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異常診断のための FDA 法の拡張

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A Simple Modification of Fisher Discriminant Analysis Method for Fault Diagnosis

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Abstract– This paper concerns about the application of the Fisher discriminant analysis(FDA) method to diagnose root causes of faults from process time series. A simple modification of the FDA method was developed so as to enhance the accuracy of the diagnosis, which uses the absolute values of the residuals in the calculation of the scatter matrices. The method was applied to a continuous stirred tank reactor(CSTR) and compared with the usual FDA method. The result shows excellant improvement in the accuracy of diagnosis for several cases.

Key Words: Fisher Discriminant Analysis, Fault Diagnosis, Process Monitoring

1 Introduction

Fault detection and diagnosis plays an important part in process engineering. The methods of fault detection and diagnosis can be categorised into three classes: quantitative model-based methods, qualitative model-based methods and process history based methods [1][2][3]. Quantitative model-based methods are the methods that develop *a priori* domain knowledge from a fundamental understanding of the process in terms of mathematical functional relationships between the inputs and outputs of the system. Qualitative model-based method requires qualitative functional relationships from past experience with the process to express the *a priori* knowledge. Process history based methods create knowledge of a diagnostic system by feature extraction processes based on the process history data.

Fisher discriminant analysis (FDA) was originally developed for pattern classification, and soon was applied to pattern recognition problems [4]. The FDA can project the multi-dimensional patterns to one-dimensional ones with the maximum ratio of between-class scatter to within-class scatter, and was used as a data-driven method for fault diagnosis [5]. Chiang *et al* [6] applied the FDA to the fault diagnosis of the Tennessee Eastman chemical plant simulator, and showed the FDA has superior diagnosis accuracy than the well known PLS or PCA.

In this work, we propose a simple modification of FDA method to improve the accuracy of fault diagnosis. The proposed method uses the absolute values of the residuals in the calculation of the scatter matrices. We named this method as the absolute-value-based FDA (AFDA). The method is demonstrated on a continuous stirred tank reactor(CSTR). The fault diagnosis results are compared with the result of usual FDA.

2 Fisher Discriminant Analysis

The FDA method is one of the promising methods to diagnose a root cause of a fault. It shows an optimal one-dimensional mapping to maximize the separability of given data classes in one-dimensional subspace. It decides the optimal projection vector which maximizes the between-class scatter matrix while minimizing the within-class scatter matrix.

Let $X \in \Re^{n\times m}$ be a set of n samples $x \in \Re^m$ and include two subsets X_1 and X_2, each of which contains n_1 and n_2 rows of X corresponding to the samples from class 1 and class 2 respectively. Let $\bar{x}_i$ be the mean of samples for class $i(i = 1, 2)$

$$\bar{x}_i = \frac{1}{n_i}\sum_{x\in X_i} x \tag{1}$$

then

$$S_w = \frac{1}{2}\sum_{\substack{x_i\in X_i \\ X_{i=1}}}^{2} \frac{1}{n_i}(x_i - \bar{x}_i)(x_i - \bar{x}_i)^T \tag{2}$$

is the within-class scatter matrix, and

$$S_b = (\bar{x}_1 - \bar{x}_2)(\bar{x}_1 - \bar{x}_2)^T \tag{3}$$

is the between-class scatter matrix.

In terms of S_b and S_w, the Fisher criterion function is defined as

$$J(w) = \frac{w^T S_b w}{w^T S_w w} \tag{4}$$

where w is a projection direction, and the vector w that maximizes $J(w)$ must satisfy

$$S_b w = \lambda S_w w \tag{5}$$

for some constant λ, which is a generalized eigenvalue problem. If S_w is nonsingular, Eq.(5) can be rewritten as

$$S_w^{-1} S_b w = \lambda w \tag{6}$$

which is a conventional eigenvalue problem, and the Fisher optimal discriminant direction is the eigenvector w corresponding to the maximal eigenvalue.

