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近似代数を用いた線形システムの教育用システム

Educational System for Linear System Theory Using Approximate Algebra

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1. Introduction

Since computers became popular, many educational system using computers have been developed. Most of them take a different role from papers and books. They can be classified, for example, as

[1] Distance Learning System

[2] Learning Management System

[3] Graphical Learning System.

The type [1] Distance Learning System is, as it is called, "e-learning system." This kind of systems are to support learners far away through computer networks[1), 2), 4)]. The type [2] Learning Management System is, used mainly by administrators in a school, to manage educational environment using computers. For example, such systems are used to give leaners training according to indivisual understanding and attitude[3), 6)]. In the type [3] Graphical Learning System, graphical environment of computers is simply applied. To present graphically objects, for example, movies and animation, this kind of systems support educational materials, textbooks and handouts and so on[5)].

Linear system theory is popularly applied and its necessity has been increasing rapidly in recent years. However, many beginneers learning for the linear system theory not only are unaware of various applications for the theory, but also finds that its difficulity to understand. Specifically, it is important for linear system theory to understand state space equation. However, the importance of the state space equation is hard to notice since it merely seems to a equation. Thus, we consider that it is necessary for learners for the linear system theory to develop an educational system.

In various disciplines, there is a case in which objects are frequently analyzed by an approximate calculation. If that calculation is well known limit such as numerical differentiation or integration, it is not difficult to calculate in computer. However, there is infrequently a case that is unable to approximately calculate with no difficulty. Then, we have to find other method to calculate.

As an educational system for the linear system theory, we have developed a software that helps learners understand the linear system theory. Since it is not necessary for learners far away to give this educational system as of the moment, our educational system does not take the type of distance learning system. Further, our educational system employ the graphical learning system. The educational system provides learners visualized and graphical applications to demonstrate effectively examples of basic materials of the linear system theory, for example, state space equation and feedback system and so on. As an example of basic materials, we have employed the inverted pendulum system. The inverted pendulum is known as the most basic experiments in the linear system theory. Since we simulate the inverted pendulum, we observe the behavior of its state space equation. The inverted pendulum system will be discussed more detail in the next section. This educational system expresses not only a behavior of the inverted pendulum by animation, but also graphs some value, for example, angle of the rod from the vertical line and so on. Additionally, in the system, we can state initialized values and some environmental values with GUI (Graphical User Interface). The system is constructed by Java[8]. It is known that Java is a very portable programming language. Additionally, execution environment of Java do not depends on Operating System such as Windows, Mac OS, Solaris, Linux, and so on. Since using Java, this educational system also works on many general operating System.

2. Model — Inverted Pendulum

In this section, we recall the inverted pendulum system, which is shown in Fig.1. In the figure, the following symbols are used:

- M: Mass of the cart (kg)
- r: Location of the cart (m)
- u: Voltage to the motor of the cart (V)
- a: Gain of the voltage to the force (N/V)
- R: Coefficient of viscosity for the motor (kg/s)
- H: Horizontal motion of center of gravity of the pendulum rod (N)
- J: Moment of inertia of the rod about its center of gravity (kg·m^2)
- θ: Angle of the rod from the vertical line (rad)
- V: Vertical motion of center of gravity of the pendulum rod (kg)
- l: Length of the pendulum rod (m)
- c: Coefficient of viscosity for the rod (kg·m^2/s)
- m: Mass of the rod of the pendulum (kg)
- g: Gravitational acceleration (m/s^2).

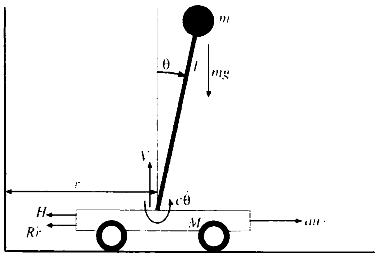

Fig. 1 An example of inverted pendulum

At first, we give a state space equation of the inverted pendulum system and its output matrix equation. The physical model can be described as follows:

$$M\ddot{r} + R\dot{r} = au - H, \tag{1}$$

$$J\ddot{\theta} = Vl\sin(\theta) - Hl\cos(\theta) - c\dot{\theta}, \tag{2}$$

$$m\frac{d^2}{dt^2}(r + l\sin(\theta)) = H, \tag{3}$$

$$m\frac{d^2}{dt^2}(l\cos(\theta)) = V - mg. \tag{4}$$

Deleting V and H from the equations above, from (1) to (4), we have

$$\begin{aligned}(M+m)\ddot{r} + ml\cos(\theta)\ddot{\theta} + \\ R\dot{r} - ml\sin(\theta)\dot{\theta}^2 &= au, \quad (5)\\ ml\cos(\theta)\ddot{r} + (J+ml^2)\ddot{\theta} + \\ c\dot{\theta} - mlg\sin(\theta) &= 0. \quad (6)\end{aligned}$$

