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Scientific Study of Road Traffic Flow (2)

-Measurement and Example of Data Analysis-

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Continued from "Scientific Study of Road Traffic Flow (I)" presented at the previous meeting of 262nd SICE Tohoku at Tohoku University on Dec. 23, 2010, when Sections 1.2 and 3 were presented. Thus in this presentation we shall begin from the Section 4.

The content of the previous presentation is as follows:

II Road Traffic Flow Fundamentals from Weber-Fechner Law

3. Weber-Fechner Law in Road Traffic Flow

3.1 Human Factors in road traffic flow
3.2 Car following model
3.3 Relationship between X and V as Weber-Fechner Law
3.4 Mathematical Verification of V-X Weber-Fechner Relationship
3.5 Fechner's Law in Road Traffic Flow and Meaning of $\beta V = 1$
Appendix 3-1
Appendix 3-2

4. Measurements and Example of Data Analyses

4.1 Parameters X and V

As mentioned in Section 3, our independent parameters to be measured are Time-Headway T(s) and Car-Speed V (m/s). The Distance-Headway X can be given from T and V as X=TV that was shown in (3-1). We have seen in the Section 3 that the car driver senses X through driver's eye and the X becomes the physical stimuli to the driver, and the car speed V is controlled by the driver's sensation for maintaining safety. Although V is a physical quantity, car driver cannot sense it as a physical quantity. Drivers can recognize the physical quantity of speed only by a speed meter that installed in front of the driver. From such a situation was taken into account, both parameters X and V have the relationship of Weber-Fechner law each other, as we saw in Section 3 by invoking the Weber-Fechner law in Section 2.

4.2 Measurements

In order to pursue the investigations for basic characterizations on the road traffic flow, we need to measure both T and V simultaneously. We used a video camera and manual watches. Simple equipment was constructed for operating the manual watches, one for T and another for V, because it was almost impossible to operate the two watches simultaneously. The equipment made possible to operate the both watches simultaneously with one lever with a manual operation. The principle of measurements and construction of the equipment is respectively shown as follows:

Fig.4-1 shows two cars, designated by i and i+1, foregoing and its following one respectively. Top figure shows the time instant when the foregoing vehicle i touched at the designated point A, middle is when i touched B, and bottom is when i+1 touched A .The point C is the position of foregoing car i, when i+1 touched A. The run distance of the car i is X_i which is the run distance within the time until i+1 touches the point A. The distance between A and B is L which is set 10m for ordinary urban or suburban road, and 20m for high speed expressway for avoiding measuring error due to high speed. The positions A and B are set in advance on the video image appropriately.

The pulse wave on the left side in this figure shows the timings when the car i and i+1 moved at the designated points A and B, respectively: t_{iA} is when i touched A, t_{iB} when i touched B, and t_{i+1A} when i+1 touched A. Thus the time-headway T_{iL} is

$$T_{iL} = t_{iB} - t_{iA} \ . \qquad (4\text{-}1)$$

The speed can be calculated by

$$V_i = \frac{L}{T_{iL}} \ . \qquad (4\text{-}2)$$

And Xi is

$$X_i = V_i T_i \ . \qquad (4\text{-}3)$$

The equipment is shown in Fig.4-2, which makes possible to operate the two watches simultaneously.

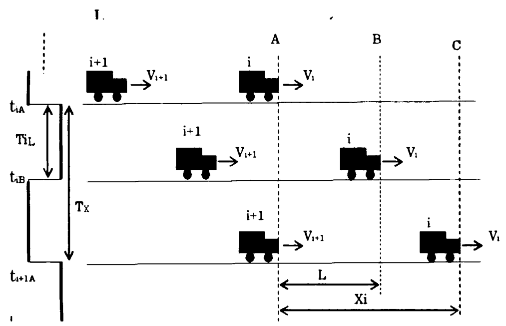


Fig.4-1 Model description of two cars successively running on road

The person, who is measuring, watches the video image in which cars continuously flow. As mentioned above, the positions A and B have been set in advance on the road in video image with the distance of 10m (for urban or suburban road) or 20m (for high speed expressway). The same pulse waveform is shown within Fig.4-2 that is shown in Fig.4-1.