3 Absolute Value Based FDA

The AFDA method is different from the FDA in the following two aspects:

- Calculation of S_w
- Calculation of S_b

We use the absolute values of the right hand side of Eq.(2) and Eq.(3) to calculate S_w^* and S_b^*.

$$S_w^* = \frac{1}{2}\sum_{\substack{x_i \in X_i \\ X_{i=1}}}^{2} \frac{1}{n_i}|x_i - \bar{x}_i||x_i - \bar{x}_i|^T \tag{7}$$

$$S_b^* = |\bar{x}_1 - \bar{x}_2||\bar{x}_1 - \bar{x}_2|^T \tag{8}$$

According to the Fisher criterion function shown in Eq.(4), we can get the same formula with Eq.(6) denoting the relation among the new within-class scatter matrix S_w^*, between-class scatter matrix S_b^*, eigenvalue λ and eigenvector w if S_w^* is nonsingular.

From Eqs.(7), (8) and (6), we can find the optimal discriminant direction w which is the eigenvector corresponding to the maximal eigenvalue, and the variable correlated to the maximum of absolute value of each element in vector w is estimated to be the root cause of the fault.

4 Case Study

We applied the AFDA method to fault diagnosis on a CSTR with temperature control.

4.1 Data preparation

The schematic diagram of the CSTR [7] is shown in Fig. 1. The outlet temperature T of the reaction mixture was controlled by regulating the flowrate of the cooling water F_j. The initial condition for the CSTR simulation is shown in Table 1, and the setpoint of the temperature T^* is $360K$. Proportional-integeral(PI) algorithm was used to control the outlet temperature T.

The following six variables were selected as sample variables :

T : outlet temperature of the reaction mixture
T_j : outlet temperature of the cooling water
C_A : outlet concentration of the reaction mixture
cv : setpoint of the flowrate of cooling water
F_j : flowrate of the cooling water
F : inlet flowrate of the reaction mixture

Zero-mean white noises having 0.25% standard deviation were added to F and F_j.

The following two faults were considered:

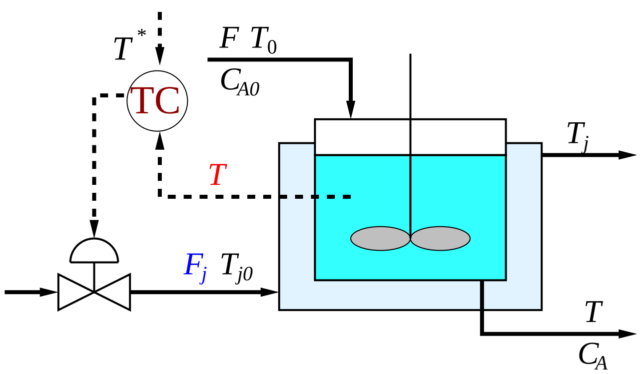

Fig. 1: Schematic diagram of the CSTR

F : leakage of $0.01\mathrm{m}^3/\mathrm{min}$ in reactant inlet;
F_j : leakage of $0.01\mathrm{m}^3/\mathrm{min}$ in coolant inlet;

For both faults, ten data sets including 100 normal samples and 50 abnormal samples were generated by ten simulation runs with different random seeds of white noise. Typical examples of the generated data are shown in Fig. 2.

Eight combinations of sample variables were investigated to evaluate the effect of selection of the variables T_j, C_A and cv for each method. To evaluate the effect of the number of fault samples, five data sets with different numbers of abnormal samples were evaluated.

The result of the fault diagnosis is summarized in Table 2 based on the accuracy of the correct recognition(%). Serial numbers in the first column of the table indicate the eight combinations of sample variables as shown in the second column. Numbers in the third column correspond to the fault of leakage in reactant inlet (I) and the fault of leakage in coolant inlet (II). The elements in the column of root cause are the variables diagnosed as the root cause of the fault I or II. The numbers 30, 35, 40, 45 and 50 in the top of the table are the numbers of abnormal samples in each data set. The numbers in FDA and AFDA columns indicate the numbers of identified fault in ten simulations. The last two columns summarize the accuracy of the fault diagnosis for each method based on the result of all the simulations.

4.2 Leakage of inlet coolant

The controller changes the value of cv based on the measured value of F_j. In case of leakage of inlet coolant flow, the controller will give larger cv than in the normal operation to compensate T to its set point. As the result, relationship between cv and F_j will be different between leakage of inlet coolant flow and normal operation.