We now assume that both θ and $\dot{\theta}$ are nearly equal to 0. Then we have

$$\sin(\theta) = \theta, \cos(\theta) = 1, \sin(\theta)\dot{\theta} = 0.$$

Now the equations (5), (6) can be modified as

$$(M+m)\ddot{r} + ml\ddot{\theta} + R\dot{r} = au, \tag{7}$$

$$ml\ddot{r} + (J+ml^2)\ddot{\theta} + c\dot{\theta} - ml\theta g = 0, \tag{8}$$

which can be further modified as

$$\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\alpha_0} \begin{bmatrix} 0 & 0 \\ -m^2l^2g & mlg\alpha_2 \\ -R\alpha_1 & mlR \\ mlc & -c\alpha_2 \\ a\alpha_1 & -mla \end{bmatrix}' \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \\ u \end{bmatrix}, \tag{9}$$

where $\alpha_0 = J(M+m) + mMl^2, \alpha_1 = J + ml^2, \alpha_2 = M + m$.

This can be described as in the state space matrix equation and the output matrix equation as follows:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \tag{10}$$

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}. \tag{11}$$

where

$$\mathbf{x} = \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix}, \mathbf{u} = [u], \mathbf{y} = \begin{bmatrix} r \\ \theta \end{bmatrix},$$

$$A = \frac{1}{\alpha_0}\begin{bmatrix} 0 & 0 & \alpha_0 & 0 \\ 0 & 0 & 0 & \alpha_0 \\ 0 & -m^2l^2g & -R\alpha_1 & mlc \\ 0 & mlg\alpha_2 & mlR & -c\alpha_2 \end{bmatrix},$$

$$B = \frac{1}{\alpha_0}\begin{bmatrix} 0 \\ 0 \\ a(J+ml^2) \\ -mla \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = O.$$

Then, $\mathbf{u}, \mathbf{y}$ are vectors of input and output, respectively.

We have discussed the state space matrix equation and the output matrix equation in inverted pendulum. Thus we can obtain the four informations of inverted pendulum, which are the location of the cart, the angle of the rod from the vertical line, angular velocity and velocity of the cart.

discretize inverted pendulum

To control the inverted pendulum system, we discretize inverted pendulum. Denote by T_s the sampling interval. Then, let us now consider a discrete state space equation by

$$\mathbf{x}_d(k+1) = A_d\mathbf{x}(k) + B_d\mathbf{u}_d(k), \tag{12}$$

$$\mathbf{y}_d(k) = C_d\mathbf{x}(k) + D_d\mathbf{u}_d(k). \tag{13}$$

Then, descrete state vector $\mathbf{x}_d(k)$ is state $\mathbf{x}$ at time kT_s that is,

$$\begin{aligned}\mathbf{x}_d(k) &= \mathbf{x}(kT_s) \\ &= \left[r(kT_s), \dot{r}(kT_s), \theta(kT_s), \dot{\theta}(kT_s)\right]^t.\end{aligned} \tag{14}$$

Similarly, descrete output $\mathbf{y}_d(k)$ is output $\mathbf{y}$ at time kT_s that is,

$$\begin{aligned}\mathbf{y}_d(k) &= \mathbf{y}(kT_s) \\ &= [r(kT_s)].\end{aligned} \tag{15}$$

Additionally, input $\mathbf{u}_d(k)$ have

$$\begin{aligned}\mathbf{u}_d(k) &= \mathbf{u}(kT_s) \\ &= [u(kT_s)].\end{aligned} \tag{16}$$

Now we ignore delayed time by calculations. Then, matrix A_d, B_d in descrete state equation (12) are given by

$$A_d = \exp(AT_s), \tag{17}$$

$$B_d = \int_0^{T_s} \exp(At)B dt, \tag{18}$$

and matrix C_d, D_d in descrete output equation (13) are directly $C_d = C, D_d = D$. Then, we can descrete inverted pendulum, and next, we control that.

