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多次元システムに関する同時安定化問題について

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On Simultaneous Stabilizability of Multidimensional System

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Abstract— The relationship between the simultaneous stabilizability and the strong stabilizability of the multidimensional system is investigated. We present that the simultaneous stabilizability over a commutative ring *cannot* be given by the simultaneous stabilizability over its local rings in general. We, however, show that if we have one-side coprime factorization, solving the simultaneous stabilizability of the multidimensional system can be recast as solving the strong stabilizability of the multidimensional system.

Keywords: Multidimensional Systems, Simultaneous Stabilizability

I. Introduction

In this paper, we consider the relationship between the simultaneous stabilizability and the strong stabilizability of the multidimensional system.

Simultaneous stabilization problem is a problem to stabilize two or more plants by a controller. When any stabilizable plant admits a doubly coprime factorization, it is known that solving the simultaneous stabilizability can be recast as solving the strong stabilizability[1, Theorem 5.4.3], [2, Theorem 3.1]. However, in the case of the multidimensional system, it seems that we do not know yet whether or not any stabilizable plant admits a doubly coprime factorization [3] (Lin gave an affirmative conjecture in [4]). Also the strong stabilizability of the multidimensional system has studied by Ying et al. in [5] and [6]. The objective of this paper is to address the relationship between the simultaneous stabilizability and the strong stabilizability of the multidimensional system.

In this paper, we have two approaches: one is to use local-global principle and the other to assume one-side coprime factorization.

It is known that using the local-global principle[7] gives the stabilizability of plants and stabilizing controllers of plants without coprime factorizability[8]–[10]. Parametrization of stabilizing controllers without coprime factorizability[11], [12] is also given by using the local-global principle. Even though the local-global principle is a powerful tool, in this paper, we will show that the simultaneous stabilizability over an original ring, which is the set of stable causal transfer functions, cannot be given by the simultaneous stabilizabilities over the local rings in general.

We will also consider the case where we assume there exists one of right- or left-coprime factorization. In this case, we will obtain the same result to the case where any stabilizable plant admits a doubly coprime factorization. Thus, we will see, under this assumption, that solving the simultaneous stabilizability can be recast as solving the strong stabilizability.

This paper is organized as follows. After this introduction, we begin on the introduction of the coordinate-free approach, we used in this paper, in Section I, including definitions. In Section II, we consider the simultaneous stabilizability with the local-global principle. Then, in Section III, we investigate the case where plant admits one-side coprime factorization.

II. Coordinate-Free Approach

We start by giving the preliminary of the coordinate-free approach. In the following we introduce the notations used in this paper. Then we give the formulation of the feedback stabilization problem.

A. Notations

Commutative Rings In this paper, we consider that any commutative ring has the identity 1 different from zero. Let $\mathcal{R}$ denote a (unspecified) commutative ring. The total ring of fractions of $\mathcal{R}$ is denoted by $\mathcal{F}(\mathcal{R})$.

We will consider that *the set of stable causal transfer functions* is a commutative ring denoted by $\mathcal{A}$. From the sense of the transfer functions we consider that the commutative ring $\mathcal{A}$ satisfies the invariant basis property (cf. [13]). In addition to $\mathcal{A}$, we will use the following three kinds of ring of fractions. The first one appears as the total ring of fractions of $\mathcal{A}$, which is denoted by $\mathcal{F}(\mathcal{A})$ or simply by $\mathcal{F}$; that is, $\mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d \text{ is a nonzerodivisor}\}$. This will be considered as *the set of all possible transfer functions*. The second one is associated with the set of powers of a nonzero element of $\mathcal{A}$. Let f denote a nonzero element of $\mathcal{A}$. Given a set $S_f = \{1, f, f^2, \ldots\}$, which is a multiplicative subset of $\mathcal{A}$, we denote by $\mathcal{A}_f$ the ring of fractions of $\mathcal{A}$ with respect to the multiplicative subset S_f; that is, $\mathcal{A}_f = \{n/d \,|\, n \in \mathcal{A},\ d \in S_f\}$. The last one is the total ring of fractions of $\mathcal{A}_f$, which is denoted by $\mathcal{F}(\mathcal{A}_f)$; that is, $\mathcal{F}(\mathcal{A}_f) = \{n/d \,|\, n, d \in \mathcal{A}_f,\ d \text{ is a nonzerodivisor of } \mathcal{A}_f\}$. If f is a nonzerodivisor of $\mathcal{A}$, $\mathcal{F}(\mathcal{A}_f)$ coincides with the total ring of fractions of $\mathcal{A}$. Otherwise, they do not coincide.

