}$ represents the set of transfer functions that are both strictly causal and stable.

Throughout the present paper, the plant we con-

[1] On the continuous-time systems, the notion of "causal" of this paper is corresponding to "proper."

sider has m inputs and n outputs, and its transfer matrix, which is also called a *plant*, is denoted by P and belongs to $\mathcal{P}^{n\times m}$. We consider the feedback system Σ shown in Figure ??. In the figure, C denotes a *controller* and is a transfer matrix of $\mathcal{F}^{m\times n}$. The stabilization problem considered in the present paper follows that developed in [2, 5, 6]. The reader is referred to [5, 17, 18, 19]. Let $H(P,C)$ denote the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,e_1^t \quad e_2^t\,]^t$ of the feedback system Σ, that is,

$$H(P,C) := \begin{bmatrix} (I_n+PC)^{-1} & -P(I_m+CP)^{-1} \\ C(I_n+PC)^{-1} & (I_m+CP)^{-1} \end{bmatrix} \in \mathcal{F}^{(m+n)\times(m+n)} \quad (1)$$

provided that $I_n + PC$ is nonsingular. The plant P is said to be *stabilizable*, P is said do be *stabilized* by C, and C is said to be a *stabilizing controller* of P if and only if $I_n + PC$ is nonsingular and $H(P,C) \in \mathcal{A}^{(m+n)\times(m+n)}$. If a causal plant is stabilizable, there always exists a causal stabilizing controller of the plant [18].

Because we investigate the set of stabilizing controllers of a certain type, we introduce some notations as follows ($P \in \mathcal{P}^{n\times m}$):

$$\begin{aligned}
\mathcal{S}(P) &:= \{C \in \mathcal{F}^{m\times n} \,|\, H(P,C) \in \mathcal{A}^{(m+n)\times(m+n)}\} \\
&\quad (=\text{“the set of all stabilizing controllers of } P\text{”}), \\
\mathcal{SP}(P) &:= \mathcal{S}(P) \cap \mathcal{P}^{m\times n} \quad (=\text{“the set of all causal stabilizing controllers of } P\text{”}), \\
\mathcal{SP}_{\mathrm{s}}(P) &:= \mathcal{S}(P) \cap \mathcal{P}_{\mathrm{s}}^{m\times n} \quad (=\text{“the set of all strictly causal stabilizing controllers of } P\text{”}), \\
\mathcal{P}_{\mathcal{B}} &:= \{x/y \in \mathcal{F} \,|\, x \in \mathcal{B},\ y \in \mathcal{A} - \mathcal{Z}\}, \\
\mathcal{SP}(P;\mathcal{B}) &:= \mathcal{S}(P) \cap \mathcal{P}_{\mathcal{B}}^{m\times n},
\end{aligned}$$

where $\mathcal{B}$ is a finitely-generated ideal of $\mathcal{A}$.

Finally, we introduce two symbols diag and Diag, where $\mathrm{diag}(a_1,\cdots,a_n)$ denotes the diagonal matrix for which the diagonal entries starting in the upper left corner are a_1, …, a_n, and $\mathrm{Diag}(A_1,\cdots,A_n)$ denotes the block diagonal matrix with the matrices A_1, …, A_n on the main block diagonal.

3 Parametrization of Strictly Causal Stabilizing Controllers

In this section, we present the main results of the present paper.

Let $\mathcal{B}$ denote a finitely-generated ideal of $\mathcal{A}$. We assume that $\mathcal{B}$ is a subset of $\mathcal{Z}$. The set $\{\zeta_1,\cdots,\zeta_l\}$ denotes the set of generators of $\mathcal{B}$ such that no element can be generated by the other elements. The set of generators of $\mathcal{B}$ is not unique in general. However, in the remainder of the present paper, we consider that the set $\{\zeta_1,\cdots,\zeta_l\}$ is arbitrary but fixed.

Our results are to obtain the set $\mathcal{SP}(P;\mathcal{B})$ with one parameter. We will present two parametrizations. The first, based on [11] and stated in Theorem 2, does not assume the existence of the coprime factorization for the plant. The second, based on the Youla-Kučera-parametrization stated in Theorem 3, assumes the existence of a coprime factorization for the plant. Before stating these parametrizations, we first prepare a result, Theorem 1, from which we obtain Theorems 2 and 3.

