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The Design of Nonlinear Servo System Using Fuzzy Method

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1. INTRODUCTION

Fuzzy theory had been introduced by Prof.L.A.Zadeh in 1965[1]. In 1974, Prof.E.Mamdani has successfully applied the first fuzzy controller on real control system [2]. After that, the method has been successfully implemented to the real-world application like cement kilns process, subway control system, and electrical appliances [3]-[5].

In early study, fuzzy control system has been introduced as model free control design. The method is easy to implement but difficult to establish the tools for stability and robustness analysis in close-loop system. Because of that, most of the recent studies are based on model based fuzzy control system.

Most of the studies in model based fuzzy control system are concentrating on regulator problem. The problem involved with the design of the controller that can drive all the initial condition to zero as required by the design specification [6]. However, only found a few study on the application of fuzzy method for nonlinear servo system in literature [7]-[11]. This type of system can regulate the control variables without steady state error against unknown disturbance.

In this paper, the controller design for nonlinear servo system by using fuzzy method will be discussed. The idea behind of this control method is to divide the operating region of nonlinear servo system into small area, and treated as a collection of local linear servo systems. The feedback co-efficient of each local linear servo systems have been calculated using Hikita Method [13]. Fuzzy method has been applied to each local linear servo system and combines it as new control law (Fig. 1).The simulation of the nonlinear mass-spring-damper system have been done to illustrate the effectiveness of the proposed method. The results shows that the proposed method can stabilize the nonlinear mass-spring-damper system.

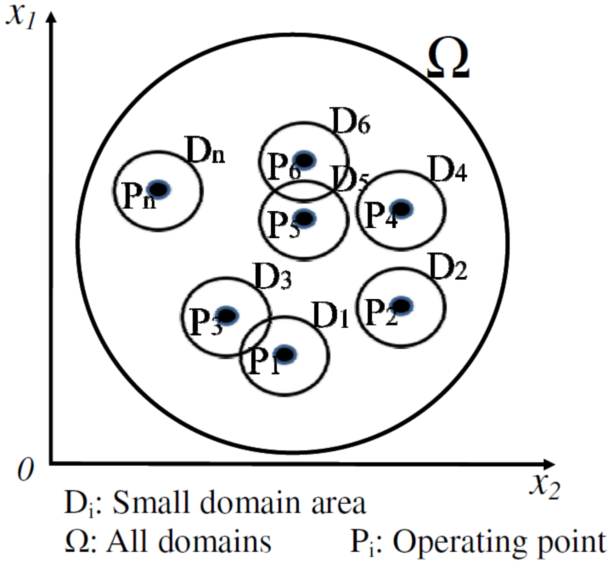


Fig.1 The idea of new fuzzy controller

2. LINEARIZATION OF THE SYSTEM

Let the original system S be a nonlinear system one as

$$\begin{aligned}\dot{x} &= f(x,u)+d \\ y &= g(x)+d_o\end{aligned} \tag{1}$$

$$x\in R^n, u\in R^m, y\in R^l\,(m\geq l)$$

where x is state vector, y is control output, u is control input, d is state disturbance and d_o as output disturbance. m, l and n are dimension of state, input and output of the system. Define an error of the system as

$$\dot{v} = y - r. \tag{2}$$

Assume that the nonlinear system operate around operating point (x_i, u_i).and

$$\begin{aligned}x &= x_i + \delta x \\ u &= u_i + \delta u,\end{aligned} \tag{3}$$

The system can be linearized by applying Taylor expansion to (1) around operating point (x_i, u_i),where

$$\delta\dot{x} = \frac{\partial}{\partial x^T} f(x_i, u_i)\delta x + \frac{\partial}{\partial u^T} f(x_i, u_i)\delta u + f(x_i, u_i) + d$$
$$y = \frac{\partial}{\partial x^T} g(x_i)\delta x + g(x_i) + d_o. \qquad (4)$$

The linear approximated system S_i can be represented as;

$$\dot{x} = A_i x + B_i u + d_{xi}$$
$$y = C_i x + d_{oi}, \qquad (5)$$

where

$$A_i = \frac{\partial}{\partial x^T} f(x_i, u_i), B_i = \frac{\partial}{\partial u^T} f(x_i, u_i), C_i = \frac{\partial}{\partial x^T} g(x_i),$$
$$d_{xi} = f(x_i, u_i) - A_i x_i - B_i u_i + d, \qquad (6)$$
$$d_{oi} = g(x_i) - C_i x_i + d_o.$$

T-S fuzzy model can be obtained as following.