Matrices The set of matrices over $\mathcal{R}$ of size $x \times y$ is denoted by $\mathcal{R}^{x \times y}$. Further, the set of square matrices over $\mathcal{R}$ of size x is denoted by $(\mathcal{R})_x$. The identity and the zero matrices are denoted by E_x and $O_{x \times y}$, respectively, if the sizes are required, otherwise they are denoted by E and O.

Matrix A over $\mathcal{R}$ is said to be *nonsingular* (*singular*) *over* $\mathcal{R}$ if the determinant of the matrix A is a nonzerodivisor (a zerodivisor) of $\mathcal{R}$. Matrices A and B over $\mathcal{R}$ are *right-coprime over* $\mathcal{R}$ if there exist matrices X and Y over $\mathcal{R}$ such that $XA + YB = E$ holds. Further, an ordered pair (N, D) of matrices N and D is said to be a *right-coprime factorization over* $\mathcal{R}$ of P if (i) D is nonsingular over $\mathcal{R}$, (ii) $P = ND^{-1}$ over $\mathcal{F}(\mathcal{R})$, and (iii) N and D are right-coprime over $\mathcal{R}$. As the parallel notion, the *left-coprime over* $\mathcal{R}$ and the *left-coprime factorization over* $\mathcal{R}$ of P are defined analogously. If a plant has both a right- and a left-coprime factorizations over $\mathcal{R}$, then the plant is said to *admit a doubly coprime factorization over* $\mathcal{R}$. For short, we may omit "over $\mathcal{R}$" when $\mathcal{R} = \mathcal{A}$.

B. Feedback Stabilization Problem

The stabilization problem follows that of Desoer *et al.* of [14], Sule in [8], and Mori and Abe in [9], who consider

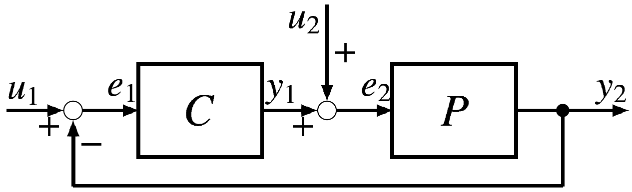

Fig. 1. Feedback system Σ.

the feedback system Σ [1, Ch.5, Figure 5.1] as in Figure 1. For further details the reader is referred to [1]. The plant we consider has m inputs and n outputs, and its transfer matrix, which is also called a *plant* itself simply, is denoted by P and belongs to $\mathcal{F}^{n\times m}$. We can always represent P in the form of a fraction $P = ND^{-1}$ ($P = \widetilde{D}^{-1}\widetilde{N}$), where $N \in \mathcal{A}^{n\times m}$ ($\widetilde{N} \in \mathcal{A}^{n\times m}$) and $D \in (\mathcal{A})_m$ ($\widetilde{D} \in (\mathcal{A})_n$) with nonsingular D ($\widetilde{D}$).