Control inverted pendulum

The inverted pendulum system can be controlled by a motor which value is represent as input $\mathbf{u}_d(k)$. The goal of the control is to keep pendulum vertical and displace the cart to the origin, that is

angular degree	$\theta = 0$,
angular velocity	$\dot{\theta} = 0$,
displacement of the cart	$r = 0$,
velocity of the cart	$\dot{r} = 0$.

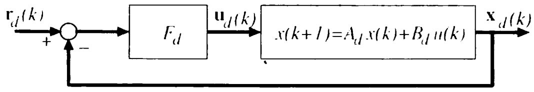

Fig. 2 block diagram of feedback system

As a way to the control, the inverted pendulum is given input $\mathbf{u}_d(k)$ that multiplying the state vector $\mathbf{x}_d$ by proper real matrix $-F_d$, that is

$$\mathbf{u}_d(k) = -F_d \mathbf{x}_d(k). \tag{19}$$

This equation is exactly a feedback system and Coefficient F_d is called feedback coefficient matrix. The relation between $\mathbf{u}$ and $\mathbf{x}$ in feedback sytem is represent by block diagram, that is shown in Fig.2 where $\mathbf{r}_d$ is called objective input and given if needed.

3. Approximate Algebra

In the previous section, we have explained the feedback coefficient matrix F_d. In this section, we discuss calculating F_d and the approximate algebra used in this calculation.

First, matrixes Q_d, R_d are defined by

$$Q_d = \mathrm{diag}(w_1, w_2, w_3, w_4), R_d = [r_1], \tag{20}$$

all elements of the matrix Q_d is called weight. Then, Feedback coefficient F_d is calculated by

$$F_d = (R_d + B_d^t P_d B_d)^{-1} B_d^t P_d A_d, \tag{21}$$

where P_d is a solution of the *Riccati equation*, that is

$$\begin{aligned}P_d &= Q_d + A_d^t P_d A_d \\ &\quad - A_d^t P_d B_d (R_d + B_d^t P_d B_d)^{-1} B_d^t P_d A_d.\end{aligned} \tag{22}$$

By this process which calculating F_d, we can control inverted pendulum. Then, a behavior of the inverted pendulum depends on values of the weight Q_d and the matrix R_d. And so, we consider weight Q_d as polynomial matrix to observe this values. However, if we now consider Q_d as polynomial matrix, we cannot calculate, in normal computational method, a inverse matrix $(R_d + B_d^t P_d B_d)^{-1}$. So now, we use *Approximate Algebra* to calculate the inverse matrix.

Suppose here that all elements of Q_d have polynomial with parameters $a_1, a_2, \ldots, a_r$. We denote by Φ_0 the inverse of the matrix $(R_d + B_d^t P_d B_d)$ with all a_i's being zero. This Φ_0 is a matrix over real numbers. Now, Consider the matrix $(R_d + B_d^t P_d B_d)\Phi_0$. If all a_i's are zero, this matrix is an identity matrix. Thus, denote by $I - \Psi$ the matrix $(R_d + B_d^t P_d B_d)\Phi_0$. Then, its inverse can be expressed as

$$I + \Psi + \Psi^2 + \Psi^3 + \ldots$$

in a formal power series. More precisely, by letting $\Psi = I - (R_d + B_d^t P_d B_d)\Phi_0$, we have

$$(R_d + B_d^t P_d B_d)^{-1} = \Phi_0(I + \Psi + \Psi^2 + \ldots) \tag{23}$$

Now, we can use the right hand side of (23) instead of $(R_d + B_d^t P_d B_d)^{-1}$ in (22). By this relation, we can rewrite (22) as

$$\begin{aligned} P_d &= Q_d + A_d^t P_d A_d \\ &\quad - A_d^t P_d B_d \Phi_0 (I + \Psi + \Psi^2 + \ldots) B_d^t P_d A_d \end{aligned} \tag{24}$$

Now, since inverse matrix do not exists in this equation (24), we can calculate Riccati equation using polynomials. Details of their calculated process refer to [7]. In the next section, we show an example of use this approximate algebra.

4. Example

Now we show actually running the educational system. In Example 1, we show basic simulation of the inverted pendulum. In Example 2, we show using the approximate algebra in Q_d.