IF $(x^T, u^T)^T \in D_i$ THEN S is S_i (7)

3. DESIGN OF SERVO SYSTEM

In this section an introduction about the design of servo system will be given [12]. The system that has ability to follow the reference given is called as servo system. This type of system can regulate the control variables without steady state error against unknown disturbance. Hikita method [13] has been used to calculate the feedback co-efficient matrix for the system.

Re-write (5) with the objective of the control is to make the output y follow the reference r.

$$\dot{x} = A_i x + B_i u + d_{xi}$$
$$y = C_i x + d_{oi} \qquad (8)$$

And the error of the system as

$$\dot{v} = y - r. \qquad (9)$$

Derive the augmented system from (8) and (9).

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u + \begin{bmatrix} d_{xi} \\ d_{oi} - r \end{bmatrix} \qquad (10)$$

Equation (10) can be re-written as,

$$\dot{z} = A_{zi} z + B_{zi} u + d_{zi} \qquad (11)$$

where

$$z = \begin{bmatrix} x \\ v \end{bmatrix}, A_{zi} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix}, B_{zi} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, d_{zi} = \begin{bmatrix} d_{xi} \\ d_{oi} - r \end{bmatrix}. \qquad (12)$$

The system in (11) is controllable if controllability condition satisfies.

$$rank\begin{bmatrix} B_{zi} & A_{zi}B_{zi} & A_{zi}^2 B_{zi} & \cdots & A_{zi}^{n+l-1} B_{zi} \end{bmatrix} = n + l \qquad (13)$$

This condition is equivalent to the next one.

$$rank\begin{bmatrix} B_i & A_i B_i & A_i^2 B_i & \cdots & A_i^{n-1} B_i \end{bmatrix} = n,$$
$$rank\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} = n + l \qquad (14)$$

Define the input of the system as,

$$u = -Kx. \qquad (15)$$

K is the feedback coefficient matrix of the input. The feedback coefficient matrix calculated using Hikita method [13]. Let,

$$f_{ij} = -(\lambda_j - A_{zi})^{-1} B_{zi} g_j$$
$$K_i = \begin{bmatrix} g_1 & \cdots & g_{n+l} \end{bmatrix}\begin{bmatrix} f_{i1} & \cdots & f_{i(n+l)} \end{bmatrix}^{-1} \qquad (16)$$
$$(j = 1,2,3,\cdots n + l),$$

where

$$g_j \in R^m, f_{ij} \in R^{n+l}.$$

λ_j in above equation are the eigenvalues and f_{ij} is eigenvector of the system. The feedback co-efficient matrix of the system can be represented as

$$K_i = \begin{bmatrix} k_{i1} & \cdots & k_{i(n+l)} \end{bmatrix}. \qquad (17)$$

4. FUZZY FEEDBACK CONTROLLER

In this section we applied fuzzy method to develop the control law for the nonlinear system. First, we divide the operating region of nonlinear system into small area, and treated as a collection of local linear servo systems. For that purpose we use the linear model that has been developed in previous chapter, as shown in (5). The feedback coefficient matrix of each local linear system is calculated using Hikita method [13]. In order to develop the control law, fuzzy method has been apply to each local linear system and combines it as new control law. The control input u is

$$u = -\frac{\sum_{i=-N}^{N} \omega_i K_i z}{\sum_{i=-N}^{N} \omega_i} \qquad (18)$$

