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２段階安定化法のパラメータ変化に関する入出力の可視化

Visualization Input-Output Relation with Parametrization of Two Stage Compensator Design

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Abstract

In this paper, we consider the two-stage compensator designs for single-input single-output plants in the framework of the factorization approach. Recently one of the authors has given the parametrization of stabilizing controllers given by the two-stage compensator design. As an investigation of the role of the two-stage compensator designs, we implement a visualization system of the parametrization of stabilizing controllers based on the two-stage compensator design, in which the discrete-time LTI systems are considered as a model of the factorization approach.

1. Introduction

The factorization approach to control systems has the advantage that it embraces, within a single framework, numerous linear systems such as continuous-time as well as discrete-time systems, lumped as well as distributed systems, one-dimensional as well as multidimensional systems, etc.[1, 2, 3]. Hence the result given in this paper will be able to a number of models in addition to the multidimensional systems. In factorization approach, when problems such as feedback stabilization are studied, one can focus on the key aspects of the problem under study rather than be distracted by the special features of a particular class of linear systems. This approach leads to conceptually simple and computationally tractable solutions to many important and interesting problems[4].

A transfer matrix of this approach is considered as the ratio of two stable causal transfer matrices. For a long time, the theory of the factorization approach had been founded on the coprime factorizability of transfer matrices, which is satisfied by transfer matrices over the principal ideal domains or the Bézout domains.

In some design problems, one uses a so-called *two-stage compensator design* for selecting an appropriate stabilizing compensator[4)]. Given a plant, the first stage consists of selecting a stabilizing compensator for the plant. The second stage consists of selecting a stabilizing controller for the new closed-loop system that also achieves some other design objectives such as decoupling, sensitivity minimization, etc. The rationale behind this procedure is that the design problems are often easier to solve when the plant is stable.

Another example is to design buildings newly attached earthquake-resistant dumpers (Figure 1).

Figure 1: Earthquake-Resistant Dumpers.

So far, the results of the two-stage compensator design use the norm algebras as well as the factorization approach. Because the analysis by the norm algebra is based on a concrete specified model, this reduces the attractiveness of the factorization approach. Recently, one of the authors has given a parametrization of stabilizing controllers of the two-stage compensator design based on the factorization approach only, which is in the form of the Youla-Kučera-parametrization[5)].

As an investigation of the role of the two-stage compensator designs, we implement a visualization system of the parametrization of stabilizing controllers based on the two-stage compensator design. In the system, we consider the discrete-time LTI systems as a model of the factorization approach. To implement a visualization system easily, we have employed Mathematica[5)].

2. Preliminaries of Factorization Approach

The stabilization problem considered in this paper follows the papers [6, 7)], in which the feedback system Σ [4)] is as in Figure 2. For further details the reader is referred to the literature[4)], [6)], [7)], and [8)].

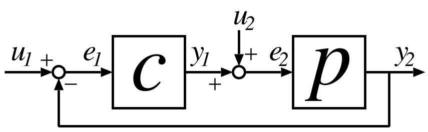

Figure 2: Feedback system Σ.

We consider that the set of stable causal transfer functions is an integral domain, denoted by $\mathcal{A}$. The total ring of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d \neq 0\}$. This $\mathcal{F}$ is considered as the set of all possible transfer functions. Matrices over $\mathcal{F}$ are transfer matrices. Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$. Define the subsets $\mathcal{P}$ and $\mathcal{P}_{\mathrm{S}}$ of $\mathcal{F}$ as follows: $\mathcal{P} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A}\backslash\mathcal{Z}\}$, $\mathcal{P}_{\mathrm{S}} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A}\backslash\mathcal{Z}\}$. Then, every transfer function in $\mathcal{P}$ ($\mathcal{P}_{\mathrm{S}}$) is called *causal* (*strictly causal*). Analogously, if every entry of a transfer matrix is in $\mathcal{P}$ ($\mathcal{P}_{\mathrm{S}}$), the transfer matrix is called *causal* (*strictly causal*).