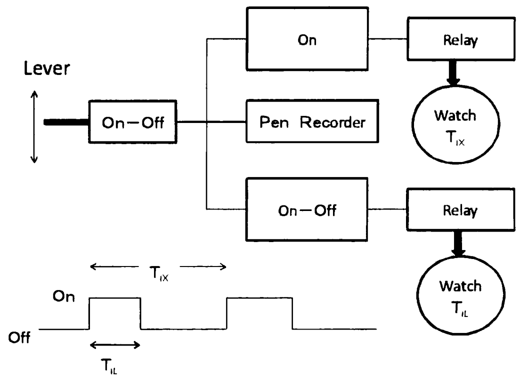


Fig.4-2 Equipment for operating two watches with one lever

The watch Tx can sense only when the lever On, then time-headways (T_i, i =1, 2, ···) are measured and stored in the memory.

The watch T_{iL} senses both On and Off, so the vehicle run time of distance L is measured and stored. The used watch is 'SEIKO Super Athlete', which can memorize 300 data. The speed can be obtained by (4-2).

An example of the actually recorded waveform with the pen recorder, together with the

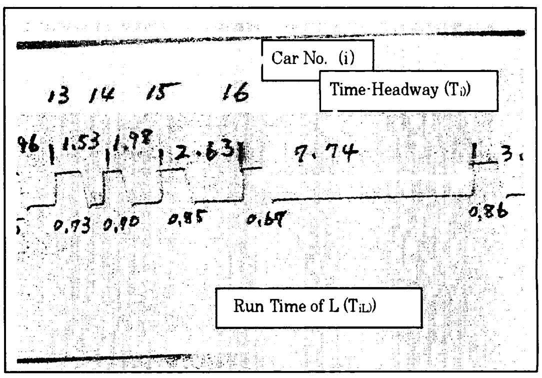


Fig.4-3 Example of recorded data

readout data written on the recording paper, is shown in Fig.4-3 which was recorded by the recorder in the equipment shown in Fig.4-2.

The numerical data indicated by arrows in Fig.4-3 are summarized in Table 4-1. Like this example, the video data can be compiled. The final data necessitated for analyzing the flow are T, V and X. These data can be obtained with respect to elapsed time that can be obtained by summing up of T_i. The elapsed time t_i when the car i just touched the point A can be obtained by

$$t_i = \sum_{i=1}^{i} T_i \ . \qquad (4\text{-}4)$$

Table 4-1 Data obtained from recorded waveform shown in Fig. 4-3

Car No.	Time-headway (T(s))	Speed (V(m/s))	10m run time (T_L(s))	Distance-headway (X(m))
16	7.74	14.9 From(4-2)	0.67	115.3 From (4-3)

The number of measured car is at most up to 300 because of the memory limit. The number of cars of 300 is enough for analyses for time domain analysis, but for the regular statistical analysis, we need more number of cars is necessary. However, since we have a plenty of investigation of statistical data analyses performed so far in the world, we need not here to concentrate to accumulate so many data.

We saw that the necessitate data are time-headway T, speed V and distance headway X (=TV). Table 4-2 is a part of EXCEL data of T, V and X with respect to elapsed time t_i, where i is the car No. The data shown in Table 4-2 were taken from the video at the parking area at the Tohoku Expressway, Azuma in Miyagi Prefecture, Japan.

In Table 4-2, i: car No., t_i: elapsed time, T(s): Time-Headway (second), V (m/s): speed (meter/second), X(m): Distance-Headway (meter).

Total numbers of cars measured are 290. All data were analyzed and graphs of t-T, t-V and t-X were made and showed in Figs. 4-4, which are shown in the next Subsection.