$$\omega_i = e^{-(z_v - z_{vi})^T Q_v (z_v - z_{vi})} \qquad (19)$$

ω_i is called as fuzzy membership function (Fig.2) , and $Q_v = Q_v^T > 0$. We take ω_i as Gaussian type fuzzy membership function. Where,

$$z_v = C_v \begin{bmatrix} x \\ u \end{bmatrix}. \qquad (20)$$

$z_v \in R^{n_v}$ is a vector of nonlinear elements, which are included in controlled system in the equation. And, $C_v \in R^{n_v \times (n+m)}$ is a matrix of which its elements are 1 or 0. For example, let

$$x^T = (x_1 \quad x_2 \quad x_3), u^T = (u_1 \quad u_2). \qquad (21)$$

In case of x_1, x_2, u_1 are nonlinear, z_v *and* C_v can be defined as

$$z_v = \begin{bmatrix} x_1 \\ x_2 \\ u_1 \end{bmatrix}, C_v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \qquad (22)$$

Apply the input in (18) into nonlinear equation (1).The simulation have been done to show the effectiveness of this new method.

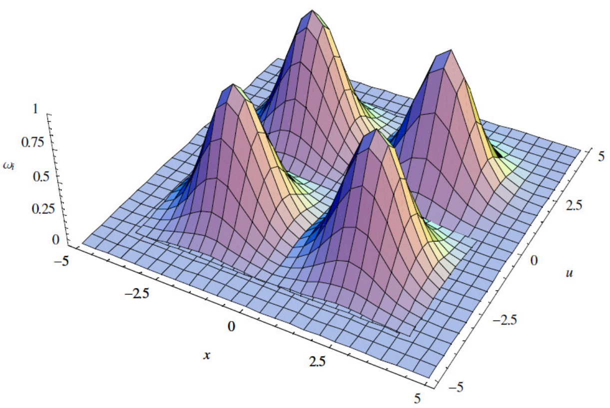


Fig.2 Gaussian type fuzzy membership function ω_i

5. SIMULATION AND RESULTS

5.1. Simulation of nonlinear mass-spring-damper system

The simulations have been done in order to illustrate the proposed method. The objective of the simulation is to show that the method can be implementing into nonlinear system. We also want to investigate the effect to the output of the system when changing poles. The proposed method has been applied to nonlinear mass-spring-damper system as shown in Fig. 3.

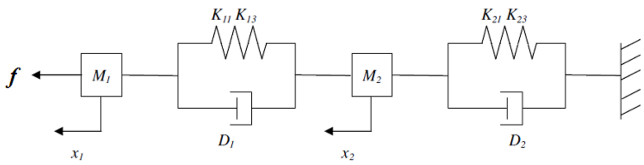


Fig.3 Nonlinear mass-spring-damper system

The mathematical model of the nonlinear mass-spring-damper system is given by

$$\begin{aligned} &K_{11}(x_1 - x_2) + K_{13}(x_1 - x_2)^3 + D_1(\dot{x}_1 - \dot{x}_2) - M_1\ddot{x}_1 = F \\ &K_{11}(x_1 - x_2) + K_{13}(x_1 - x_2)^3 + D_1(\dot{x}_1 - \dot{x}_2) \\ &-(K_{21}x_2 + K_{23}x_2^3 + D_2\dot{x}_2) - M_2\ddot{x}_2 = 0. \end{aligned} \tag{23}$$

Each of parameters is defined as shown in Table 1.

