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Scientific Study of Road Traffic Flow (3)
5. Development of Theory for Time-Domain Analysis
-Fundamentals of Road Traffic Characterizations-

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Continued from "Scientific Study of Road Traffic Flow (1)" presented at the previous meeting of 262nd SICE Tohoku at Tohoku University on Dec. 23, 2010, Second version (2) subtitled "-Measurement and Example of Data Analysis-" was presented at Tohoku Bunka Gakuen University on March 11 when devastating earthquake and tsunami hit around northern Tohoku area, the oral presentation ceased due to the disaster. This paper begins with Section 5 because the present series have finished Chapters 1, 2, 3 in (1) and Chapter 4 in (2). Thus in this understand presentation (3) mentions Section 5. T

The content (subtitle) of the previous presentation is as follows:

(1) Road Traffic Flow Fundamentals
from Weber-Fechner Law
(2) Measurement and Example of Data Analysis

5. Development of Theory for Time-Domain Analysis -Fundamentals of Road Traffic Characterizations-

5.1 Example of Time-Domain Data

First of all, we will see the graph Fig.5-1 which is a part of Fig.5-4 in this Section.

The reason why the author shows Fig.5-1 first of all is that the readers may be easily able to

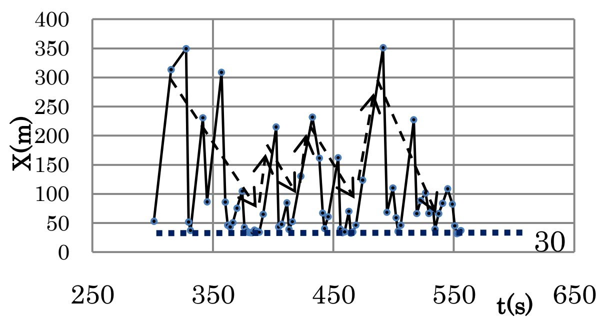


Fig.5-1 Variation of X with respect to elapsed time t (The original data is shown in Fig.5-4)

understand the traffic flow dynamics. The meaning of the time-domain analysis is that it makes possible to see the dynamical phenomena on the axis of time (denotedby t) of which unit we use in our study is always *second*. The dynamical phenomena should be expressed on the graph of which abscissa is scaled with the elapsed time t in seconds. Fig.5-1 is the typical example of this type of graph (t − X graph).

We are able to understand that this type of t − X graph has full of contents, which is able to characterize the car movement behaviors against the elapsed time t(s). Clearly we can see from Fig.5-1 that the distance-headways (X's) between successively moving cars have a tendency like that shown by the arrows in the figure. That is to say, every driver in average has a habit to chase foregoing car with faster speed than that of foregoing car, and reaches the shortest distance-headway (theoretically eX_0) about 30m as shown in Fig.5-1 when the car speed becomes the same with the speed of foregoing car ($\Delta V = 0$). The state of the minimum distance-headway in this case could not be sustained long time and the phenomena like Fig.5-1 cyclically occur, which may depend on car density. In case where car density increased, the state of shortest distance headway may continue and the cyclical movement of X should become slow.

Readers may remember when they read **4.5.3** in Section 4 where the distance-headway becomes eX_0 ($\approx$27m, when X_0 is assumed to be 10m), but in case of Fig.5-1 $eX_0 \approx 30\text{m}$ ($X_0 \approx 11\text{m}$).

Theoretically this condition appears at the condition of $\beta V = 1$, which is the condition of maximum flow (the minimum time-headway T_{min}).

Following subsections will discuss the details of the above mentioned.

5.2 Fechner's Law under condition of $\beta V = 1$

5.2.1 Prolog

To begin with, we shall admit that the analyses ever made in the world on the road traffic flow were almost all statistical ones and also we should admit this because no appropriate ways exist in analyses without statistics. We cannot forget that almost all physical laws were born from the results of limitless number of experiences or investigations, and the Weber-Fechner Law was also the same. However, so far as we concern the road traffic flow, we inevitably have to discuss it with respect to elapsing time i.e. time-domain. Our interest in our live activity is always bound by time and we cannot be indifference to time. Nevertheless, we have not yet a good achievement in dealing with the traffic flow in terms of time domain. Present study tries for the first time to make a theoretical approach to formulate the traffic flow behavior with respect to elapsing time. The key is the condition of $\beta V = 1$.