Theorem 1 *Let P be a plant of $\mathcal{P}^{n\times m}$ and $\Phi = [\,\zeta_1 I_n \ \cdots \ \zeta_l I_n\,]^t \in \mathcal{A}^{nl\times n}$. Then,*

$$\mathcal{SP}(P;\mathcal{B}) = \{\Gamma\Phi \,|\, \Gamma \in \mathcal{SP}(\Phi P)\}. \quad (2)$$

□

Theorem 2 *Let P and Φ be as in Theorem 1. Assume that $\Gamma \in \mathcal{P}^{m\times nl}$ is a stabilizing controller of ΦP. Let*

$$\begin{aligned}
\Omega(Q) &= (H(\Phi P,\Gamma) - \mathrm{Diag}(I_{nl}, O_{m\times m}))\,Q \\
&\quad (H(\Phi P,\Gamma) - \mathrm{Diag}(O_{nl\times nl}, I_m)) \\
&\quad + H(\Phi P,\Gamma)
\end{aligned}$$

where Q is a parameter matrix of $\mathcal{A}^{(m+nl)\times(m+nl)}$. Then,

$$\mathcal{SP}(P;\mathcal{B}) = \left\{ H_{22}^{-1}H_{21}\Phi \;\middle|\; \begin{matrix} & nl & m \\ \begin{matrix} nl \\ m \end{matrix} & \multicolumn{2}{c}{\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}} \end{matrix} = \Omega(Q),\ Q \in \mathcal{A}^{(m+nl)\times(m+nl)} \right\}. \quad (3)$$

□

Theorem 3 *Let P and Φ be as in Theorem 1. Suppose that ΦP admits right- and left-coprime factorizations such that*

$$\begin{cases} \Phi P = ND^{-1} = \widetilde{D}^{-1}\widetilde{N}, \\ \widetilde{Y}N + \widetilde{X}D = I_m, \quad \text{and} \\ \widetilde{N}Y + \widetilde{D}X = I_{nl} \end{cases} \quad (4)$$

where N, D, $\widetilde{N}$, $\widetilde{D}$, $\widetilde{Y}$, $\widetilde{X}$, Y, and X are matrices over $\mathcal{A}$ of appropriate sizes. Then,

$$\mathcal{SP}(P;\mathcal{B}) = \{(\widetilde{X} - R\widetilde{N})^{-1}(\widetilde{Y} + R\widetilde{D})\Phi \,|\, R \in \mathcal{A}^{m\times nl}\}. \quad (5)$$

□

The proof of Theorem 1 will be given later. Theorems 2 and 3 are proved in the following using Theorem 1.

Proof of Theorem 2: Applying the parametrization of [11] to ΦP, we have

$$\mathcal{SP}(\Phi P) = \left\{ H_{22}^{-1}H_{21} \;\middle|\; \begin{matrix} & nl & m \\ nl & H_{11} & H_{12} \\ m & H_{21} & H_{22} \end{matrix} = \Omega(Q), \quad Q \in \mathcal{A}^{(m+nl)\times(m+nl)} \right\}.$$

Applying this equation to (2), we obtain (3). Since ΦP is strictly causal, the matrix H_{22} is always nonsingular. ∎

Proof of Theorem 3: Applying the Youla-Kučera-parametrization to ΦP, we have

$$\mathcal{SP}(\Phi P) = \{(\widetilde{X} - R\widetilde{N})^{-1}(\widetilde{Y} + R\widetilde{D}) \,|\, R \in \mathcal{A}^{m\times nl}\}.$$

The remaining proof is analogous to the proof of Theorem 2. ∎

In the remainder of this section, our task is to prove Theorem 1. Before giving the proof, we prepare two lemmas.

Lemma 1 *Let A be a matrix of $\mathcal{P}^{n\times m}$. Let $\Phi_{\text{diag}} = \text{diag}(\phi_1, \cdots, \phi_n) \in \mathcal{A}^{n\times n}$, where each ϕ_i is any one of $\zeta_1, \cdots, \zeta_l$. Then, A is over $\mathcal{A}$ if and only if $\Phi_{\text{diag}}A$ is over $\mathcal{A}$.*

Proof: Denote by a_{ij} the (i, j)-entry of the matrix A. We show the "if" part only. The "only if" part is obvious.