Table 4-2 Example of a part of compiled data

i	t_i(s)	T (s)	V(m/s)	X(m)
50	161.47	2.06	11.11	22.96
51	164.18	2.71	9.70	26.35
52	171.41	7.23	9.90	71.58
53	175.10	3.69	11.49	42.41
54	180.35	5.24	10.36	54.35
55	183.48	3.13	8.52	26.67
56	187.52	4.04	10.10	40.87
57	193.27	5.74	8.84	50.82
58	196.87	3.60	10.83	38.98
59	198.74	876	12.04	22.61
60	200.13	390	9.20	12.79

4-3 Data plots via elapsed time

Fig. 4-4 shows the measured data which include the data of Table 4-2 plotted with respect of elapsed time t (= t_i in Table 4-2) (a) t-T, (b) t-V and (c) t-X, respectivley. The abscissas of the above three figures are of the same scale so that we can compare their features each other.

If we look (a), (b) and (c), we will be able to see something interested as follows:

1) From variations of the Time-Headway T with respect to time t, we may not be able to deduce useful information available for practical applications, although T is one of the most essential parameters. The essential reason is, as mentioned again in the following 2), that T becomes large in both cases of heavy or congesting traffic condition and of low density and high speed condition.
2) Above 1) suggest that another parameter V is needed and, if we look at the figure (b), we will get more information about the traffic flow, e.g. when the low speed continues we can see it as the situation of congestion (high density). We shall note that time-headway T (Fig.4-4(a)) becomes large in case of congestion, but it also happens to become large when flow-density is low (high speed).
3) Figure (c) may have compiled all information on the road traffic flow,

because the two essential parameters T and V are involved in (c) as the product that provides the distance-headway X (=TV).

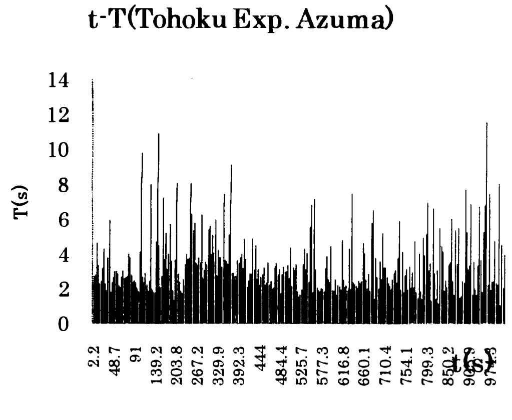


Fig.4-4 (a) t-T plot

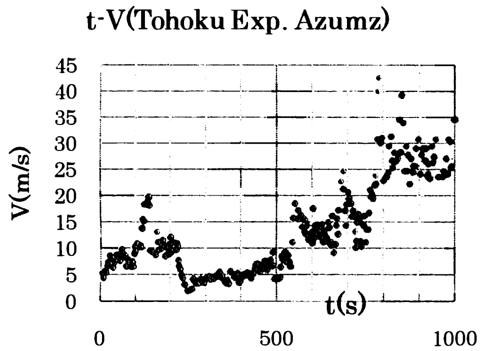


Fig.4-4 (b) t-V plot

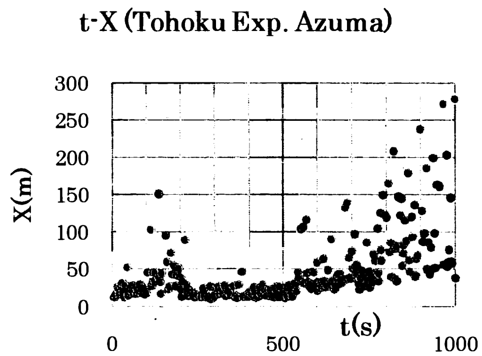


Fig.4-4 (c) t-X plot

4.4 Embedding data into Weber' law and Fechner' law

As mentioned in Section 3, the relationship between T and V obeys the Weber-Fechner law, and this was mathematically proved if we admit $\beta V = 1$, the maximum flow condition as we have seen in Subsection 3.5. This condition should be scrutinized later as the special subject (Section 5).