5.2.2 Fechner's exponential formula under $\beta V = 1$

The Fechner's law in road traffic flow was shown again here as follows;

$$X = X_0 \exp(\beta V). \quad (3.7) \text{ (from Section 3)}$$

Since the condition of $\beta V = 1$ ($X = eX_0$) is very strict condition and as we saw in **5.1** in this Section, this condition cannot be sustainable for long time in the practical flow like the case of not so dense flow. The speed V can be thought in general as $V + \Delta V$, then (3.7) is written like

$$\begin{aligned} X &= X_0 \exp(\beta V) \\ &= X_0 \exp\{\beta(V + \Delta V)\} \\ &= X_0 \exp\left\{\beta V(1 + \frac{\Delta V}{V})\right\}. \qquad (5.1) \end{aligned}$$

We have noted in (3.4) of Section 3 that $\Delta V/V$ is

$$\frac{\Delta V}{V} = \frac{\Delta X}{X}. \qquad (3.4) \text{ (from Section 3)}$$

Thus (5.1) can be written as

$$X = X_0 \exp\left\{\beta V\left(1 + \frac{\Delta X}{X}\right)\right\}. \qquad (5.2)$$

5.3 Relationship between X, T and V

5.3.1 $X = X(t)$ as a time function

We have to clarify the formula (5.2) which can connect the statistical data to the time domain behavior of the road traffic flow. As we have seen and mentioned before, the averaged driver has a habit of chasing the foregoing car with higher speed than that of foregoing car. Should we say a little mathematically, we can write X as a function of time, i.e.$X(t)$, and X can be written like

$$X(t) = X'_0 + \Delta X(t), \qquad (5.3)$$

where X'_0 is an arbitral initial distance-headway at $t = 0$. $\Delta X(t)$ can be written as

$$\Delta X(t) = T(t)\Delta V(t), \qquad (5.4)$$

where T is the time-headway.

Referring Fig.3-1 ((2) in this series) along with related sentences, $\Delta V(t)$ can be written as

$$\Delta V(t) = V_A(t) - V_B(t), \qquad (5.5)$$

where $V_A(t)$ and $V_B(t)$ are foregoing and following car speed, respectively. From driver's chasing habit, the relationship between $V_A(t)$ and $V_B(t)$ is $V_A(t) < V_B(t)$ ($\Delta V(t) < 0$) in average. The approaching speed of the distance-headway to its shortest of eX_0 depends on the averaged slopes of the arrows in Fig.5-1. The averaged $\Delta V(t)$ is negative when distance-headway X is large, but finally becomes equal to zero ($\Delta V(t) = 0$) at the shortest limit of eX_0. Fig.5-2 shows this situation conceptually. As shown in Fig.5-1, the actual distance-headway in this case is $eX_0 \approx 30\text{m}$, that means averaged $X_0 \approx 11$m, This is the highest efficiency state.

After the highest efficiency state, there are two cases that are shown by A and B in Fig.5-2, where A is the case going to the flow of less density and B to congestion.

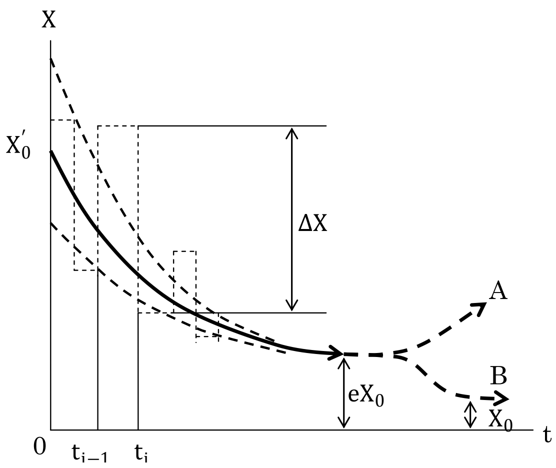


Fig.5-2 Conceptual tendency of $X(t)$ approaching to the shortest distance-headway eX. After that: A: goes to high speed with less density flow. B: goes to congestion $(V \approx 0,\ X \approx X_0)$

5.3.2 Time-domain effect of V and T on X

We consider here about how speed V and time-headway T affect on the movement of $X\ (= X(t))$ in a time-domain.

Here, we shall put the timing t by writing t_i. In Fig.5-2, $\Delta X(t_i) = \Delta X_i$ and $T_i(t_i) = T_i$ $(i = 1,2,3,\text{----})$ where i denotes the car's number and $t_i = \sum_1^i T_i$. The following matters come into being:

$\frac{\Delta x_i}{\Delta t_i}$: slope of X at $t = t_i$,

$\Delta X_i = X_i - X_{i-1}$,

$T_i = \Delta t_i = t_i - t_{i-1}$.

Since $X = VT$ and $\Delta X_i = T_i \Delta V_i$ $(\frac{\Delta X_i}{T_i} = \Delta V_i)$, then

at $t = t_i$,

$$\frac{\Delta X_i}{\Delta t_i} = \Delta V_i. \tag{5.6}$$

As totally ΔV_i $(i = 1,2,3,\text{-------})$ is negative $(\Delta V_i < 0)$, X shrinks with respect to time until ΔV_i becomes zero which means all X_i's approach to eX_0 that is simultaneously all T_i's become equal value of T_{min} in theory (but the actual data are different as shown in Fig.5-4).