Example 1

First, we here use the following environmental values:

$$\begin{aligned} M &= 5.01[\mathrm{kg}], \\ a &= 30.9[\mathrm{N/V}], \\ R &= 24.5[\mathrm{kg/s}], \\ l &= 0.115[\mathrm{m}], \\ c &= 59.8 * 10^{-5}[\mathrm{kg \cdot m^2/s}], \\ m &= 0.1[\mathrm{kg}], \\ g &= 9.8[\mathrm{m/s^2}]. \end{aligned}$$

Now we here use the following initialized values:

$$\begin{aligned} r_0 &= 3[\mathrm{m}], \\ \theta_0 &= 0.3[\mathrm{rad}], \\ \dot{r} &= -8[\mathrm{m/s}], \\ \dot{\theta} &= 0[\mathrm{rad/s}]. \end{aligned}$$

Also, we here use the following values in controller:

$$\begin{aligned} Q_d &= \mathrm{diag}\,(1, 1, 1, 1) \\ R_d &= 1 \end{aligned}$$

The window when we input their values is shown in Fig.3.

Now, we show a window of the simulation in their condition as Fig.4.

Additionally, we show a graph that is a behaviors of inverted pendulum, that is shown as

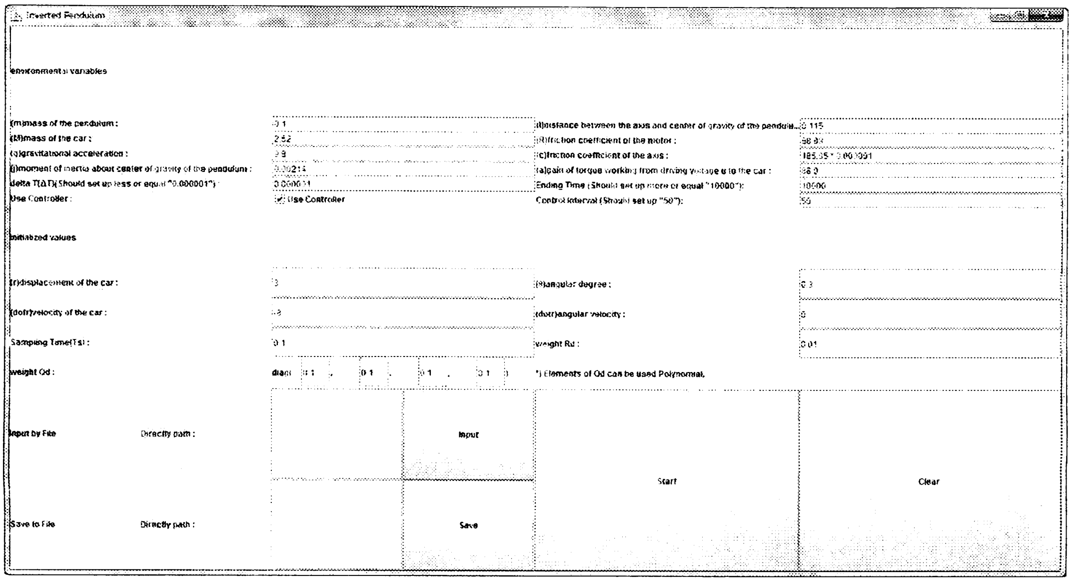

Fig. 3 window that input initialize values

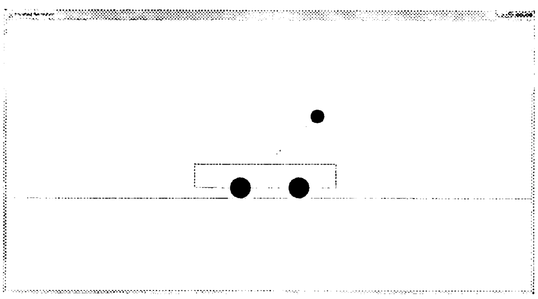

Fig. 4 simulation window as Example 1

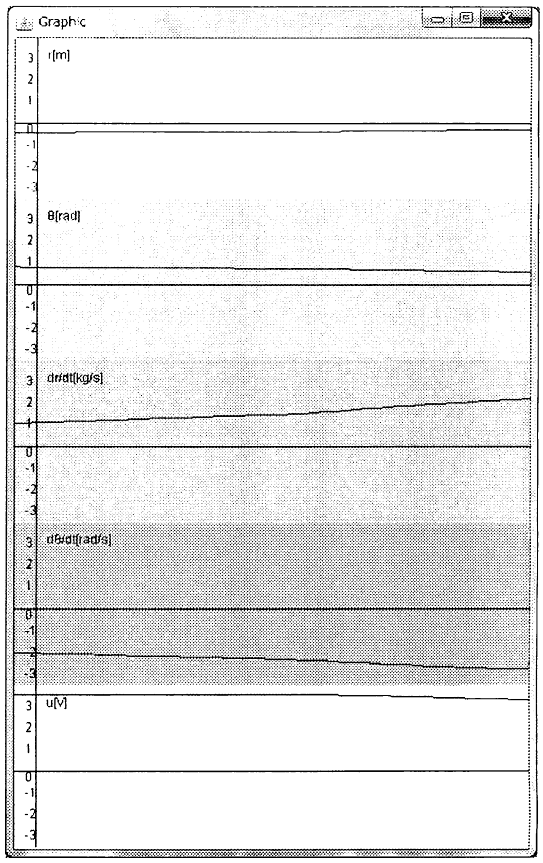

Fig. 5 graph window as Example 1

Fig.5. The figure is combined 5 graphes, each represent from the top are a displacement of the cart r, an angular degree θ, a velocity of the cart $\dot{r}$ and an angular velocity $\dot{\theta}$.

Example 2

In this example, we show using polynomial in matrix Q_d. As this example, we also use the same environmental values:

$$M = 5.01[\text{kg}],$$
$$a = 30.9[\text{N/V}],$$
$$R = 24.5[\text{kg/s}],$$
$$l = 0.115[\text{m}].$$

$$c = 59.8 * 10^{-5}[\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}],$$
$$m = 0.1[\mathrm{kg}],$$
$$g = 9.8[\mathrm{m/s}^2].$$