In this subsection, we will see the graphs in which the data like Table 4-2 are embedded into both Weber' law (3-3) and Fechner's law (3-7) shown in Section.3.

4.4-1 Weber's law

The Weber's law in the present case was formula (3-3) in Section 3, which is shown again here as follows:

$$\frac{\Delta X}{X} = \beta \Delta V. \quad \text{(3-3) (from Section 3)}$$

A part of the data to be embedded is shown in Table 4-3 as an example which is calculated from Table 4-2 by

$$\Delta V_i = V_i - V_{i+1} \qquad (4\text{-}5),$$

and

$$\Delta X_i = X_i - X_{i+1} \qquad (4\text{-}6).$$

Table 4-3 Example of calculations of ΔV, ΔX and $\Delta X/X$

i	V(m/s)	X(m)	ΔV	ΔX	$\Delta X/X$
50	11.11	22.96	1.41	-3.39	-0.147
51	9.70	26.35	-0.2	-45.23	-1.716
52	9.90	71.58	-1.59	29.17	0.407
53	11.49	42.41	1.13	-11.94	-0.281
54	10.36	54.35	1.84	27.68	0.509
55	8.52	26.67	-1.58	-14.2	-0.532
56	10.10	40.87	1.26	-9.95	-0.243
57	8.84	50.82	-1.99	11.84	0.232
58	10.83	38.98	-1.21	16.37	0.419
59	12.04	22.61	2.84	9.82	0.434
60	9.20	12.79	1.41	-3.39	-0.147

For example, in case of i=50 and i+1=51 in Table 4-2, V_i=11.11, V_{i+1}=9.7, then ΔV_i =1.41., ΔX_i = -3.39, $\Delta X/X$ =-0.147. The number of data is 290 (i=1~290).

All data i=1~290 were embedded into the

Weber's law, which is shown in Fig.4-5.

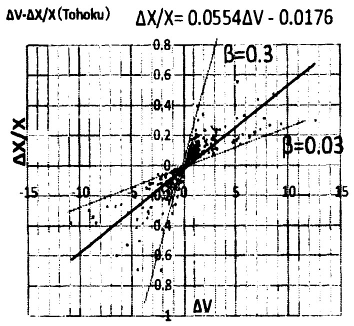


Fig. 4-5 Data embedded into Weber's law ($\frac{\Delta X}{X} = \beta \Delta V$)

As shown in Fig.4-5, all data are confined within the lines of β=0.03 and 0.3. The gothic line between β=0.03 and 0.3 is the averaged line of which β is 0.0554 ($\beta = 0.0554$) as shown by the coefficient of ΔV in the formula in the figure. Note that number 0.0176 in the formula must be equal to zero which is the error occurred from our measurements.

- Weber's law doesn't care the existing of $\Delta V = 0$, which means the flow is laminating, so called *laminate flow*.
  The laminate flow condition is the most essential condition when we considered the road quality, of which details will be mentioned in Section 5.

4.4-2 Fechner's law

The Fechner's law was shown in formula (3-7) of Section 3. Again we shall show here, that is

$X = X_0 \exp(\beta V)$ (3-7) (from Section 3).

Fig.4-6 shows the result of the V-X plots, in which the approximated exponential curve is shown.

The curve in Fig. 4-6 is the approximated one, of which β=0.0686. From the one glance of Fig. 4-6, comparing with the case of Weber's law shown in Fig.4-5, we see that the Fechner's law contains more comprehensive and useful information for the road traffic flow. Thus, we should make the approximated curve more concrete way in the meaning of the accuracy; that is to say, we should examine the accuracy of exponential function for approximation since the formula shown in Fig.4-6 is the one automatically calculated by computer..