The state of $X = eX_0$ at the speed $V_\beta\ (= 1/\beta)$ gives the highest transportation efficiency with highest car density. Like that shown in Fig.5-2, this condition can be separated due to the condition of flow in near future, so that it may go to congestion or high speed lower density flow. Such flow dynamics will be later discussed by using time-space diagram (t-s diagram).

5.3.3 Approach to laminar flow in actual flow

In the actual flow, what process does involve in approaching the laminar flow will be shown here. The most comprehensive result of examination is shown in Fig.5-3. As we have seen (3.4) in Section 3, the variation of $\Delta X/X$ can be substituted by $\Delta V/V$. The situation that ΔV is going to toward zero in the normal flow (no congestion) means ΔX is also going toward zero, where the perfect laminar flow is established. We have the smallest distance-headway eX_0 with the speed V_β at this situation, and the time-headway becomes T_{min} as shown in **3.5** and **Appendix 3-2.**

We have examined in Fig.5-1 that the shortest distance-headway (eX_0) is about 30m in average. In this condition, the speeds of all cars become the same with V_β, and the flow becomes laminar flow.

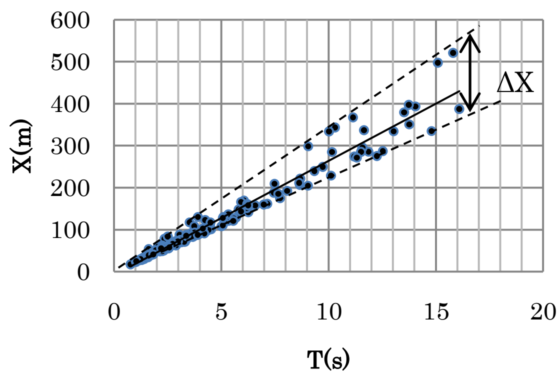


Fig. 5-3 $T - X$ plots shows ΔX decreases as T decreases (Tohoku Expressway)

In Fig.5-3, both smallest values of T and X are T_{min} and eX_0. The details will be shown in Fig.5-4 which is the same data with that of Fig.5-3 but the data of T are limited less than 5 seconds $(T < 5s)$.

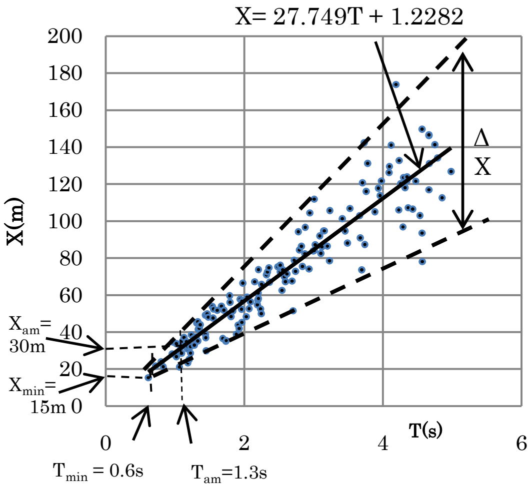


Fig.5-4 $T - X$ plots ($T < 5s$)

From Fig.5-4, we can see rather precise values of the small parts of T and X, where the both minimum T and X ($T_{min} =$ 0.6s, X_{min} =15m) and both T_{am} and X_{am} are shown ($T_{am} = 1.3s$, $X_{am} = 30m$). The value of $X_{am} = 30m$ was decided from measured data of Fig.5-1 (theoretically, X_{am} should be equal to eX_0). To meet with $X = 30m$, T_{am} corresponded to the average of T_i less than 3 seconds, and it became 1.3s.

The value of 30m for X_{am} is not so different for any other roads in spite of urban, suburban roads. However T_{am} depends of roads, the typical value should be around 1.5 s for expressway and 2s for urban and suburban roads.

As references, the minimum values of T and X (T_{min}, X_{min}) are shown, but they do not have the essential meaning because they are occasional.

- **Definition of Laminar Flow**

  Theoretically the definition of laminar flow should be the case when $\Delta V = 0$. However, perfect state of $\Delta V = 0$ does not appear as shown in Fig.5-4, so that we should define it by the limitation of the time-headway. The author defined the flow less than 3 seconds of time-headway should be the laminar flow for any style of roads, although no definite reason exists. Roughly speaking, if we look at Fig.5-3, the laminar flow nearly exists among an entire flow independently of the time-headway when the flow is smooth.

5.4 Bunching flow and density

5.4.1 Periodical phenomena of distance-headway

Fig.5-1 was the part (t=300~550s) of Fig.5-5, where the same phenomena can be seen that occurs periodically. We can naturally assume the phenomena are the driver's natural habit. The phenomena shown here must stem from the result of driver's unconscious behavior to follow the foregoing car as close as possible. The closest distance-headway is eX_0 in the theory, where the time-headway T becomes minimum which is denoted byT_{min}. As we see details in Fig.5-5, T_{min}'s appear periodically. In Fig.5-5, the arrows indicate the variations of X. Like that shown in Fig.5-1, almost all occasions X goes to be small from the peak values. We should note that the slopes of upward arrows are steeper than those of the downward arrows, which means, as we naturally consent, that X is seldom to increase one after another, on the contrary to this, X decreases generally one after another. The bunching phenomena in road traffic flow can be interpreted from the above behaviors.