Table 1 Parameters value of nonlinear mass-spring-damper system

Mass M_1=1.0kg	Mass M_2=0.5kg
Spring Coefficient K_{11}=100N/m	Spring Coefficient K_{21}=50N/m
Spring Coefficient K_{13}=10N/m^3	Spring Coefficient K_{23}=3N/m^3
Damping coefficient D_1=8.0Ns/m	Damping coefficient D_2=3.0Ns/m

Re-arranged the equation in term of $\dot{x}_1, \dot{x}_2, \dot{x}_3$ and $\dot{x}_4$ where $x_1 = x_1, x_2 = x_2, x_3 = \dot{x}_1, x_4 = \dot{x}_2$ and $F = u$. We get,

$$\begin{aligned} \dot{x}_1 &= x_3, \\ \dot{x}_2 &= x_4, \\ \dot{x}_3 &= \tfrac{1}{M_1}(u - K_{11}x_1 - K_{13}x_1^3 + K_{11}x_2 + 3K_{13}x_1^2 x_2 \\ &- 3K_{13}x_1x_2^2 + K_{13}x_2^3 - D_1x_3 + D_1x_4), \\ \dot{x}_4 &= \tfrac{1}{M_2}(K_{11}x_1 + K_{13}x_1^3 - K_{11}x_2 - K_{21}x_2 - 3K_{13}x_1^2x_2 \\ &+ 3K_{13}x_1x_2^2 - K_{13}x_2^3 - K_{23}x_2^3 + D_1x_3 - D_1x_4 - D_1x_2). \end{aligned} \tag{24}$$

With output as

$$y = x_1. \tag{25}$$

Define an error of the system as

$$\dot{v} = y - r. \tag{26}$$

The linearization has been done to mathematical model of the nonlinear mass-spring-damper system. The state space equation for linear servo system can be represent as

$$\begin{aligned} &\dot{z} = A_{zi}z + B_{zi}u + d_{zi} \\ &A_{zi} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ a_{13} & a_{23} & a_{33} & a_{43} & 0 \\ a_{14} & a_{24} & a_{34} & a_{44} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, B_{zi} = \begin{bmatrix} 0 \\ 0 \\ b_{13} \\ 0 \\ 0 \end{bmatrix} \\ &d_{zi} = \begin{bmatrix} 0 \\ 0 \\ d_{13} \\ d_{14} \\ -r \end{bmatrix}. \end{aligned} \tag{27}$$

Each co-efficient can be described as the following

$$\begin{aligned} a_{13} &= \frac{-K_{11} - 3K_{13}x_{1i}^2 + 6K_{13}x_{1i}x_{2i} - 3K_{13}x_{2i}^2}{M_1}, \\ a_{23} &= \frac{K_{11} + 3K_{13}x_{1i}^2 - 6K_{13}x_{1i}x_{2i} + 3K_{13}x_{2i}^2}{M_1}, \\ a_{33} &= \frac{-D_1}{M_1}, a_{43} = \frac{D_1}{M_1}, \\ a_{14} &= \frac{K_{11} + 3K_{13}x_{1i}^2 - 6K_{13}x_{1i}x_{2i} + 3K_{13}x_{2i}^2}{M_2} \\ a_{24} &= \frac{-K_{11} - K_{21} - 3K_{13}x_{1i}^2 + 6K_{13}x_{1i}x_{2i} - 3K_{13}x_{2i}^2 - 3K_{23}x_{2i}^2}{M_2} \\ a_{34} &= \frac{D_1}{M_2}, a_{44} = \frac{-D_1 - D_2}{M_2}, b_{13} = \frac{1}{M_1}. \end{aligned} \tag{28}$$

The nonlinear variable in this simulation can be determined as

$$z_v = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C_v \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}, \tag{29}$$

with the fuzzy membership function ω_i can be represented as

$$\omega_i = e^{-(z_v - z_{vi})^T Q_v (z_v - z_{vi})} \quad (30)$$
$$= e^{-q_1 (x_1 - x_{1i})^2 - q_2 (x_2 - x_{2i})^2},$$

where

$$Q_v = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}. \quad (31)$$

The simulation has been done with initial condition $z_0 = [0 \quad 0 \quad 0 \quad 0 \quad 0]^T$ .The parameter used for simulation are shown in Table 1.The poles have been selected from negative real number and the feedback coefficient matrices are calculated using Hikita method as describe in section before.