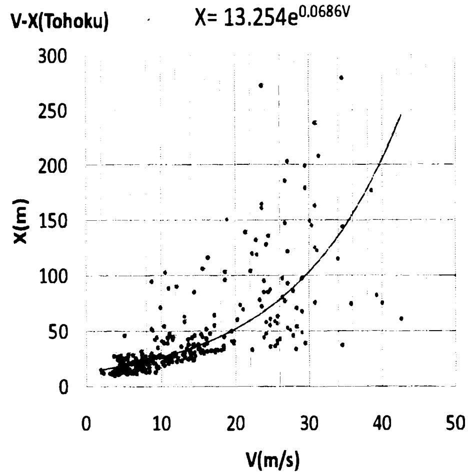


Fig.4-6 Data embedded into Fechner's law ($X = X_0 \exp(\beta V)$)

One of the most comprehensive ways showing the accuracy is to show the averaged characteristics of the data. When the curve is exponential, semi-logarithmic graph examination is available. In the present case, we take the natural log (ln) for X_{av} and plot on the V-ln(X_{av}) plane (semi-logarithmic section paper). If the Fechner's exponential relationship is preserved for the road traffic flow between speed V and ln(X_{av}), the V-ln(X_{av}) relationship should become linear on the semi-logarithmic section paper. Then, we try this for examination of the linearity. Fig. 4-8 is the result, which shows very good relationship with more than 0.95 of correlation factor. Fig.4-7 is the original V-X_{av} relationship.

Any other road traffic flows were measured and examined by the same way mentioned above. The other examples are shown in Appendix 4-1, where the urban road and suburban road are examined. The results show that all relationships between V and ln(X) are nearly linear with correlation factor of more than 0.85.

Xav-Vav(Tohoku Exp. Azumz)

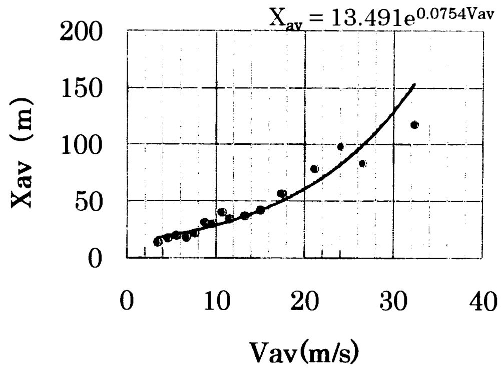


Fig. 4-7 V_{av} - X_{av} relationship on linear section paper

Vav-ln(Xav)(Tohoku Exp. Azumza)

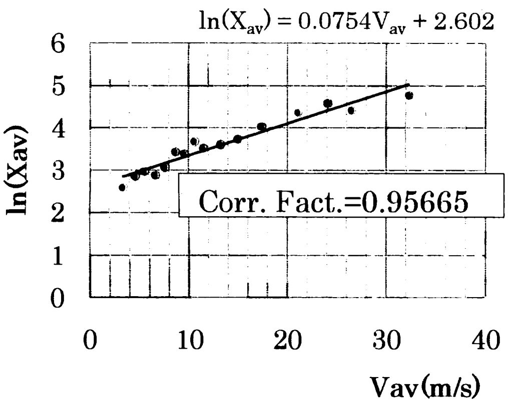


Fig.4-8 V_{av} - $\ln(X_{av})$ relationship on semi-log section paper

From those results, we are convinced of saying that the relationship between the distance-headway (X) and speed (V) obeys the Weber-Fechner law from the data analyses (see Appendix 4-1, the end of this Section), which is the practical proof from the measurements. Mathematically, we have seen it in the Subsection 3.4 under the condition of $\beta V = 1$.

If we compare Fig.4-6 and 4-7, we notice that the numerical values of β are different each other, for which we have to scrutinize in detail, but right now it can be put off, because this matter is not crucial. The difference may come from averaging procedures.

As a remark, we can say that the relationship between speed V and distance-headway X is always expressed by exponential function.

4.5. Discussions on Traffic Flow and β

4.5.1 β as scaling factor

We have seen in Section 3 that the relationships between the distance-headway X and the speed V obeys the exponential functions which can be written in general as

$X = X_0 \exp(\beta V)$ (5-1) ((3-7) in Section 3).

which first appeared in (3-7) in Section 3. In this simple equation, β is the scaling factor. However, in the actual traffic flow, the coefficient β has an important role.