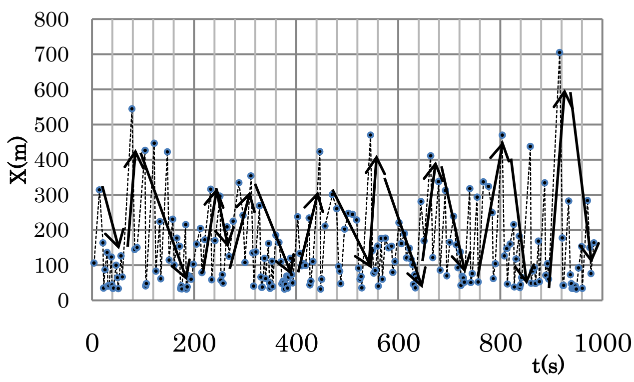


Fig.5-5 $t - X$ data showing waving phenomena (Tohoku Expressway. Hitokita)

However, more visible way to comprehend the bunching phenomena can be seen from the car density.

5.4.2 Flow density and bunching phenomena

The above periodical variation of X must be comprehensively shown by the car density variation. The density denoted by k will be defined as

$$k = \frac{X_0}{X} = \exp(-\beta V), \qquad (5.7),$$

which comes from the Fechner's law in (3.7) in Section 3. The definition of (5.7) is rigorous because X_0 is the shortest distance-headway when cars are stopping ($V \approx 0$) to make a queue, and X is the distance-headway when cars are running ($V > 0$). The typical case is $V = 0$ which can be seen at intersection when signal is red. At this condition k becomes one ($k = 1$) (see more detail discussions in **5.8.1** in this Section). At present discussions on the flow in Fig.5-5, the speed was not so different for all cars, so that we need not take into account of the effect of speed on the density (see **Appendix 5.1**). In order to get clear characteristics, we took the moving average method of n=5 (5 data are averaged sequentially, k in this case is denoted by k_{n5}) was taken for extracting the features of Fig.5-5. The density of cars was observed at one point on the road with respect to time. Fig.5-6 is the result of density variations. As shown in Fig.5-6, periodic or cyclic variations of the density are clearly shown, which means in the actual flow, the bunching occurs cyclically.

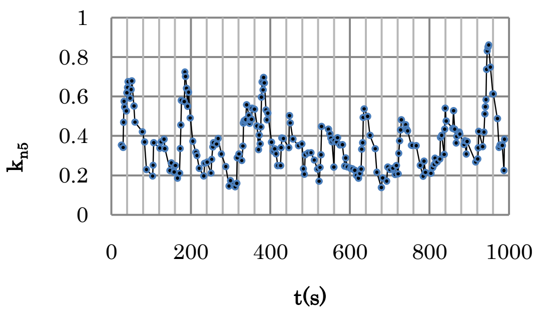


Fig.5-6 $t - k_{n5}$(car-density variation of flow in Fig.5-5) The other $t - X_{n5}, t - V_{n5}$ and $t - T_{n5}$ are shown in **Appendix 5-1**)

5.5 Traffic flow in time-space diagram (t-s diagram)

The group flow dynamics for cars can be clearly shown by using so called *t-s diagram*. By using it, we can intuitively comprehend the time-domain movements of cars on a two-dimensional paper.

Here we introduce the time-space diagram (abbreviated by *t-s diagram*), which is inevitable for analyzing traffic flow, and this will be used in later Section for qualitative (some time quantitative) analyses for the fundamental behaviors of road traffic flow. (V. Section 8)

For the time being, the t-s diagram will be introduced here for understanding the conceptual traffic flow phenomena. Fig.5-7 is indicating how the traffic flow goes to the minimum time-headways.