The results of the simulation are shown below. The reference *r*, used in simulation is *r* = 1.0m. The set of poles used for simulation is shown in Table 2.

Table 2 Set of poles for simulations

Poles 1	-1.1,-1.2,-1.3,-1.4,-1.5
Poles 2	-5.1,-5.2,-5.3,-5.4,-5.5
Poles 3	-10.1,-10.2,-10.3,-10.4,-10.5
Poles 4	-25.1,-25.2,-25.3,-25.4,-25.5

5.2. The results of simulation

The results in Fig. 5 show the input and output of nonlinear mass-spring-damper system. The vertical axes represent displacement of system in unit meter *m* and input of the system in unit Newton *N*. The horizontal axis represents time in unit second *s*. The output for pole-placement method in case of simple linear approximation method is shown in Fig.4. From the result, we can say that the nonlinear mass-spring-damper system cannot be controlled by using simple linear approximation method.

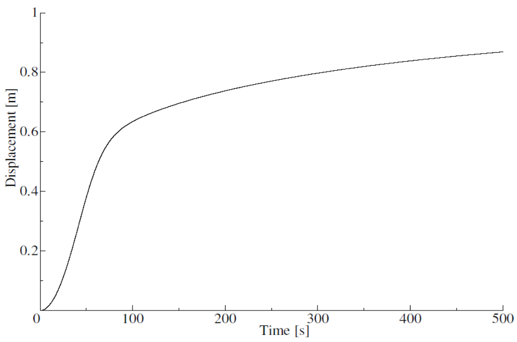
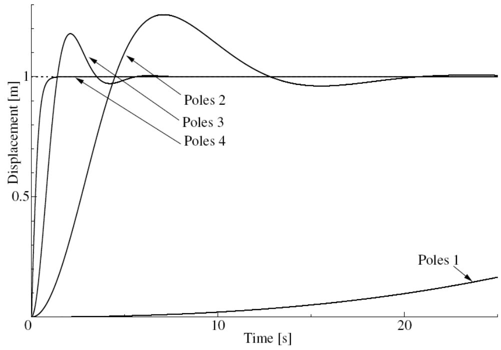
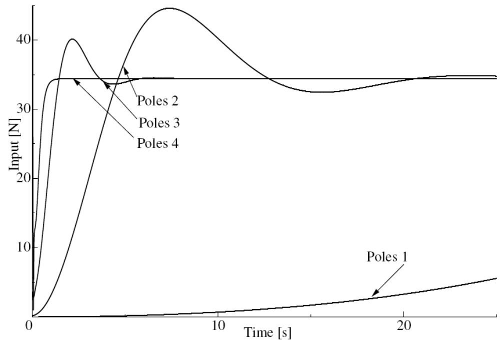


Fig.4 Output in case of simple linear approximation method.

The results shown that method proposed can stabilize the mass-spring-damper system .The result is not proper in case of using set of poles 1 of which poles are near the origin of s-plane. For set of poles 2, the overshoot from reference point is 27% with settling time 18.87s. For set of poles 3, the overshoot from reference point is 18.6% with settling time 4.92s. For set of poles 4, the overshoot from reference point is 0% with settling time 0.83s. The results shows that, when the set of poles chosen further to the left from origin in s-plane the settling time and overshoot of the system output going fastest and smallest. The result also shows that smaller input need to stabilize the system when the poles chosen further to the left from origin in s-plane.



(a)



(b)

Fig. 5 Result for simulation using different set of poles: (a) the outputs and (b) inputs of the simulations.

6. CONCLUSION

In this paper, we introduced new fuzzy control method for nonlinear servo system. The implementations of the new control method have been discussed. The simulations have been done to mass-spring-damper system and the results shown that method proposed can stabilize the system. The result also shows the effect of set of poles λ .As shown in the result, the output of the system follows the reference given and by choosing set of poles further to the left from origin in s-plane the settling time and overshoot of the system output going fastest and smallest. For future work, study of characteristic of the proposed method will be done by doing the simulation using other system together with the study of the stability issue.

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