For this fault, Table 2 shows that the accuracy of the detection of the AFDA is completely identical to that of the usual FDA for all the combination of sample variables. Moreover, estimated root causes of these two methods are also the same in the ten simulations no matter which variable combination or which abnormal dataset is used.

As the result, we conclude that the two methods have the same performance for the diagnosis of the fault of the leakage of inlet coolant flow.

Table 1: Initial condition for the CSTR simulation

Variable	Value	Unit
F	0.0524	$\mathrm{m}^3/\mathrm{min}$
T_0	294.4	K
C_{A0}	8.24	$\mathrm{kmol}/\mathrm{m}^3$
F_j	0.099	$\mathrm{m}^3/\mathrm{min}$
T_{j0}	294.4	K
T	340	K
T_j	340	K
C_A	3.0	$\mathrm{kmol}/\mathrm{m}^3$

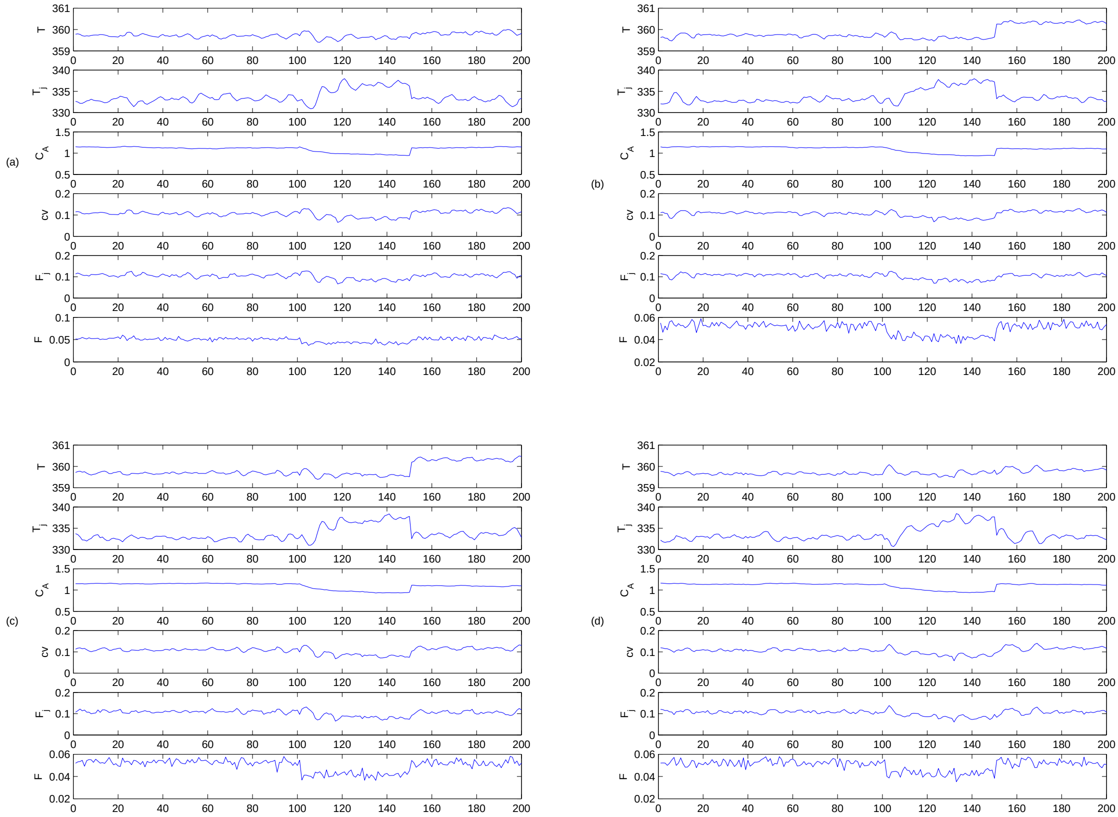

Fig. 2: Examples of generated data sets. (Data between the sample 1 and 100 are normal samples; Data between 101 and 150 are samples with a leakage of inlet reactant; Data between 151 and 200 are samples with a leakage of inlet coolant.)