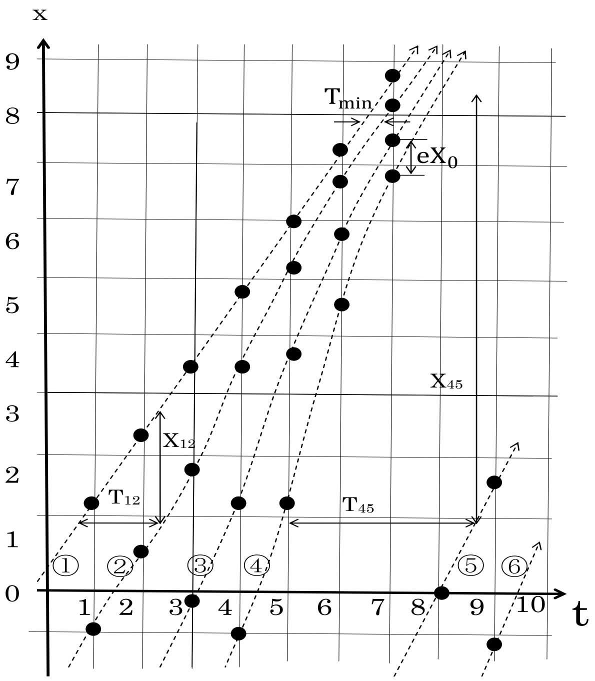


Fig.5-7 Example of t-s diagram explaining how each car moves with a group

Fig.5-7 shows how both the distance-headway X and the time-headway T are reaching to the shortest one i.e. eX_0 and T_{min}, respectively on the time-space (t-s) plane. This type of graph (t-s diagram) is a very important tool for traffic analyses. In Fig.5-7, six cars are indicated by black points, ①～④ are one group and ⑤, ⑥,--are of another group. In the graph, the ordinate x denotes distance and its scale (0~9) is arbitrary and also the abscissa t denotes time t and its scale (0~10) is also arbitrary. The dotted lines are indicating the trajectories of individual car movement. The slope of trajectory indicates the speed V, which can be derived by

$$V = \frac{dx}{dt} \qquad (5.8).$$

The time-headway T is the distance between trajectories, for instance T_{12} indicates the time-headway between car ① and ②. The distance-headway X_{12} is shown on the ordinate along with T_{12} on abscissa.

As we see in Fig.5-7, the speed of car ① is set constant (slope = dx/dt of ① is constant), and ② follows ①, ③ follows ②, and ④ follows ③. The cars ⑤, ⑥,---, are another group, and the time-headway T_{45} between ④ and ⑤ is wider than those of the others (group ①~④), and we see the same thing of the distance headway X_{45} as discussed in Fig.5-8. The two groups symbolize the phenomena in the case of Fig.5-5 or Fig.5-6 (both show the cyclic changes of distance-headway X and density k respectively as we will see next **5.6**).

5.6 Process going to high density flow

We will be able to visibly see that how the traffic flow goes to bunching state of which ultimate is that of the shortest time-headway and simultaneously of the shortest distance-headway. Both the shortest time-headway and the shortest distance-headway have been denoted by T_{min} and eX_0 respectively. If we look at Fig.5-7, we easily understand the process of how T_{min} and eX_0 are produced by T and X.

Fig.5-7 was drawn for that the car ① is moving with constant speed (its trajectory is a straight line) and is followed by ② of which speed is faster than that of ① (trajectory of ② is steeper than that of ①). The relationships of speed of ②, ③ and ④ are the same as the relationship between ① and ②, respectively. So the speed of ④ is the fastest. Each car speed of ② to ④ is being kept nearly constant until t=6, but from t=6 to 7, each speed becomes the same with that of ①, then all cars move with the same speed after t=7, then the differences of speed become 0 (($\Delta V = 0$), ($\Delta V \propto \Delta X$), see also Fig.5-4). At this situation, the ultimate state of car flow of $\beta V = 1$ is realized, which is the case of highest car density in the normal flow. But when the density is not so high, flow becomes a bunching row as mentioned in **5.4**, which can be easily understand if we look at another group of cars of ⑤,⑥--- in Fig.5-7, which are far apart from the group of ①~④. The situation of cars ⑤,⑥--- will be the same with ①~④. If we draw this situation of Fig.5-7 at the timing of t=7 on road, it will be like Fig.5-8.

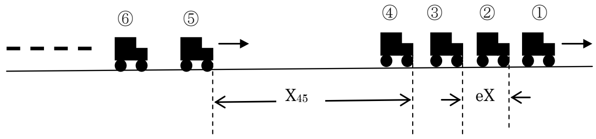


Fig.5-8 Final situation of Fig.5-7 is shown as on road situation of t-s diagram in t=7

Naturally, if the car density increases, X_{45} in Fig.5-8 becomes short (its limit is eX_0). We should notice again that the above mentioned phenomena are independent of the car speed, because the condition of $\beta V = 1$ can be satisfied at any run-speed for any roads.