4.5.2 β in Weber's law and Fechner's law

We have seen the data embedded figures in Fig.4-5 for Weber's law and Fig.4-6 for Fechner's law. We should notice that there are big differences between the two figures not only the shape of graph but also the numerical values of β.

(1) Shape of graph

The shape difference comes from the data to be embedded. We should notice that in case of the Weber's law in Fig.4-5, the data of $\Delta V = 0$ are omitted, because no data exist when $\Delta V = 0$. On the contrary, Fig.4-6 for Fechner's law includes all data are embedded even in the case of $\Delta V = 0$. What we can see from both cases of Weber's and Fechner's is that the Weber's law deals the flow dynamics which excludes the static state such as stopped row of cars in the congestion. On the contrary, the Fechner's law does not exclude the static flow which is the case of $X =$ constant that means $\Delta V = 0$. As we will see later that the static state of flow is important in the road traffic.

(2) Numerical value of β

Although the same data are embedded, the numerical values of β of Weber's law (Fig.4-5, $\beta = 0.0554$) and of Fechner's law (Fig.4-6, $\beta = 0.0686$) are not the same. This discrepancy can be considered due to omitting the data of $\Delta V = 0$ in the case of Weber's law.

4.5.3 Condition of $\beta V = 1$

$\beta V = 1$, first appeared in (3.6) in Section 3, was the condition for the Weber's law in traffic flow shown in (3.3) comes into being. In the other word, we may be able to say that this condition is the solution of the Weber's law of

(3.3).

We have defined V as V_β(= 1/β). At V=V_β, the distance-headway X becomes

$$X = eX_0 = 2.718X_0 \quad (4.7)$$

because exp(βV_β)=exp(1)=e (=2.71828--).

We have to scrutinize what (4.7) means. We wonder that (4.7) is independent of speed V. If we assume X_0=10m which was mentioned in 3.5 in Section 3, always X becomes about 27m (about 30m) independently of speed. It is difficult to agree this theoretical fact, and we have a big question on this matter in the road traffic flow analysis.

Making clear the above mentioned question became the essence in our theoretical research on the road traffic flow. This will be mentioned in the next Section 5.

4.5.4 Optional remarks

We said in this Section that the car speed is a sensation of the driver. There may be argues on this matter, but we cannot afford but feel it, if we actually be the drivers. When we drive the cars, we want to run as fast as possible, but this cannot always be satisfied because of safety or existing foregoing car. Exactly we can see in the next Session 5 of the driver's habit to follow the foregoing car as close as possible. In such a case, the driver almost all time does not recognize how fast he is running with a physical unit of km/h.

Another thing we should consider the Weber-Fechner law in road traffic flow is that of the limit of speed V. In the V-X plotting field shown in for example Fig.4-6, X increases exponentially with respect to linear increase of V. But this explanation cannot be accepted philosophically, because in the V-X plot, V must be a dominant parameter to determine X. But the above mentioned is not right, because the above mentioned should be inversely said as the distance-headway X is the dominant parameter to determine the speed V. From this view, we rewrite Fig.4-6 as Fig.4-9.

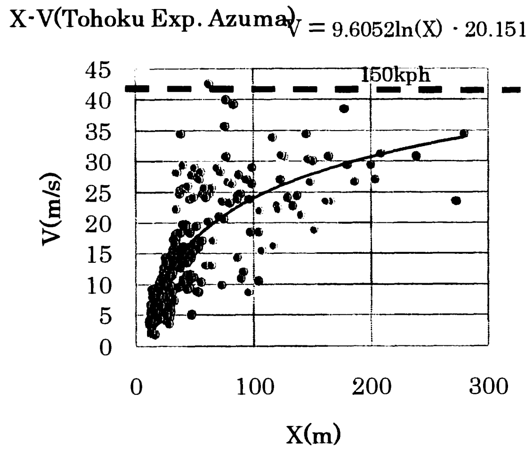


Fig.4-9 X-V plot of Tohoku Expressway, Azuma

Fig.4-9 is of course logarithmic expression of the Weber-Fechner law, but this figure offers the persuasive data for upper limit of speed. In the above case, the highest speed was 150 kph (km/h)(about 42m/s). Actual speed limit is legally set with 100kph (=28m/s) .