Suppose here that the matrix A is not over $\mathcal{A}$. Then, there exists at least one entry a_{ij} of A with $a_{ij} \in \mathcal{P} - \mathcal{A}$. Let a_{ijn} and a_{ijd} be in $\mathcal{A}$ and in $\mathcal{A} - \mathcal{Z}$, respectively, such that $a_{ij} = a_{ijn}/a_{ijd}$. Since ϕ_i is one of $\zeta_1, \zeta_2, \cdots, \zeta_l$, we have that ϕ_i and a_{ijd} have no common nonunit factor of $\mathcal{A}$. Thus, $\phi_i a_{ij}$ is not in $\mathcal{A}$, which is the (i, j)-entry of the matrix $\Phi_{\text{diag}}A$. Hence, $\Phi_{\text{diag}}A$ is not over $\mathcal{A}$. ∎

Lemma 2 *Let $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$. Let Φ_{diag} and ϕ_i be as in Lemma 1. Then, P is causal and $H(\Phi_{\text{diag}}P, C)$ is over $\mathcal{A}$ if and only if C is causal and $H(P, C\Phi_{\text{diag}})$ is over $\mathcal{A}$.*

Proof: First we let

$$\begin{bmatrix} H_{a11} & H_{a12} \\ H_{a21} & H_{a22} \end{bmatrix} := H(\Phi_{\text{diag}}P, C),$$

which is equal to

$$\begin{bmatrix} (I_n + \Phi_{\text{diag}}PC)^{-1} & -\Phi_{\text{diag}}P(I_m + C\Phi_{\text{diag}}P)^{-1} \\ C(I_n + \Phi_{\text{diag}}PC)^{-1} & (I_m + C\Phi_{\text{diag}}P)^{-1} \end{bmatrix}$$

and let

$$\begin{bmatrix} H_{b11} & H_{b12} \\ H_{b21} & H_{b22} \end{bmatrix} := H(P, C\Phi_{\text{diag}}),$$

which is equal to

$$\begin{bmatrix} (I_n + PC\Phi_{\text{diag}})^{-1} & -P(I_m + C\Phi_{\text{diag}}P)^{-1} \\ C\Phi_{\text{diag}}(I_n + PC\Phi_{\text{diag}})^{-1} & (I_m + C\Phi_{\text{diag}}P)^{-1} \end{bmatrix}.$$

"Only if" part. Suppose that P is causal and $H(\Phi_{\text{diag}}P, C)$ is over $\mathcal{A}$, that is, each H_{aij} $(1 \le i, j \le 2)$ is over $\mathcal{A}$.

Since P is causal, $\Phi_{\text{diag}}P$ is strictly causal. By Proposition 1 of [12], we see that C is causal. We now show that $H(P, C\Phi_{\text{diag}})$ is over $\mathcal{A}$, that is, each H_{bij} $(1 \le i, j \le 2)$ is over $\mathcal{A}$.

H_{b22}. Since $H_{b22} = H_{a22}$, H_{b22} is over $\mathcal{A}$.

H_{b12}. The matrix $-\Phi_{\text{diag}}P(I_m + C\Phi_{\text{diag}}P)^{-1}$, which is equal to H_{a12}, is over $\mathcal{A}$. By Lemma 1, the matrix $-P(I_m + C\Phi_{\text{diag}}P)^{-1}$ is also over $\mathcal{A}$, which is equal to H_{b12}.

H_{b21}. Since H_{a21} is over $\mathcal{A}$, by Lemma 1, the matrix $C(I_n + \Phi_{\text{diag}}PC)^{-1}\Phi_{\text{diag}}$ is also over $\mathcal{A}$, which is equal to H_{b21}.

H_{b11}. Recall that $H_{a11} = I_n - \Phi_{\text{diag}}P(I_m + C\Phi_{\text{diag}}P)^{-1}C$ and $H_{b11} = I_n - P(I_m + C\Phi_{\text{diag}}P)^{-1}C\Phi_{\text{diag}}$. Since H_{a11} is over $\mathcal{A}$, the matrix $\Phi_{\text{diag}}P(I_m + C\Phi_{\text{diag}}P)^{-1}C$ is also over $\mathcal{A}$. By applying Lemma 1 twice, we see that the matrix $P(I_m + C\Phi_{\text{diag}}P)^{-1}C\Phi_{\text{diag}}$ is over $\mathcal{A}$. Hence, H_{b11} is over $\mathcal{A}$.