Now we here use the following initialized values:

$$r_0 = 10[\mathrm{m}],$$
$$\theta_0 = -0.3[\mathrm{rad}],$$
$$\dot{r} = -4[\mathrm{m/s}],$$
$$\dot{\theta} = 0[\mathrm{rad/s}].$$

Also, we here use the following values in controller:

$$\begin{aligned} Q_d &= \mathrm{diag}\,(1+a, 1, 1, 1) \\ R_d &= 1 \end{aligned}$$

pushing the "start" button, the approximation of feedback coefficient F_d is calculated as polynomial, that is

$$F_d = [f_1, f_2, f_3, f_4]^t, \tag{25}$$

where

$$\begin{aligned} f_1 &= -0.3793782236677563 \\ &\quad -0.04068595702061475a \\ &\quad +0.03399998151796067a^2 \\ &\quad -0.02854649130713098 8a^3 \\ &\quad +0.02408554750186012a^4, \\ f_2 &= -5.3199396702536394 \\ &\quad -0.09806363258531675a \\ &\quad +0.08143882984597903a^2 \\ &\quad -0.06792412209210291a^3 \\ &\quad +0.05690922046547561a^4, \\ f_3 &= -1.4529846522187073 \\ &\quad -0.03278973818992042a \\ &\quad +0.02718117291519567 6a^2 \\ &\quad -0.02262955770804005a^3 \\ &\quad +0.01892626447567905a^4, \\ f_4 &= -1.0105944335948485 \\ &\quad -0.01361821843278648 2a \\ &\quad +0.01127449708172908 5a^2 \\ &\quad -0.00937433234672930 8a^3 \\ &\quad +0.00782986807782843 1a^4. \end{aligned}$$

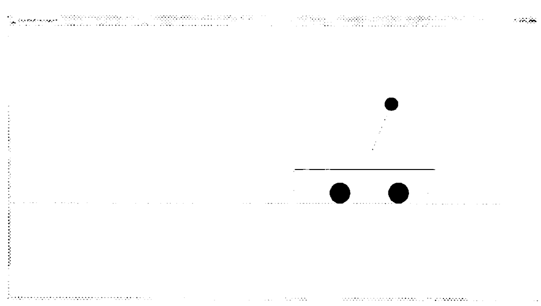

Fig. 7 simulation window 2 at $a = 0$

Then, this matrix is not visible from users. As we decide a value of variable a, we can determine feedback coefficient F_d as shown Fig.6. Then, we substitute $a = 0$, we have

$$F_d = \begin{bmatrix} -0.3793782236677563 \\ -5.3199396702536394 \\ -1.4529846522187073 \\ -1.0105944335948485 \end{bmatrix}.$$

Now, we start simulation. Then, a behavior of then is shown as Fig.7 of a simulation and Fig.8 of a graph.