Table 2: Summary of the diagnosis

Case	Variables	Numbers of abnormal samples		30		35		40		45 , 50		Accuracy(%)	
		Fault	Root cause	FDA	AFDA	FDA	AFDA	FDA	AFDA	FDA	AFDA	FDA	AFDA
1	T, F_j, F	I	F	9	9	9	10	10	10	10	10	96	98
			F_j	1	1	1							
		II	F_j	10	10	10	10	10	10	10	10	100	100
2	T, T_j, F_j, F	I	F	9	9	10	10	10	10	10	10	98	98
			F_j	1	1								
		II	F_j	10	10	10	10	10	10	10	10	100	100
3	T, C_A, F_j, F	I	F	8	9	9	9	10	10	10	10	94	96
			F_j	2	1	1	1						
		II	F_j	10	10	10	10	10	10	10	10	100	100
4	T, T_j, C_A, F_j, F	I	F	8	9	9	9	10	10	10	10	94	96
			F_j	2	1	1	1						
		II	F_j	10	10	10	10	10	10	10	10	100	100
5	T, cv, F_j, F	I	F	9	9	9	10	10	10	10	10	96	98
			F_j	1	1								
			cv			1							
		II	F_j	8	8	8	8	7	7	7	7	74	74
			cv	2	2	2	2	3	3	3	3		
6	T, T_j, cv, F_j, F	I	F	9	10	9	10	10	10	10	10	96	98
			F_j	1	1								
			cv			1							
		II	F_j	8	8	8	8	7	7	7	7	74	74
			cv	2	2	2	2	3	3	3	3		
7	T, C_A, cv, F_j, F	I	F	7	9	7	9	8	10	8	10	76	96
			F_j	1	1		1						
			cv	2		3		2		2			
		II	F_j	8	8	8	8	7	7	7	7	74	74
			cv	2	2	2	2	3	3	3	3		
8	T, T_j, C_A, cv, F_j, F	I	F	7	9	8	9	9	10	10	10	88	96
			F_j	1	1		1						
			cv	2		2		1					
		II	F_j	8	8	8	8	7	7	7	7	74	74
			cv	2	2	2	2	3	3	3	3		

4.3 Leakage of inlet reactant

The value of F_j changes with cv in the normal operation. In case of leakage of inlet reactant flow, the controller will give smaller cv than in the normal operation to keep T to its set point, and F_j will become smaller with the change of cv. As the result, relationship between cv and F_j will be the same between leakage of inlet reactant flow and normal operation. At the same time, T_j will become larger with the deduction of F_j, and C_A smaller with the deduction of F.

For this fault, Table 2 shows that the accuracy of the detection of the AFDA is better than that of the usual FDA in most cases, although the accuracy of these two methods is identical for the case 2.

For the comparison of these two methods, we divide the variable combinations into the following groups:

Related to T_j

case 1 & case 2

case 3 & case 4

case 5 & case 6

case 7 & case 8

Related to C_A

case 1 & case 3

case 2 & case 4

case 5 & case 7

case 6 & case 8

Related to cv

case 1 & case 5

case 2 & case 6

case 3 & case 7

case 4 & case 8

From the comparison of the diagnosis result, we conclude the following two differences between the two methods:

1. The AFDA method always have high accuracy in the diagnosis of root cause, and the accuracy does not change by the difference of variable combinations. However, the accuracy of the usual FDA method is obviously decreased when both cv and C_A are included in the variable combination. These results show that the selection of sample variables for fault diagnosis requires much attention in the usual FDA than the AFDA.
2. The estimated root causes of these two methods are not the same. The AFDA method diagnoses F and F_j as the root causes in all the cases. But the usual FDA method diagnoses F, F_j and cv as the root causes when cv is used for fault diagnosis.

5 Conclusions

A simple modification of the FDA method was proposed for improving the accuracy of fault diagnosis of process plant. The method uses the absolute values of residuals rather than the residuals of sample variables in the calculation of the between-class scatter matrix and within-class scatter matrix. The case study of a CSTR showed that the AFDA method is identical or superior than the usual FDA method for the isolation of faults.

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