5.7 From maximum flow to congestion

5.7.1 Cue to congestion

The car flow shown by the row shown in Fig.5-8 (cars ①~④) is the situation of the condition of $\beta V = 1$. The highest density at the normal flow without congestion can be seen as the row of cars moving with the same speed shown like ①~④ in Fig.5-7 at the timing of t=7. If we could have this situation with many cars, the flow of better efficiency will be realized, The maximum flow rate should be able to obtain when all distance-headways in a row of moving cars become the shortest in average, i.e. eX_0 (about 30m). Some drivers may have had the experience closed to this situation. But in many cases, this situation often changes and goes to tight congestion

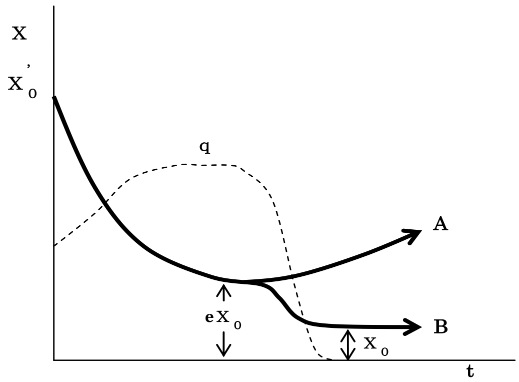


Fig.5-9 Arrow A is the case of going sparse flow, and B is the case of going to complete congestion.

After the flow got the shortest distance-headway eX_0 as we saw in Fig.5-2, to where the arrow goes? There are two ways, one is the case where no congestion occurs, and another case is that traffic goes to congestion. The former case, (indicated by A in Fig.5-9), occurs when the traffic density decreased. The latter case, indicated by B (in Fig.5-9), occurs when the flow density increased or some disturbance happened, for example a car in the row made its speed down even if it is small, and the flow goes to dead. From the theoretical point of view, the destiny of the flow, death or alive, is clear, but from practical situation, the cue is very critical and not known here.

So far as the flow rate is concerned, it is indicated by **q** (dotted line) in Fig.5-9, which is defined in (5.9) in the next **5.7.2**).

5.7.2 Distance-headway in congestion

The maximum flow condition was the case where the distance-headway X was the shortest ($X = eX_0$), and simultaneously the time-headway T became the shortest ($T = T_{min}$) as we saw above.

The ultimate congestion is the case where cars in a row cannot move i.e. $V = 0$.

Again we refer the Fechner's formula of

$X = X_0 \exp(\beta V)$. (3.7) (from Section 3)

If $V = 0$ in (3.7), X becomes equal to X_0 ($X = X_0$), because $\exp(0) = 1$.

The arrow in Fig.5-2 goes to two ways that can be shown in Fig.5-9. Two arrows A and B; A goes to apart away from shortest distance-headway eX_0, and B goes to congestion. X_0 is the distance-headway at speed $V = 0$. The flow rate q (dotted curve) is conceptual flow variation. q is defined as

$$q = \frac{1}{T}\ . \qquad (5.9).$$

- The flow stops in the case of ultimate congestion where the average distance-headway becomes smallest value of X_0 of which value is assumed to be about 10m in case of passenger car. Time-distance T at this condition ($V = 0$) becomes infinity because all cars stop (see (Ap 3-2) in Section **3**).

5.8 30seconds averaged parameters in time-domain

5.8.1 Car density k (k_{30})

We have seen the definition of car density k briefly in (5.7). Here, we describe it in details.

Traditionally car density has been defined as the number of cars per 1km for each lane. But this is difficult to measure and almost impossible in practical, because it should be done from high attitude observation and count the number of cars which varies with time to time. Thus, we cannot pursue it in the time-domain.

In this study, the density k is defined as

$$k = \frac{X_0}{X}, \qquad (5.10)$$

then becomes as we have seen in (5.7) in **5.4.1**;

$k = \exp(-\beta V)$. (5.7) (from **5.4.1**)

The above (5.7) is of course the formula of statistical base, and becomes 1 ($k = 1$) when $V = 0$, because the argument βV in the exponential function becomes equal to zero ($\beta V = 0$). k is dimensionless number and, at the highest density occurs when $V = 0$, and $k = 1$.

- (5.7) is the relationship between k and V. This relationship has been called $k - V$ correlation which had been discussed in the early days in the research of traffic flow as the one of the main research subjects. Many model formulae have been proposed by researchers in the world, but all the model cannot be fitted with the actual data of $k - V$ relationship. We should notice all models proposed so far are empirical ones and not scientific. The researches on this subject had been made in 1930's to 60's. Today, this item looks out of interest.

We denote the density k_{30} which is averaged density in 30 seconds. As an example, we have shown the moving averaged data in Fig.5-6. Here we will show the 30seconds averaged density denoted by k_{30} instead of the moving average (k_{n5} was used in Fig.5-6).

Fig.5-10 shows the statistical, original $V - k$ relationship of the same data with Fig.4.4 in Section 4.