Next, we decide a value of variable a as 0.3 in the same condition and start simulation. Values of F_d then have

$$F_d = \begin{bmatrix} -0.3890996747678 5173 \\ -5.343402251953867 \\ -1.4608329634291801 \\ -1.0138548794292601 \end{bmatrix},$$

where is shown as Fig.9 A behavior of then is shown as Fig.10, Fig.11.

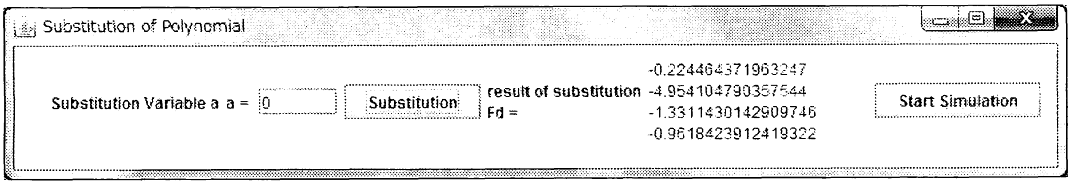

Fig. 6　substitution window at $a = 0$

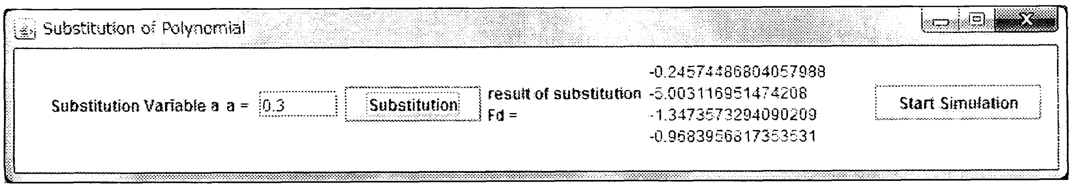

Fig. 9　substitution window at $a = 0.3$

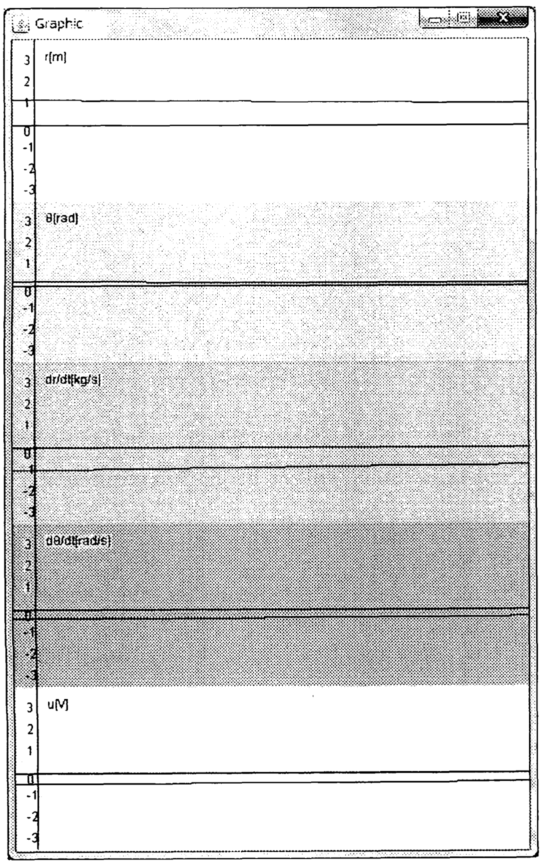

Fig. 8　graph window 2 at $a = 0$

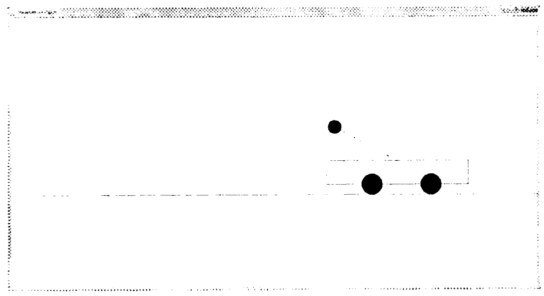

Fig. 10　simulation window 2 at $a = 0.3$

Incidentally, making the value of a more or less than a certain value, the inverted pendulum is unable to be controlled since elements of the vector $\mathbf{x}$ diverge. Then, this system is automatically stopped.