Finally, the relationship between physical stimuli and sensation can be said as "Sensation is proportional to the logarithm of physical stimulus". This expression is more common in general, but since the exponential function is convenient for analyzing the data, we are consistently using it.

4.6 Conclusion of this Section 4

In this Section we confirmed the relationship between distance-headway (denoted by X) can be seen as the stimulus and the speed (denoted by V) can be seen as the human sensation in car driving system. To say more concretely, Weber-Fechner Law can well fit to the car driving phenomena.

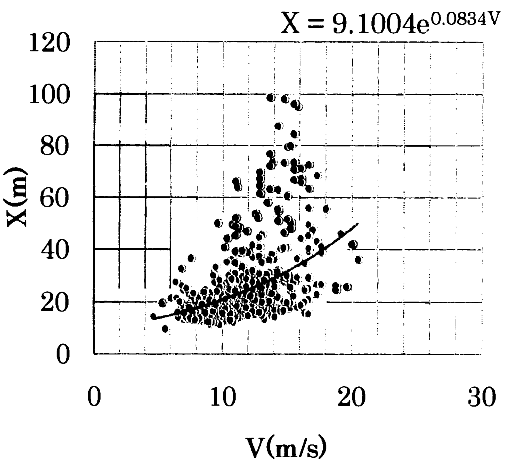
Acknowledgments

The author wishes to express his thanks to Professors Shosuke Suzuki and Masanari Taniguchi of Tohoku Bunka Gakuen University for their long time association with this work and encouraging him to pursue it.

Appendix 4-1

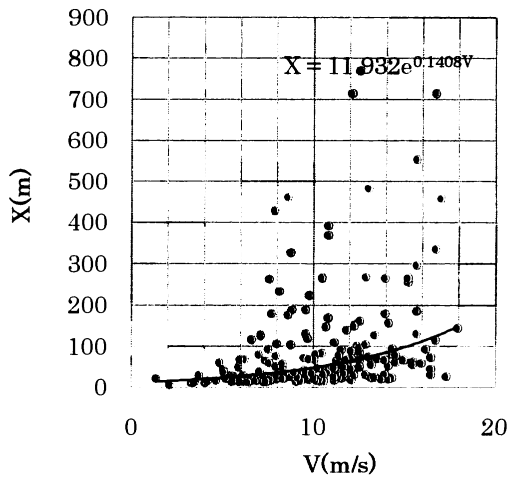
App. 4-1 (a-1) Urban Raw Data
Inside City. Sendai

V-X(Kotodai, Sendai)



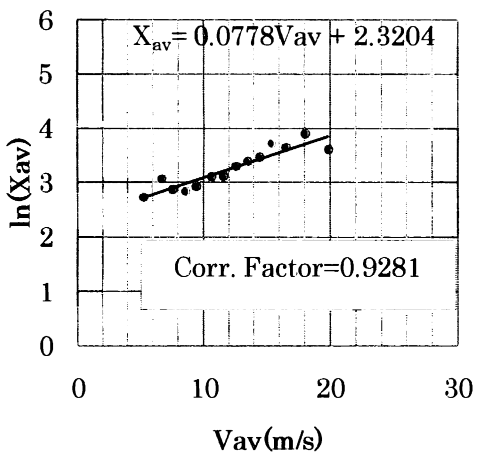
App. 4-1 (b-1) Residential Area Road

V-X(Nakayama,Sendai)



App. 4-1 (a-2)

Vav-ln(Xav)(Kotodai, Sendai)



App. 4-1 (b -2)

V-ln(L)(Nakayama,Sendai)