"If" part. Since we have

$$H(\Phi_{\text{diag}}C^t, P^t) = \begin{bmatrix} -I_m & O_{m\times n} \\ O_{n\times m} & I_n \end{bmatrix} H(P, C\Phi_{\text{diag}})^t \times \begin{bmatrix} -I_m & O_{m\times n} \\ O_{n\times m} & I_n \end{bmatrix},$$

this part can be proved analogously to the "only if" part. ∎

Now that we have the lemmas we can prove Theorem 1.

Proof of Theorem 1: We prove this theorem by showing two relations "$\supset$" and "$\subset$" of (2). We denote by Π the matrix $\begin{bmatrix} \overbrace{P^t \;\; P^t \;\; \cdots \;\; P^t}^{l} \end{bmatrix}^t$ and by Φ_{diag} the diagonal matrix $\text{Diag}(\zeta_1 I_n, \cdots, \zeta_l I_n) \in \mathcal{A}^{nl\times nl}$.

$\supset$: Let $\Gamma \in \mathcal{SP}(\Phi P)$. By noting that $\Phi P = \Phi_{\text{diag}}\Pi$ and by Lemma 2, $\Gamma\Phi_{\text{diag}}$ is in $\mathcal{SP}_{\text{s}}(\Pi)$. Let $C = \Gamma\Phi$. This C is strictly causal. A straightforward but tedious computation shows that

$$H(P, C)$$

$$= \begin{bmatrix} [\, I_n \;\; O_{n\times n(l-1)} \,] & O_{n\times m} \\ O_{m\times nl} & I_m \end{bmatrix} H(\Pi, \Gamma\Phi_{\rm diag}) \begin{bmatrix} [\, \overbrace{I_n \;\; I_n \;\; \cdots \;\; I_n}^{l}]^t & O_{nl\times m} \\ O_{m\times n} & I_m \end{bmatrix}.$$

Since the right-hand side of the above equation is over $\mathcal{A}$, $H(P, C)$ is also over $\mathcal{A}$. Now the (2, 2)-block of $H(P, C)$ is equal to the (2, 2)-block of $H(\Pi, \Gamma\Phi_{\rm diag})$. Hence, $I_n + PC$ is nonsingular. Thus, $C \in \mathcal{SP}_{\rm s}(P)$.

$\subset$: Let $C \in \mathcal{SP}_{\rm s}(P)$. Then, by Theorem 4.1 (p.889) of [6], Diag(C, P) admits both right- and left-coprime factorizations (this holds even if P does not admit right- and/or left-coprime factorizations). In addition, Diag(P, C) is a stabilizing controller of Diag(C, P). Let $P_0 =$ Diag(C, P) and $C_0 =$ Diag(P, C). Furthermore let N_0, D_0, $\widetilde{Y}_0$, and $\widetilde{X}_0$ be matrices over $\mathcal{A}$ such that

$$P_0 = N_0 D_0^{-1}, \quad C_0 = \widetilde{X}_0^{-1}\widetilde{Y}_0, \quad \widetilde{Y}_0 N_0 + \widetilde{X}_0 D_0 = I_{m+n}.$$

Since C_0P_0 is equal to Diag(PC, CP) and C is strictly causal, C_0P_0 is strictly causal. Thus, all entries of $\widetilde{Y}_0 N_0$ are in $\mathcal{Z}$. By Lemma 3.4 of [6], $\det(\widetilde{X}_0)$ is in $\mathcal{A}\backslash\mathcal{Z}$.

Decompose N_0 and $\widetilde{Y}_0$ as

$$N_0 := \begin{bmatrix} N_{01} \\ N_{02} \end{bmatrix}, \quad \widetilde{Y}_0 := [\, \widetilde{Y}_{01} \;\; \widetilde{Y}_{02} \,]$$

where $N_{01} \in \mathcal{A}^{m\times(m+n)}$, $N_{02} \in \mathcal{A}^{n\times(m+n)}$, $\widetilde{Y}_{01} \in \mathcal{A}^{(m+n)\times m}$, and $\widetilde{Y}_{02} \in \mathcal{A}^{(m+n)\times n}$. Then, let $P_{00} =$ Diag(C, Π). Since $[\, O_{n\times n} \;\; C^t \,]^t$ is strictly causal, every entry of $\widetilde{Y}_{02}$ is in $\mathcal{Z}$. Thus, there exist matrices $\widetilde{Y}_{02i}$ $(1 \le i \le l)$ over $\mathcal{A}$ such that $\widetilde{Y}_{02} = \sum_{i=1}^{l} \widetilde{Y}_{02i}\zeta_i$. Now let

$$\widetilde{Y}_{00} = [\, \widetilde{Y}_{01} \;\; \widetilde{Y}_{021}\zeta_1 \;\; \cdots \;\; \widetilde{Y}_{02l}\zeta_l \,], \quad C_{00} = \widetilde{X}_0^{-1}\widetilde{Y}_{00}.$$

