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Research on the Stabilization Control for Inverted Pendulum using Fuzzy Method

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***Abstract*—Inverted pendulum is suitable to use for investigation and verification of various control methods for dynamic systems. In this paper, Lagrange method has been applied to develop the mathematical model of the system. The objective of the simulation to be shown using the fuzzy control method can stabilize the nonlinear system of inverted pendulum.**

1. INTRODUCTION

Inverted pendulum is the abstract model of many control problems which the gravity center is upper and the fulcrum is lower and it is unstable object. In the control process it can reflect a number of key issues effectively such as the system stability, non-linear problem, controllability, robustness and so on. As a controlled object it is a higher order, nonlinear, multi-variable, strong coupling and unstable rapid control system. Walking robot joint control, the vertical degree of control in rocket launch, satellite attitude control, those all related to the stability of the control problem that of upside-down objects. Therefore, the inverted pendulum control strategies can be applied to aerospace, military, robotics, industry and others` areas, to solve the balance problems.

2. ESTABLISHING MATHEMATICAL MODEL

Inverted pendulum is composed by cart, pendulum, guide and so on as in Fig.1.

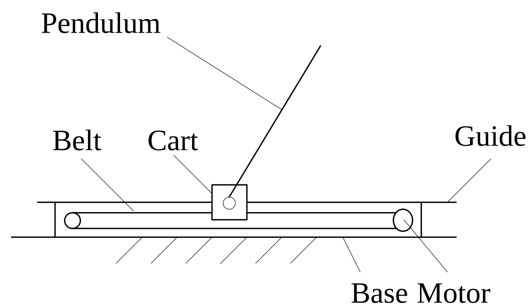


Fig.1 Configuration of inverted pendulum

Lagrange method is adopt to establish the mathematical model because the number of equations equal to the degree of freedom system and modeling process can be simplified. Lagrange equation is shown in Eq. (1).

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \\ L(q,\dot{q}) = T(q,\dot{q}) - V(q) \end{cases} \tag{1}$$

By calculating, the mathematical model equation of inverted pendulum is shown as in Eq.(2)

$$\begin{cases} (m+M)\ddot{x} + Da\dot{x} + mlcos\theta\ddot{\theta} - ml\dot{\theta}^2 sin\theta = Gu \\ \frac{4}{3}ml^2\ddot{\theta} + mlcos\theta\ddot{x} + C\dot{\theta} = gmlsin\theta \end{cases} \tag{2}$$

So let $X = (x \quad \dot{x} \quad \theta \quad \dot{\theta})^{\mathrm{T}} = (x_1 \quad x_2 \quad x_3 \quad x_4)^{\mathrm{T}}$.

Using appropriate mathematical transformation, the model equation can be written in another form as Eq.(3).

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \frac{Gu - Fx_2 + mlsin\,x_3 x_4{}^2 - gmcos\,x_3 sin\,x_3}{m + M + mcos^2 x_3} \\ \dot{x_3} = x_4 \\ \dot{x_4} = \frac{Gucos\,x_3 + Fx_2 + g(M+m)sin\,x_3 - mlsos\,x_3 sin\,x_3 x_4{}^2}{l(m + M + mcos^2 x_3)} \end{cases} \tag{3}$$

3. STABILITY ANALYSIS

Inverted pendulum is a multi-variable, complex nonlinear system. In order to facilitate control strategy, linear the model near the balance point. Then could get inverted pendulum linear equation of state:

$$\begin{cases} \dot{X} = AX + Bu \\ Y = CX \end{cases} \quad (4)$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -5.55 & -0.827 & 0.052 \\ 0 & 0 & 0 & 1 \\ 0 & 20.8 & 39.8 & -2.54 \end{pmatrix} \; B = \begin{pmatrix} 0 \\ 76.4 \\ 0 \\ -286 \end{pmatrix} \; C = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}^{T}$$

Base on the Lyapunov Law: The non-linear control system can be analyzed by the linearization system. The controllability matrix of the control system

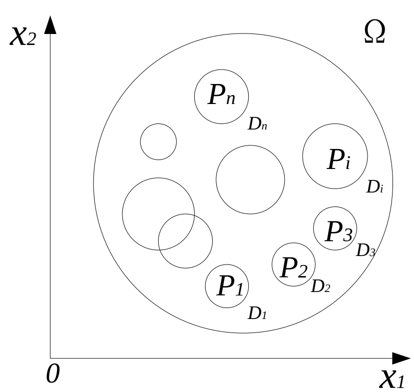
$$S = [B \;\; AB \;\; A^2B \;\; \cdots \;\; A^{n-1}B\,]$$

is full-rank. rank(S) =4. So the inverted pendulum system is controllable.