Throughout the paper, the plant we consider has single-input and single-output, and its transfer function, which is also called a *plant* itself simply, is denoted by p and belongs to $\mathcal{P}$. We can always represent p in the form of a fraction $p = nd^{-1}$, where $n \in \mathcal{A}$ and $d \in \mathcal{A}^{m\times m}$ with nonzero d.

For $p \in \mathcal{P}$ and c, a matrix $H(p,c) \in \mathcal{F}^{2\times 2}$ is defined as

$$H(p,c) := \begin{bmatrix} (1+pc)^{-1} & -p(1+pc)^{-1} \\ c(1+pc)^{-1} & (1+pc)^{-1} \end{bmatrix} \qquad (1)$$

provided that $1+pc$ is a nonzero of $\mathcal{A}$. This $H(p,c)$ is the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,e_1^t \quad e_2^t\,]^t$ of the feedback system Σ. If $1+pc$ is a nonzero of $\mathcal{A}$ and $H(p,c) \in \mathcal{A}^{2\times 2}$, then we say that the plant p is *stabilizable*, p is *stabilized* by c, and c is a *stabilizing controller* of p. In the definition above, we do not mention the causality of the stabilizing controller. However, it is known that if a causal plant is stabilizable, there always exists a causal stabilizing controller of the plant [7].

We will denote by $\mathcal{S}(p)$ the set of stabilizing controllers of P.

It is known that $W(p,c)$ defined below is over $\mathcal{A}$ if and only if $H(p,c)$ is over $\mathcal{A}$:

$$W(p,c) := \begin{bmatrix} c(1+pc)^{-1} & -pc(1+cp)^{-1} \\ pc(1+pc)^{-1} & p(1+cp)^{-1} \end{bmatrix}. \qquad (2)$$

This $W(p,c)$ is the transfer matrix from u_1 and u_2 to y_1 and y_2.

3. Two-Stage Compensator Design

In some design problems, one uses a so-called *two-state procedure* for selecting an appropriate stabilizing compensator[4]. Given a plant p, the first stage consists of selecting a stabilizing compensator for p. Let $c_0 \in \mathcal{S}(p)$ denote this compensator (that is, an arbitrary but fixed compensator of p) and define $p_1 = p(1+c_0p)^{-1}$. The second stage consists of selecting a stabilizing controller for p_1 that also achieves some other design objectives such as decoupling, sensitivity minimization, etc. The resulting configuration with its inner and outer loops is shown in Figure 3.

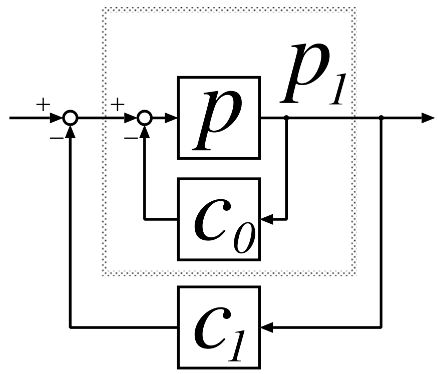

Figure 3: Two-Stage Compensator Design (y_2 to u_2).

Here we give that the two-stage compensator design based on Figure 3 cannot give all stabilizing controllers. The following two theorems have been given by one of the authors[5]. The first Theorem 1 is same as Theorem 5.3.10 of [4]. The second is a detailed version of Theorem 1 with coprime factorizability.

Theorem 1 *Let p denote a causal plant of $\mathcal{P}$ and c_0 a causal stabilizing controller of p ($c_0 \in \mathcal{P}$). Further let p_1 be $p(1+c_0p)^{-1}$. Denote by $c_0 + \mathcal{S}(p_1)$ the following set:*

$$\{c_0 + c_1 \,|\, c_1 \in \mathcal{S}(p_1)\}.$$

Then

$$c_0 + \mathcal{S}(p_1) \subset \mathcal{S}(p), \tag{3}$$

with equality holding if and only if $c_0 \in \mathcal{A}^{m\times n}$.