Definition 1 *For $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$, a matrix $H(P,C) \in (\mathcal{F})_{m+n}$ is defined as*

$$H(P,C) \;=\; \begin{bmatrix} (E_n + PC)^{-1} & -P(E_m + CP)^{-1} \\ C(E_n + PC)^{-1} & (E_m + CP)^{-1} \end{bmatrix} \quad (1)$$

provided that $\det(E_n + PC)$ *is a nonzerodivisor of $\mathcal{A}$. This $H(P,C)$ is the transfer matrix from $[\,u_1^t \quad u_2^t\,]^t$ to $[\,e_1^t \quad e_2^t\,]^t$ of the feedback system Σ. If (i)* $\det(E_n + PC)$ *is a nonzerodivisor of $\mathcal{A}$ and (ii) $H(P,C) \in (\mathcal{A})_{m+n}$, then we say that the plant P is* stabilizable*, P is* stabilized *by C, and C is a* stabilizing controller *of P.*

Since the transfer matrix $H(P,C)$ of the stable causal feedback system has all entries in $\mathcal{A}$, we call the above notion *$\mathcal{A}$-stabilizability*. One can further introduce the notion of *$\mathcal{A}_f$-stabilizability* as follows.

Definition 2 *Let f be a nonzero element of $\mathcal{A}$. If (i)* $\det(E_n + PC)$ *is a nonzerodivisor of $\mathcal{A}_f$ and (ii) $H(P,C) \in (\mathcal{A}_f)_{m+n}$, then we say that the plant P is* $\mathcal{A}_f$-stabilizable*, P is* $\mathcal{A}_f$-stabilized *by C, and C is an* $\mathcal{A}_f$-stabilizing controller *of P.*

The causality of transfer functions is an important physical constraint.

Definition 3 (Definition 3.1 of [15]) *Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$, with $\mathcal{Z} \neq \mathcal{A}$, including all zerodivisors. Define the subsets $\mathcal{P}$ and $\mathcal{P}_S$ of $\mathcal{F}$ as follows:*

$$\begin{aligned} \mathcal{P} &= \{n/d \in \mathcal{F} \mid n \in \mathcal{A},\ d \in \mathcal{A}\backslash\mathcal{Z}\}, \\ \mathcal{P}_S &= \{n/d \in \mathcal{F} \mid n \in \mathcal{Z},\ d \in \mathcal{A}\backslash\mathcal{Z}\}. \end{aligned}$$

Then every transfer function in $\mathcal{P}$ ($\mathcal{P}_S$) is called causal (strictly causal). *Analogously, if every entry of a transfer matrix F is in $\mathcal{P}$ ($\mathcal{P}_S$), the transfer matrix F is called* causal (strictly causal). *A matrix over $\mathcal{A}$ is said to be* $\mathcal{Z}$-nonsingular *if the determinant is in $\mathcal{A}\backslash\mathcal{Z}$, and* $\mathcal{Z}$-singular *otherwise.*

To apply the coordinate-free approach to the multidimensional system with structural stability, $\mathcal{A}$ and $\mathcal{Z}$ are given as

$$\begin{aligned} \mathcal{A} &= \{a/b \mid a, b \in \mathbb{R}[z_1, \ldots, z_l],\ b \neq 0 \text{ in } \overline{U}^l\}, \\ \mathcal{Z} &= \sum_{i=1}^{l} z_i\mathcal{A} = \{a/b \in \mathcal{A} \mid a, b \in \mathbb{R}[z_1, \ldots, z_l], \\ &\qquad \text{the constant term of } a \text{ is zero.}\}, \end{aligned}$$

where $\overline{U}^l$ denotes the closed unit polydisc.

Finally, we introduce the notion of simultaneous stabilization.

Definition 4 *Let P_0 and P_1 be plants in $\mathcal{P}^{n\times m}$. If C in $\mathcal{F}^{m\times n}$ is a stabilizing controller of P_0 and P_1, then C is said to be a* simultaneously stabilizing controller *of P_0 and P_1, and that P_0 and P_1 are* simultaneously stabilized *by C. If there exists a simultaneously stabilizing controller of P_0 and P_1, then they are said to be* simultaneously stabilizable.

For the case we consider $\mathcal{A}_f$-stabilizability instead of $\mathcal{A}$-stabilizability, we will use "*simultaneously $\mathcal{A}_f$-stabilizing controller*," "*simultaneously $\mathcal{A}_f$-stabilized*," and "*simultaneously $\mathcal{A}_f$-stabilizable*" analogously.