4. FUZZY CONTROL

4.1 Fuzzy method

Fuzzy method has been used as alternative method to develop control rule for complex system. The method has been successfully implemented to the real-world application like engine control system of subway train. The idea behind of control method in this paper is to divide the operating region of nonlinear system into small area, and treated as a collection of local linear systems. After that , in order to develop the control law, fuzzy method has been apply to each local linear system and combines it as new control law as shown in Fig.2.



Di : small domain Ω: all domain
Fig.2 Driving domain

4.2 Linearization

As the mathematical model equation of inverted pendulum is nonlinear equation, let $x = x_i + \delta x,\; u = u_i + \delta u$, then equation Eq.(3) is linearized by Eq.(5)

$$\dot{\delta x} = \frac{\partial}{\partial x^T} f(x_i, u_i)\delta x + \frac{\partial}{\partial u^T} f(x_i, u_i)\delta u + f(x_i, u_i) + d \quad (5)$$

So the linear equation of inverted pendulum is shown as Eq.(6)

$$\begin{cases} \dot{x} = A_i x + B_i u + d_{xi} \\ y = C_i x + d_{yi} \end{cases} \quad (6)$$

$$A_i = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}, B_i = \begin{pmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{pmatrix}, d_{xi} = \begin{pmatrix} 0 \\ d_2 \\ 0 \\ d_4 \end{pmatrix}$$

$a_{22} = -4Da/(4m + 4M - 3m(cosx_{3i})^2)$

$a_{23} = -3gm(cosx_{3i})^2/(4m + 4M - 3m(cosx_{3i})^2)$

$a_{24} = 3Ccosx_{3i}/(4ml + 4Ml - 3ml(cosx_{3i})^2)$

$a_{42} = 3Dacosx_{3i}/(4ml + 4Ml - 3ml(cosx_{3i})^2)$

$a_{43} = 3g(m + M)cosx_{3i}/(4ml + 4Ml - 3ml(cosx_{3i})^2)$

$a_{44} = -3C(m + M)/(4m^2l^2 + 4mMl^2 - 3m^2l^2(cosx_{3i})^2)$

$b_2 = 4/(4m + 4M - 3m(cosx_{3i})^2)$

$b_4 = 3cosx_{3i}/(4ml + 4Ml - 3ml(cosx_{3i})^2)$

$d_2 = -3gmcosx_{3i}sinx_{3i}/(4m + 4M - 3m(cosx_{3i})^2)$

$b_4 = 3g(m + M)sinx_{3i}/(4ml + 4Ml - 3ml(cosx_{3i})^2)$

4.3 Fuzzy rule

The state equation of inverted pendulum can be shown as

$$\begin{cases} \dot{x} = f(x, u) + d \\ y = g(x) + d_0 \end{cases} \quad (7)$$

Set $z_v = C_v \begin{bmatrix} x \\ u \end{bmatrix} \in R^{n_v}$, so $z_v \in D_i$, the fuzzy rule is

$$if \;\; z_v \in D_i \;\; then\; S\; is\; S_i \;\; (-N \le i \le N) \quad (8)$$

And in this part, S and S_i are following.

S: $\dot{x} = f(x, u) + d$, $\quad y = g(x) + d_0$

S_i: $\dot{x} = A_i x + B_i u + d_{xi}$, $\quad y = C_i x + d_{yi}$

4.4 Simulation

Comprehensive analysis part B and C, feedback coefficient K_i can be obtained from every small domain D_i with pole assignment. Fuzzy input u is

$$u = -\frac{\sum_{i=-N}^{N} \omega_i K_i x}{\sum_{i=-N}^{N} \omega_i} \quad (9)$$

$$\omega_i = e^{-q_i(x - x_i)^2} \quad (10)$$

The initial angle of θ is 25° and initial condition of simulation $x(0) = (0 \quad 0 \quad 0.44 \quad 0)^T$. The system simulate with λ_1=λ_2=λ_3=λ_4=-0.80, and $q = 5$. The results of simulation are shown in Fig.3