Theorem 2 *Let* p, c_0, p_1 *be as in Theorem 1. Let* n, d, y, x *be in* $\mathcal{A}$ *such that*

$$\begin{cases} p = nd^{-1}, \quad c_0 = yx^{-1}, \\ ny + dx = 1. \end{cases} \tag{4}$$

Then we have

$$\begin{aligned} &c_0 + \mathcal{S}(p_1) \\ &= \{(x - rn)^{-1}(y + rd) \mid r = r_1 x^2, r_1 \in \mathcal{A}\}. \end{aligned}$$

By Theorem 1, we see that the sum of c_0 and a stabilizing controller of p_1, say c_1, is again a stabilizing controller of p. This sum, a stabilizing controller of p, is the parallel allocation of c_0 and c_1, as shown in Figure 4.

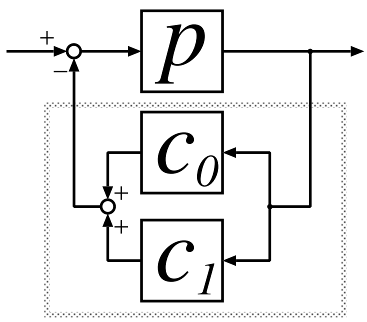

Figure 4: Composite Stabilized Feedback with c_0 and c_1.

The theorems were based on the feedback from y_2 to u_2 (cf. Figures 2 and 3). Even so, we note that, from Figure 2, we have two inputs u_1 and u_2 and two outputs y_1 and y_2. Thus we can consider alternative two-stage compensator design based on other input(s) and other output(s). Let us consider the two-stage compensator design based on the feedback from y_1 to u_1. In this case, the feedback system is as in Figure 5.

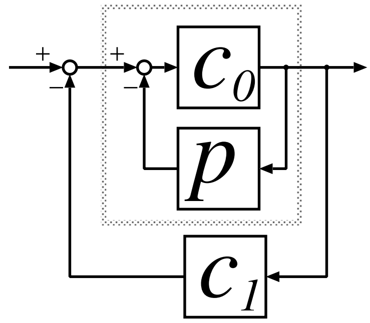

Figure 5: Feedback from y_1 to u_1.

The configuration is as in Figure 6.

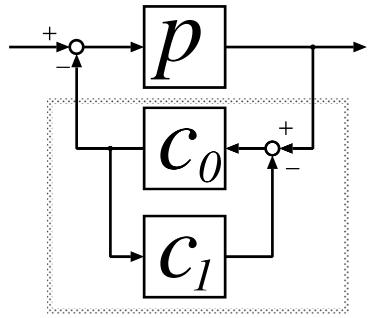

Figure 6: Composite Stabilized Feedback with c_0 and c_1 based on Feedback from y_1 to u_1.

Based on this feedback, the following result has also been given in [5].

Corollary 1 *Let* $P_{y_1u_1}$ *denote* $c_0(1 + pc_0)^{-1}$. *Then we have*

$$\begin{aligned} &\{(1 + c_0c_1)^{-1}c_0 \mid c_1 \in \mathcal{S}(p_{y_1u_1}), \quad 1 + c_0c_1 \neq 0\} \\ &= \{(x - rn)^{-1}(y + rd) \mid r = -r_1 y^2, r_1 \in \mathcal{A}, \\ &\qquad x - rn \neq 0\}. \end{aligned} \tag{5}$$

4. Visualization of Two-Stage Compensator Design

Now that we have a parametrization of two-stage compensator design, we are able to visualize the parameterization of the two-stage compensator design. To implement this visualization, we employ Mathematica[9].

In this implementation, we consider the discrete-time LTI systems as a model of the factorization approach.