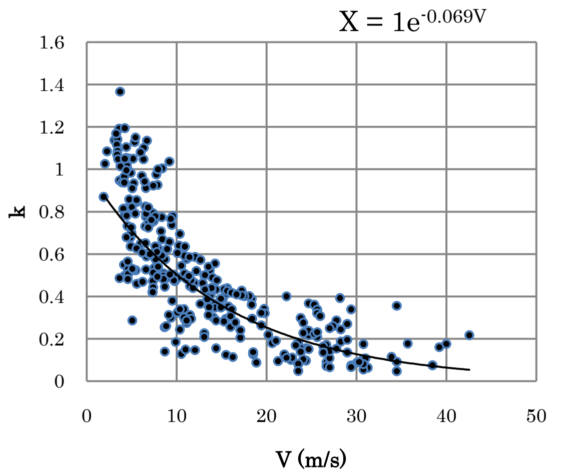


Fig.5-10 $V-k$ relationship of Tohoku Expressway (Azuma)

As shown in Fig.5-10, data are widely dispersed but as shown by the centered line which computer approximated, it looks very good approximation that can be expressed by (5.11). The numerical formula became

$$k = \exp(-0.069V) \qquad (5.11),$$

where β=0.069 is the same with that of $V-X$ relationship of this data shown in Fig.4-6 where $\beta = 0.686$ of which round off number is 0.069.

Fig.5-11 shows the time-domain variation of k_{30} with respect to time t.

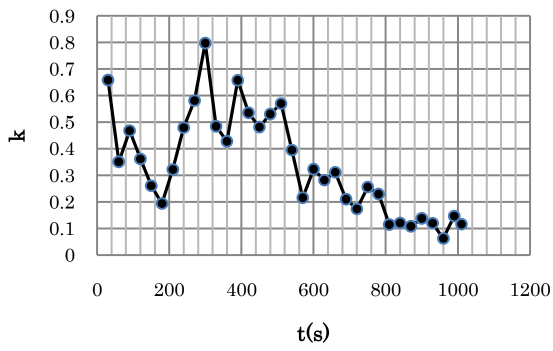


Fig. 5-11 t-k_{30} (Tohoku Expressway, Azuma)

As shown in Fig.5-11, the variation of the density can be clearly seen by 30seconds averaged data where high region of k_{30}(>0.6) may be recognized as the congestion state.

5.8.2 Time variation of flow rate (q_{30}) along with T_{30}, V_{30} and X_{30}

The flow rate with 30seconds averaged q_{30} is applied for Tohoku Expressway, Azuma (Fig.4-4), where q_{30} is

$$q_{30} = \frac{1}{T_{30}} \qquad (5.12).$$

Fig.5-12 shows $t-q_{30}$ graph along with $t-T_{30}$, $t-V_{30}$ and $t-X_{30}$.

Fig.5-12(a) t-q_{30}

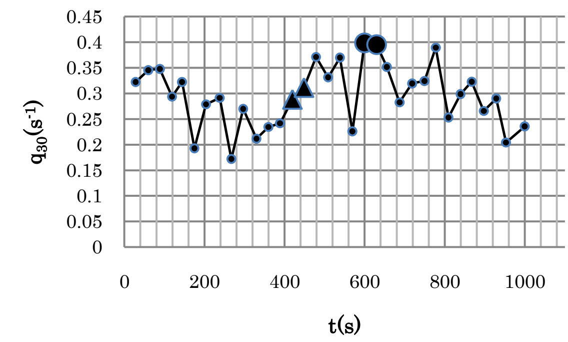


Fig.5-12(b) t-V_{30}

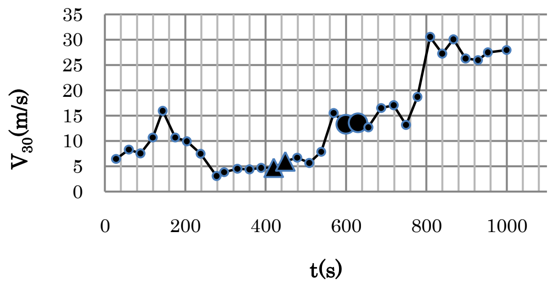


Fig.5-12(c) t-T_{30}

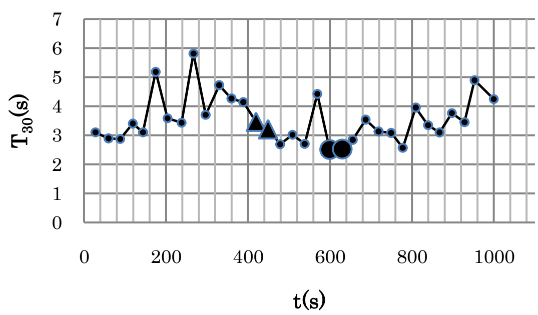
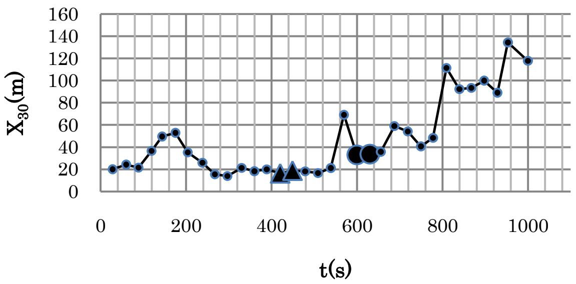