III. Local-Global Principle and Simultaneous Stabilizability

We consider in this section the simultaneous stabilization problem and the local-global principle. The following is a local-global principle with a fixed stabilizing controller.

Theorem 1 *Let P_0 and P_1 be plants in $\mathcal{P}^{n\times m}$. Let C denote a transfer matrix of $\mathcal{F}^{m\times n}$.*

Then the following statements are equivalent:

- *(i) The plants P_0 and P_1 are simultaneously stabilized by C.*
- *(ii) There exists a finite subset Λ of $\mathcal{A}$ such that (a) $\sum_{\lambda\in\Lambda} \lambda = 1$ and (b) for each $\lambda \in \Lambda$, both P_0 and P_1 are simultaneously $\mathcal{A}_\lambda$-stabilized by C.*

The proof is relatively easy, so that it is omitted.

Then we have another problem whether or not the simultaneous stabilizability over $\mathcal{A}$ is equivalent to the simultaneous stabilizabilities over local rings of $\mathcal{A}$. We give the result that they are not equivalent as follows.

Theorem 2 *Let P_0 and P_1 be plants in $\mathcal{P}^{n\times m}$. Consider the following statements:*

- *(i) The plants P_0 and P_1 are simultaneously stabilizable.*
- *(ii) There exists a finite subset Λ of $\mathcal{A}$ such that (a) $\sum_{\lambda\in\Lambda} \lambda = 1$ and (b) for each $\lambda \in \Lambda$, both P_0 and P_1 are simultaneously $\mathcal{A}_\lambda$-stabilizable.*

Then, (i) implies (ii). However, (ii) does not imply (i) in general.

Proof: Because "(i)⇒(ii)" is obvious, we will prove only that (ii) does not imply (i) in general. To prove this, it is sufficient to show an example that (ii) holds but (i) does not hold. As an example, we employ Anantharam's example [16]. He considered the case $\mathcal{A} = \mathbb{Z}[\sqrt{-5}] = \{u + v\sqrt{-5} \mid u, v \in \mathbb{Z}\}$, where $\mathbb{Z}$ denotes the set of integers (This ring [17, pp.134–135] is isomorphic to $\mathbb{Z}[x]/(x^2+5)$ and is an integral domain but not a unique factorization domain. In fact, $6 \in \mathbb{Z}[\sqrt{-5}]$ has two factorizations, $2 \cdot 3$ and $(1+\sqrt{-5}) \cdot (1-\sqrt{-5})$). We let $\mathcal{Z} = \{0\}$. Anantharam [16] showed that a plant $(1+\sqrt{-5})/2$ does not admit a coprime factorization but is stabilized by $(1-\sqrt{-5})/(-2)$. We use these transfer functions. Let $p_0 = (1+\sqrt{-5})/2$ and $p_1 = (1-\sqrt{-5})/(-2)$.

In the following, we first present the controller parametrizations of p_0 and p_1, next show that they are not simultaneously stabilizable and then show that they are simultaneously stabilizable over $\mathcal{A}_\lambda$ for every $\lambda \in \Lambda$.

Controller Parametrization Let $\mathcal{S}(p)$ denote the set of all stabilizing controllers of p.

The controller parametrization of p_0 is given in [18], which is given as

$$\mathcal{S}(p_0) = \left\{ \frac{2r_0 + (1-\sqrt{-5})}{-(1+\sqrt{-5})r_0 - 2} \;\middle|\; r_0 \in \mathcal{A} \right\}. \qquad (2)$$

Similarly, the controller parametrization of p_1 is given as

$$\mathcal{S}(p_1) = \left\{ \frac{2r_1 - 1 - \sqrt{-5}}{(1-\sqrt{-5})r_1 - 2} \;\middle|\; r_1 \in \mathcal{A} \right\}. \qquad (3)$$