Let the plant p be

$$\frac{-3d+d^2}{2+2d}$$

and the inputs u_1 and u_2 be $\frac{9}{d-7}$ and $\frac{2+d}{d^2+3}$, respectively. In this case, the coprime factorization is given as

$y=1,\ x=2-d/2,\ n=1/(4(d-3)d,\ x=(1+d)/2.$

4.1 Visualization of Youla-Kučera-parametrization

Figure 7 is the visualization based on Youla-Kučera-parametrization:

$$\frac{y+rd}{x-rn},$$

where $1 \leqq r \leqq 2$.

Figure 7 is the visualization with $r=\frac{1}{d-a}$ and $1<a\leqq 2$.

4.2 Visualization of Two-Stage Compensator Design

Figure 9 is the visualization based on Theorem 1, where $1 \leqq r \leqq 2$.

Figure 10 is the visualization based on Theorem 1, where with $r=\frac{1}{d-a}$ and $1<a\leqq 2$.

4.3 Visualization of Alternative Two-Stage Compensator Design

Figure 9 is the visualization based on Theorem 1, where $1 \leqq r \leqq 2$.

Figure 10 is the visualization based on Theorem 1, where with $r=\frac{1}{d-a}$ and $1<a\leqq 2$.

5. Conclusion and Future Works

This paper reports a visualization of two-stage compensator design. We have obtained visual input-output relations with varying parameter. We have used two types of two-stage compensator design. By usiging them, we consider to design further robust stabilizning controller.

参考文献

1) C. A. Desoer, R. W. Liu, J. Murray, and R. Saeks, "Feedback system design: The fractional representation approach to analysis and synthesis," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 399–412, 1980.

2) M. Vidyasagar, H. Schneider, and B. A. Francis, "Algebraic and topological aspects of feedback stabilization," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 880–894, 1982.

3) K. Mori, "Parametrization of all strictly causal stabilizing controllers," *IEEE Trans. Automat. Contr.*, vol. AC-54, pp. 2211–2215, 2009.

4) M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, Cambridge, MA: MIT Press, 1985.

5) K. Mori, "Multi-stage compensator design — a factorization approach —," in *Proceedings of The 8th Asian Control Conference (ASCC 2011)*, 2011, 1012-1017.

6) V. R. Sule, "Feedback stabilization over commutative rings: The matrix case," *SIAM J. Control and Optim.*, vol. 32, no. 6, pp. 1675–1695, 1994.

7) K. Mori and K. Abe, "Feedback stabilization over commutative rings: Further study of coordinate-free approach," *SIAM J. Control and Optim.*, vol. 39, no. 6, pp. 1952–1973, 2001.

8) S. Shankar and V. R. Sule, "Algebraic geometric aspects of feedback stabilization," *SIAM J. Control and Optim.*, vol. 30, no. 1, pp. 11–30, 1992.

9) P. Wellin, *Programming with Mathematica: An Introduction*, Cambridge University Press, 2013.