By letting

$$N_{00} = \begin{bmatrix} N_{01}^t & \overbrace{N_{02}^t \;\; \cdots \;\; N_{02}^t}^{l} \end{bmatrix}^t,$$

we have $P_{00} = N_{00}D_0^{-1}$ and $\widetilde{Y}_{00}N_{00} + \widetilde{X}_0 D_0 = I_{m+n}$. Hence, C_{00} is a stabilizing controller of P_{00}.

Now let

$$\Gamma_0 = [\, O_{m\times n} \;\; I_m \,]\, C_{00}\, [\, O_{nl\times m} \;\; I_{nl} \,]^t$$

and

$$V = \begin{bmatrix} O_{nl\times m} & I_{nl} & O_{nl\times n} & O_{nl\times m} \\ O_{m\times m} & O_{m\times nl} & O_{m\times n} & I_m \end{bmatrix}.$$

Since $\det(\widetilde{X}_0)$ is in $\mathcal{A}\backslash\mathcal{Z}$, Γ_0 is strictly causal. Then, a straightforward but tedious computation shows that $H(\Pi, \Gamma_0) = VH(P_{00}, C_{00})V^t$. On the other hand, the (2, 2)-block of $H(\Pi, \Gamma_0)$ is equal to $(I_m + CP)^{-1}$. Here $I_m + CP$ is nonsingular. Thus, Γ_0 is a stabilizing controller of Π. Decompose Γ_0 as $\Gamma_0 = [\, \Gamma_{01} \;\; \cdots \;\; \Gamma_{0l} \,]$ with $\Gamma_{0i} \in \mathcal{P}_{\rm s}^{m\times n}$. One more straightforward but tedious computation shows that C is equal to $\sum_{i=1}^{l} \Gamma_{0i}$. On the other hand, we can factorize Γ_0 as $\Gamma_0 = \Gamma\Phi_{\rm diag}$, where the matrix Γ is equal to $[\, O_{m\times n} \;\; I_m \,]\, \widetilde{X}_0^{-1}\, [\, \widetilde{Y}_{021} \;\; \cdots \;\; \widetilde{Y}_{02l} \,]$, which is causal. Hence, $C = \Gamma\Phi$. By Lemma 2, $\Gamma \in \mathcal{SP}(\Phi_{\rm diag}\Pi)$. Since $\Phi_{\rm diag}\Pi = \Phi P$, we have $\Gamma \in \mathcal{SP}(\Phi P)$. ∎

4 Example

Let us consider the classical continuous-time systems. The set $\mathcal{A}$ of stable proper transfer functions is given by

$$\mathcal{A} = \{ f(s) \in \mathbb{R}(s) \mid \sup_{s\in C_{+e}} |f(s)| < \infty \}.$$

It is known that this $\mathcal{A}$ is a *Euclidean domain* with the degree function $\delta : (\mathcal{A} - \{0\}) \to \mathbb{Z}_+$:

$$\delta(f) = \text{"number of zeros of } f \text{ in } C_{+e}\text{"}.$$

The ideal $\mathcal{Z}$ for the definition of the properity is given as

$$\mathcal{Z} = \{ f \in \mathcal{A} \mid \; f = n/d, \; n, d \in \mathbb{R}[s], \; \deg(n) < \deg(d) \},$$

which is a prime and principal ideal.

Now let

$$P = s/(s-1) \in \mathcal{P}^{1\times 1}.$$

Consider to obtain the set of all stabilizing controllers of this P with relative degree more than 1. First consider

$$\mathcal{Z} = (\frac{1}{s+1}) \text{ and } \mathcal{B} = (1/(s+1)^2) \;\; (\zeta_1 = 1/(s+1)^2).$$

Thus, let

$$\Phi = [\, 1/(s+1)^2 \,].$$

Then we have

$$P\Phi = [\, s/((s-1)(s+1)^2) \,].$$

We also have

$$ny + dx = u,$$

where

$$\begin{aligned} P\Phi &= [\, n/d \,], \\ n &= \frac{s}{(s+1)^3}, \quad d = \frac{s-1}{s+1}, \\ y &= \frac{12}{s+1}, \quad x = \frac{s^3 + 3.5s^2 + 8s - 0.5}{(s+1)^3}, \\ u &= \frac{2s^3 + 3s^2 + 6s + 1}{2(s+1)^3}. \end{aligned}$$