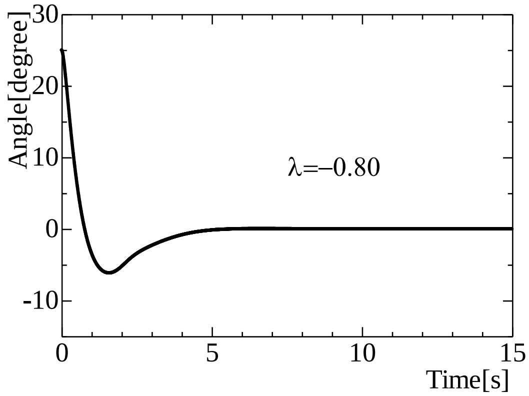


Fig.3 Result for simulation of θ

As the result of simulation, the system is stable with this fuzzy input, but it is stable which initial angle of θ is between ±25°. It is necessary for enlarging controllable angle to change fuzzy input. So set

$$\omega_i = e^{-q_1(x_1-x_{1i})^2-q_2(x_2-x_{2i})^2} \quad (11)$$

The initial angle of θ is 50° and initial condition of simulation $x(0) = (0 \quad 0 \quad 0.87 \quad 0)^T$.The system simulate 3 times which λ_1= λ_2= λ_3= λ_4 were -0.80, -0.78 and -0.82 separately and $q_1 = 5,\ q_2 = 1$. The results of simulation are shown in Fig.4,Fig.5,Fig.6.

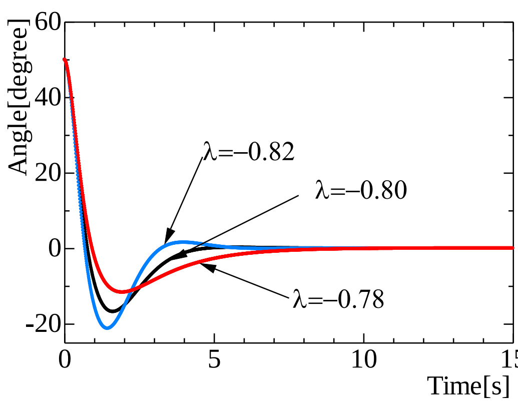


Fig.4 Result for simulation of θ

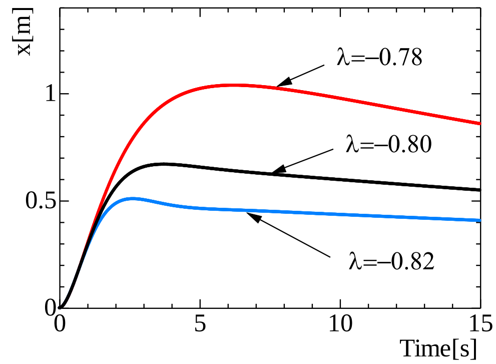


Fig.5 Result for simulation of x from 0s to 15s

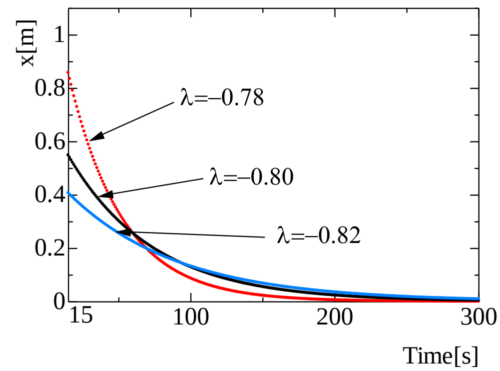


Fig.6 Result for simulation of x from 15s to 300s

The simulation results show that with different poles, the system response different characteristics. With smaller pole, the overshoot of pendulum is smaller and settling time of pendulum is shorter. However, overshoot of cart is larger. Settling time of cart has the approximate same performance.

5. Conclusion

In this paper, mathematical model of inverted pendulum is established based on Lagrange method, and from the simulation results, the fuzzy method can make nonlinear system stable. This fuzzy method adapt to most of nonlinear system. For different needs, the system can provide different response characteristics. In the future, servo system will be established and simulated to improve the system performance by changing the parameters.

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TABLE 1

Symbol	Quantity	Units
x	Is 0 when cart be in the equilibrium position	*m*
θ	Angle between Vertical axis and pendulum	*degree/ rad*
u	Output voltage of DC torque motor	*v*
$l=0.2$	Length of pendulum's center of gravity to its rotation axis	*m*
$m=0.08$	Mass of pendulum	*kg*
$M=0.7$	Mass of cart	*kg*
$G=55$	Coefficient of force by DC torque motor	*N/v*
$Da=4.0$	Coefficient of friction between cart and system	*Ns/m*
$C=0.01$	Coefficient of friction between pendulum and rotation axis	Kgm^2/s