5. Conclusion

In this research, we have developed an educational system for the linear system theory with Java. This educational system simulates the inverted pendulum system, which is controlled in form of state feedback system. Since the educational system represents a behavior of the inverted pendulum by not only animation but also graph, the system supports learners for linear system theory. Further, this educational system enables us a calculation that the feedback coefficient matrix in form of polynomial. Then, in the calculation, the approximate algebra has been empoloyed. By the approximation, we can calculate a inverse of polynomial matrix effectively.

Future works are to evaluate the educational system on precision of approximate algebra and on users view and so on, and to enable the educational system to run on tablet computers and smartphones.

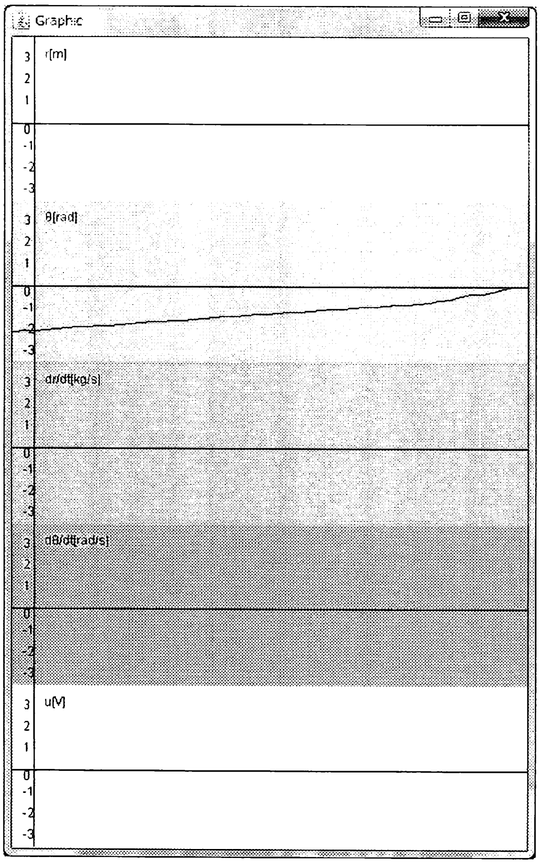

Fig. 11 graph window 2 at $a = 0.3$

References

1) N.Sugawara, K.Oda, H.Akaike, H.Kakuda: SHoes: Realtime e-learning System for Classroom Education and Organizational Learning, 情報処理学会論文誌, Vol.48, No.8, pp.2791/2801, (2007).

2) K.Emi: Development of E-Learning Materials on Linear Algebra for Computer Course Students, 工学教育研究, 10, pp.69/85, (2005), Kanazawa Institute of Technology.

3) K.Yonemitsum, T.Fujii, Y.Ehara, S.Hoyashita, K.sumi, M.Takasaki, M.Otani, M.Ohtsuki, T.Minamoto, Y,Okazaki, K.Watanabe, H.Kondo: Development and Operation of a New Learning Management System Based on Moodle and XOOPS Customized for Japanese Academic Affairs, 情報処理学会論文誌, Vol.48, pp.1710/1720, (2007).

4) H.Shigeno, N.Mashita, T.Narahara, Y.Matsushita: An XML-based Distance Learning System Capable of Conveying Information on "Lecture Event", 情報処理学会論文誌, Vol.42, pp.2319/2327, (2001).

5) Y.Nishino, Y.Tanaka, T.Kawaguchi, E.Hayakawa: Education and Evaluation of OS Education Support Environment in Technical High School 情報処理学会論文誌, Vol.48, pp.2802/2813, (2007).

6) H.Taguchi, H.Itoga, K.Mouri, T.Yamamoto, H.Shimakawa: Programming Training of Student According to Individual Understanding and Attitude 情報処理学会論文誌, Vol.48, pp.958/968, (2007).

7) K.Mori: Approximate Riccati Equation and Its Application to Optimal Control in Discrete-Time Systems, Intelligent Control

and Innovative Computing, 2012(To be published in 2012), Springer.

8) K.Arnold, J.Gosling, and D.Holmes, "*The Java Programming Language, Fourth Edition*," Prentice Hall, 2005.