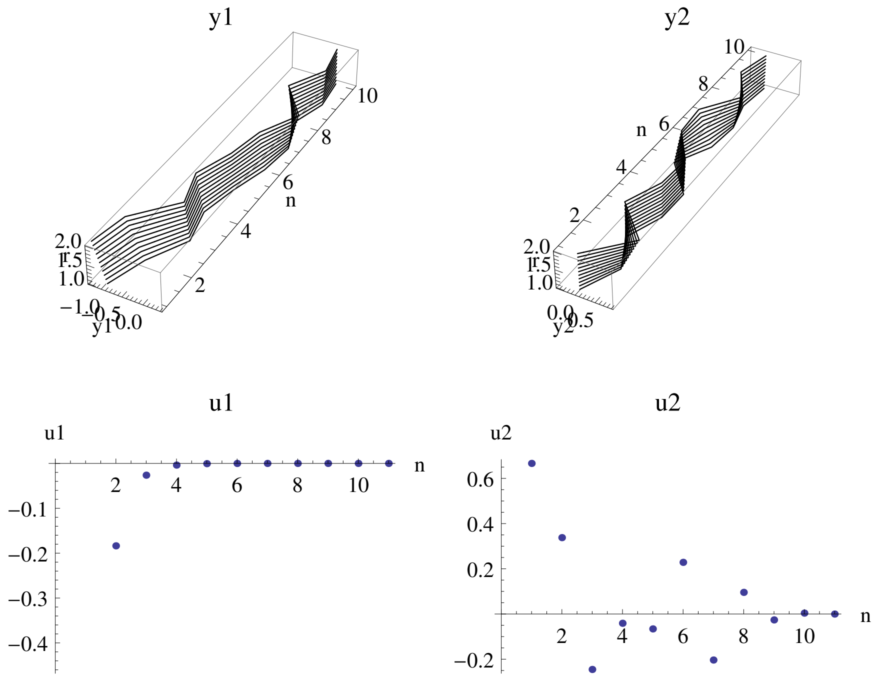

Figure 7: Visualization based on Youla-Kučera-Parametrization with $1 \leqq r \leqq 2$.

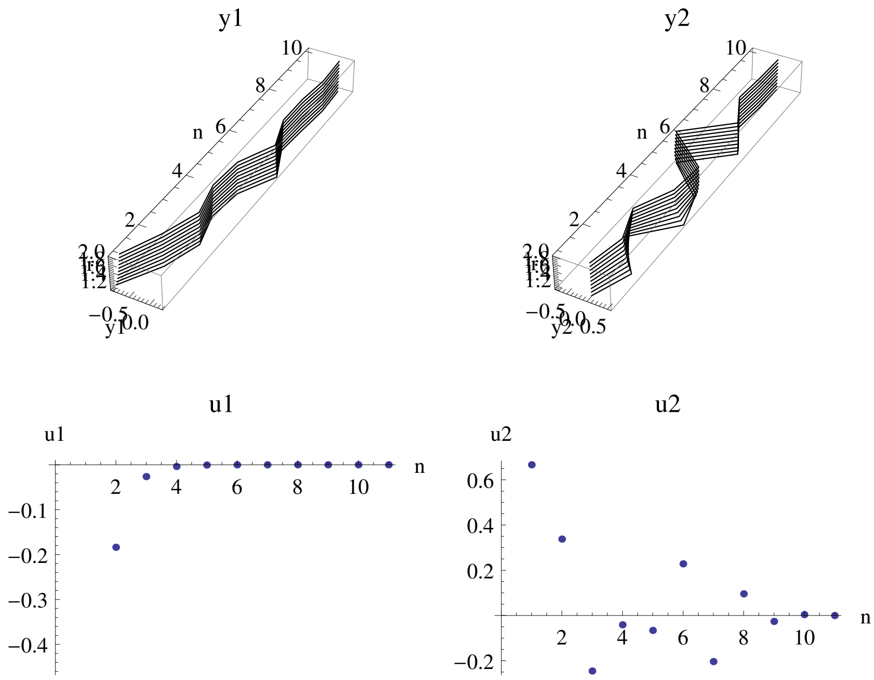

Figure 8: Visualization based on Youla-Kučera-Parametrization with $1 \leqq r \leqq 2$.