Then u is a unit of $\mathcal{A}$ because the zeros of u are

$$-0.66 \pm 1.53i \text{ and } -0.18.$$

Now $\mathcal{SP}(P;\mathcal{B})$ can be obtained by virtue of Theorem 3. For example, letting

$$R = [\,7/(s+2)\,],$$

we obtain the following stabilizing controller:

$$\frac{2(17+19s)}{2s^4+11s^3+30s^2+17s-2}.$$

This relative degree is 3.

5 Conclusion

In the present paper, we have presented the parametrization $\mathcal{SP}(P;\mathcal{B})$ of all causal stabilizing controllers with some conditions with $\mathcal{B} \subset \mathcal{Z}$ beging a finitely-generated ideal of $\mathcal{A}$. The results of the presented paper make no assumption of coprime factorizability in principle. Since the factorization approach has been used, the result can be applied to numerous linear system models.

References

[1] K. Mori, "Parametrization of all strictly causal stabilizing controllers," *IEEE Trans. Automat. Contr.*, vol. AC-54, pp. 2211–2215, Sep. 2009.

[2] C. A. Desoer, R. W. Liu, J. Murray, and R. Saeks, "Feedback system design: The fractional representation approach to analysis and synthesis," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 399–412, 1980.

[3] V. Kučera, "Stability of discrete linear feedback systems," in *Proc. of the IFAC World Congress*, 1975, Paper No.44-1.

[4] V.R. Raman and R. Liu, "A necessary and sufficient condition for feedback stabilization in a factor ring," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 941–943, 1984.

[5] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, Cambridge, MA: MIT Press, 1985.

[6] M. Vidyasagar, H. Schneider, and B. A. Francis, "Algebraic and topological aspects of feedback stabilization," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 880–894, 1982.

[7] D.C. Youla, H.A. Jabr, and J.J. Bongiorno, Jr., "Modern Wiener-Hopf design of optimal controllers, Part II: The multivariable case," *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 319–338, 1976.

[8] V. Anantharam, "On stabilization and the existence of coprime factorizations," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 1030–1031, 1985.

[9] A. Quadrat, "On a generalization of the Youla-Kučera parametrization, Part I," *Systems and Control Letters*, vol. 50, no. 2, pp. 134–148, 2003.

[10] A. Quadrat, "On a generalization of the Youla-Kučera parametrization, Part II," *Mathematics of Control, Signals, and Systems*, vol. 18, no. 3, pp. 199–235, 2006.

[11] K. Mori, "Parameterization of stabilizing controllers over commutative rings with application to multidimensional systems," *IEEE Trans. Circuits and Syst. I*, vol. 49, pp. 743–752, 2002.

[12] K. Mori, "Elementary proof of controller parametrization without coprime factorizability," *IEEE Trans. Automat. Contr.*, vol. AC-49, pp. 589–592, 2004.

[13] V. Kučera, *Analysis and Design of Discrete Linear Control Systems*, London, United Kingdom: Prentice Hall, 1991.

[14] V. Kučera, "Parametrization of stabilizing controllers with applications," in *Advances in Automatic Control*, M. Voicu, Ed. 2003, pp. 1301–1317, Boston, MA: Kluwer.

[15] Z. Lin, "Feedback stabilization of multivariable two-dimensional linear systems," *Int. J. Control*, vol. 48, no. 3, pp. 1301–1317, 1988.

[16] K. Mori, "Parametrization of all strictly causal stabilizing controllers of multidimensional systems — single-input single-output case —," in *Proceedings of 18th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2008)*, 2008, 88–99.

[17] V. R. Sule, "Feedback stabilization over commutative rings: The matrix case," *SIAM J. Control and Optim.*, vol. 32, no. 6, pp. 1675–1695, 1994.

[18] K. Mori and K. Abe, "Feedback stabilization over commutative rings: Further study of coordinate-free approach," *SIAM J. Control and Optim.*, vol. 39, no. 6, pp. 1952–1973, 2001.

[19] S. Shankar and V. R. Sule, "Algebraic geometric aspects of feedback stabilization," *SIAM J. Control and Optim.*, vol. 30, no. 1, pp. 11–30, 1992.