Fig.5-12(d) t-X_{30}



The gothic marks shown in Fig.5-12 mean as follows:

▲: congestion states

●: maximum flow states.

a) From $t-q_{30,}$ q_{30max} (maximum flow rate) is clear, but congestion state is unclear. The value of q_{30max} is about 0.4 (s^{-1}), X_{30min} =33m and T_{30min} =2.52s. $V_{\beta 30}$(=X_{30min}/T_{30min})=13.2m/s (47.6km/h), while the result from the statistical analysis was 14.5m/s (52km/h). Both values are almost the same.

b) From $t-V_{30,}$ congestion state is clear, but flow maximum state is unclear..

c) From $t-T_{30}$, minimum T_{30} (T_{30min}) is clear of which inverse is the flow maximum ($q_{30max}=1/T_{30min}$), but congestion state is unclear.

d) From t-X_{30}, congestion state is clear, but maximum flow state is unclear. (See Fig.4-4(c))

Table 5-1 is the summary of the above a), b), c) and d).

Table 5-1 Parameters and their information

	Maximum Flow(q_{max})	Congestion State	$q_{max}=\frac{1}{T_{min}}$
$t-V_{30}$	Unclear	Clear	
$t-T_{30}$	Clear	Unclear	$q_{30}=\frac{1}{T_{30}}$
$t-X_{30}$	Unclear	Clear	$X=TV$

We will see from the above Table that the information given by the parameters V and T is essential to know the flow situation, if we observe the time variations of those parameters. That is;

$t-V_{30}$ and $t-X_{30}$ can indicate Congestion State, and $t-T_{30}$ can indicate Maximum Flow State.

5.9 Concluding remarks

We discussed in this Section about the time-domain behavior of the road traffic flow. Basing upon the Weber-Fechner law, the statistically maximum flow condition of $\beta V=1$ was clearly explained in the time-domain. The process with respect to time for both going to flow maximum state and congestion state could be comprehensively explained. Those mentioned concepts may be useful in future optimizations of road traffic flow.

The flow $q(=1/T)$, and speed V should be the final essential parameters for evaluating the road quality which will be how fast, how many cars or vehicles and how much loads can be carried to the destinations. Those practical matters will be discussed in the next Section 6. We have had, from the discussions in the present Section 5, the basic tools for discussing the matters mentioned above.

Acknowledgments

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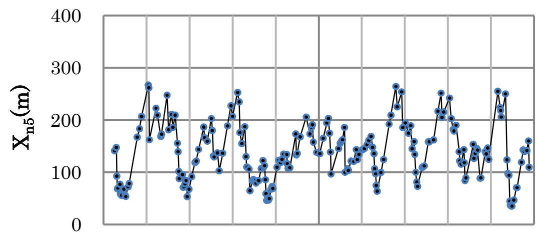
Appendix 5.1

$t-X_{n5}, t-V_{n5}$ and $t-T_{n5}$ (Tohoku Expressway, Hitokita)

Fig. (a), (b) and (c) show the time-domain characteristics of the moving averaged data (n=5) for X, V and T (X_{n5}, V_{n5} and T_{n5}) with respect to time t. The flow was very smooth and no congestion existed.

(a) t-X_{n5}



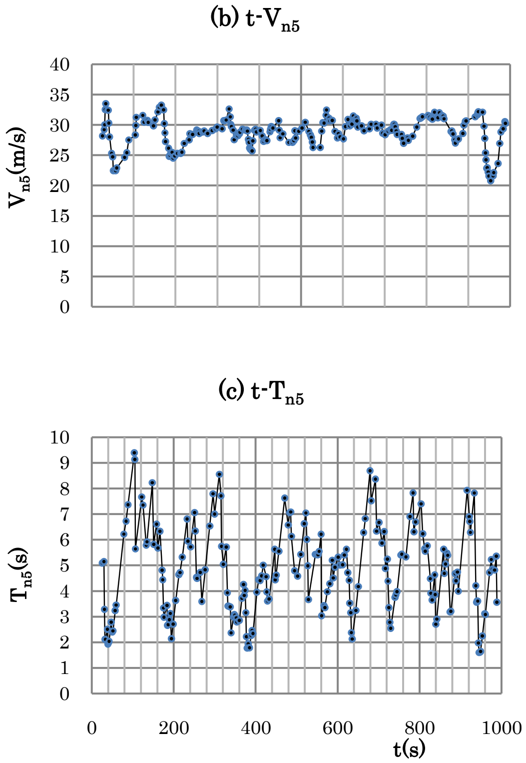


Fig. App. 5-2 $t-X_{n5}$, $t-V_{n5}$ and $t-T_{n5}$, at smooth flow (Tohoku Expressway (Hitokita))

Here we see that the distance-headway X is governed by almost the time-headway T when the flow is smooth (speed does not change drastically so the variation of X and T with respect to time are almost the same).