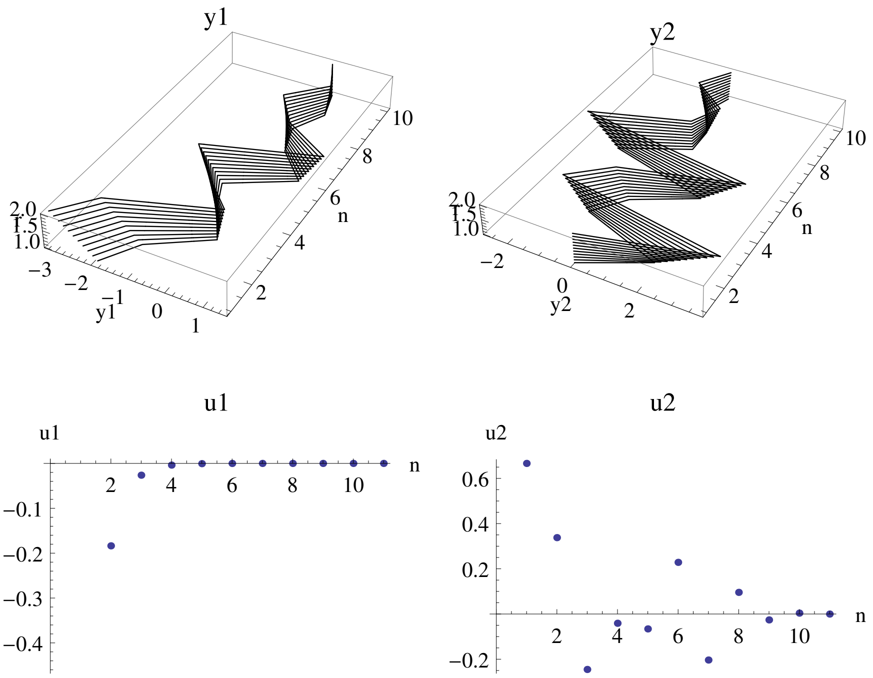

Figure 9: Visualization with $1 \leqq r \leqq 2$.

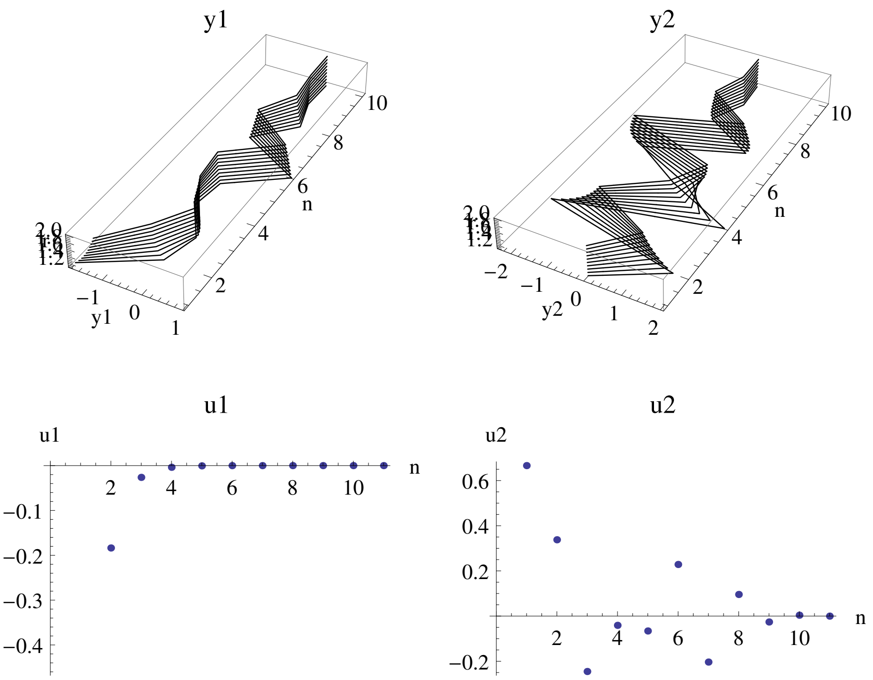

Figure 10: Visualization with $r = \frac{1}{d-a}$ and $1 < a \leqq 2$.

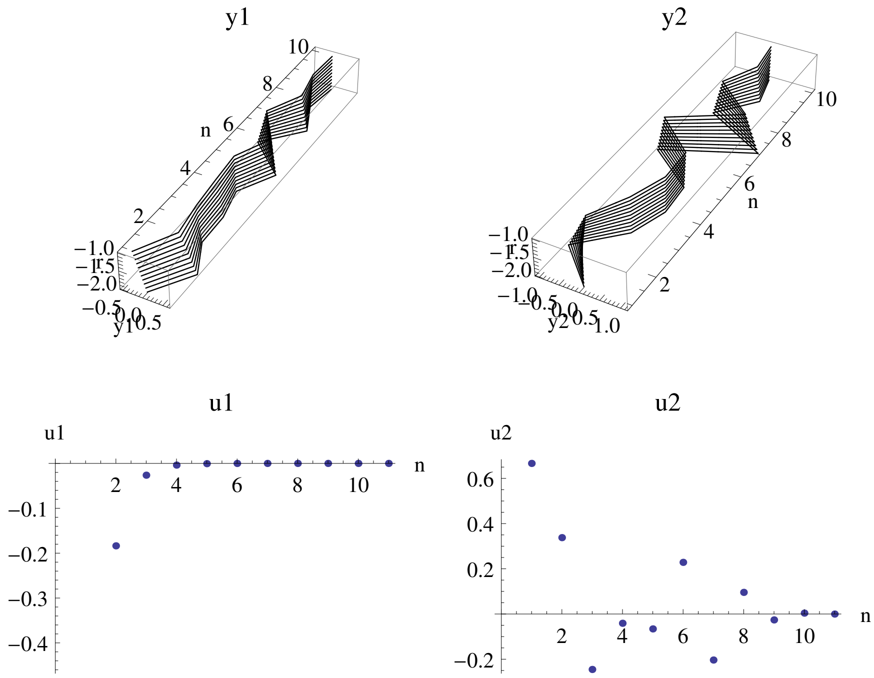

Figure 11: Visualization of Alternative Two-Stage Compensator Design with $1 \leqq r \leqq 2$.

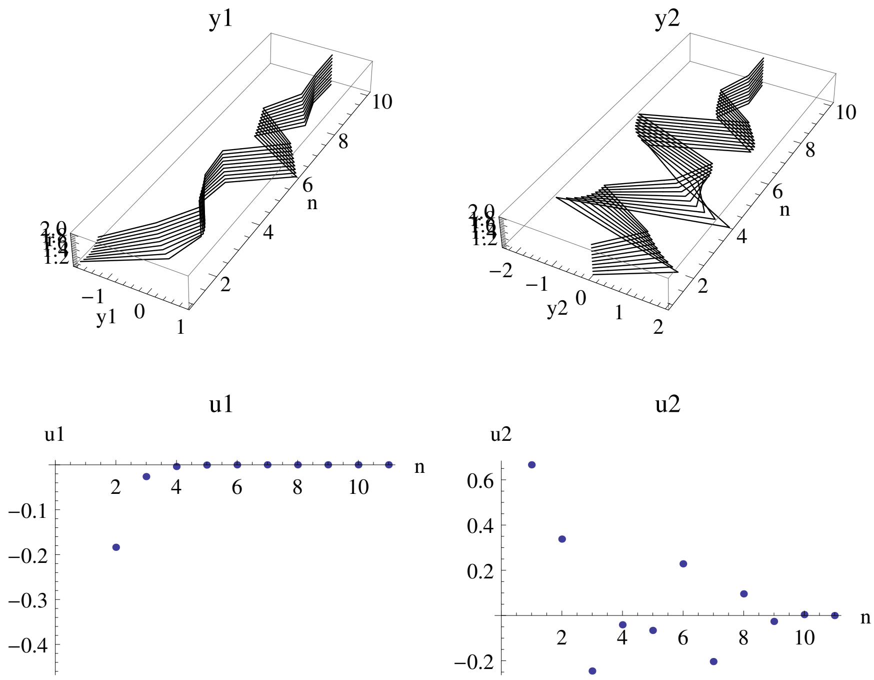

Figure 12: Visualization of Alternative Two-Stage Compensator Design with $r = \frac{1}{d-a}$ and $1 < a \leqq 2$.