Non-Simultaneous-Stabilizability Suppose that there exists a simultaneously stabilizing controller of p_0 and p_1. Then, from (2) and (3), there exist r_0 and r_1 in $\mathcal{A}$ such that

$$\frac{2r_0 + (1-\sqrt{-5})}{-(1+\sqrt{-5})r_0 - 2} = \frac{2r_1 - 1 - \sqrt{-5}}{(1-\sqrt{-5})r_1 - 2} \qquad (4)$$

holds. Let a_0, b_0, a_1, b_1 be integers with $r_0 = a_0 + \sqrt{-5}b_0$ and $r_1 = a_1 + \sqrt{-5}b_1$. Then (4) can be decomposed into the real part and the imaginary part equations as follows:

$$\begin{cases} -4 + 4a_0a_1 + 10b_0 + 10b_1 - 20b_0b_1 = 0, \\ -2a_0 - 2a_1 + 4a_1b_0 + 4a_0b_1 = 0. \end{cases} \qquad (5)$$

By solving (5), b_0 can be expressed by a_1 and b_1 as follows:

$$b_0 = \frac{2 + 2a_1^2 - 9b_1 + 10b_1^2}{5 + 4a_1^2 - 20b_1 + 20b_1^2}. \qquad (6)$$

Recall now that the variables b_0, a_1, b_1 have some integer values. Even so, we here show that the right hand side of (6) cannot be integer. Now decompose each of the numerator and the denominator of (6) into two parts within parentheses:

$$\frac{(1 + 2a_1^2) + (1 - 9b_1 + 10b_1^2)}{(2 + 4a_1^2) + (3 - 20b_1 + 20b_1^2)}.$$

Then we have $0 < 1 + 2a_1^2 < 2 + 4a_1^2$ for every $a_1 \in \mathbb{Z}$ also $0 < 1 - 9b_1 + 10b_1^2 < 3 - 20b_1 + 20b_1^2$ for every $b_1 \in \mathbb{Z}$. Thus the right hand side of (6) is greater than zero and less than one for any $a_1, b_1 \in \mathbb{Z}$. This implies that b_0 cannot be an integer and that the equation (4) of r_0 and r_1 does not have a solution.

Hence we conclude that the plants p_0 and p_1 are not simultaneously stabilizable, that is, p_0 and p_1 do not satisfy (i) of Theorem 2.

Simultaneous Stabilizability over $\mathcal{A}_\lambda$ for $\lambda \in \Lambda$ First, we let $\Lambda = \{-2, 3\}$. Then $\sum_{\lambda\in\Lambda} \lambda = 1$.

Because the denominators of p_0 and p_1 are 2 and -2, respectively, both p_0 and p_1 are in $\mathcal{A}_{-2}$. Thus p_0 and p_1 are $\mathcal{A}_2$-stabilized by zero, which is a simultaneously $\mathcal{A}_{-2}$-stabilizing controller.

Consider $\mathcal{A}_3$. Observe that p_0 can be rewrite as $3/(1-\sqrt{-5})$ and p_1 as $-3/(1+\sqrt{-5})$. Thus $(1, p_0^{-1})$ and $(1, p_1^{-1})$ are coprime factorizations of p_1 over $\mathcal{A}_3$.

Let $\mathcal{S}(p)_\lambda$ denote the set of all $\mathcal{A}_\lambda$-stabilizing controllers of p. Then we have, by Youla-parametrization,

$$\begin{aligned} \mathcal{S}(p_0)_3 &= \{(1 + p_0^{-1}r_0)/r_0 \mid r_0 \in \mathcal{A}_3\backslash\{0\}\}, \\ \mathcal{S}(p_1)_3 &= \{(1 + p_1^{-1}r_1)/r_1 \mid r_1 \in \mathcal{A}_3\backslash\{0\}\}. \end{aligned}$$

If $\mathcal{S}(p_0)_3 \cap \mathcal{S}(p_1)_3$ is not empty, then p_0 and p_1 are simultaneously $\mathcal{A}_3$-stabilizable. Hence, consider the equation

$$(1 + p_0^{-1}r_0)r_1 = (1 + p_1^{-1}r_1)r_0$$

over $\mathcal{A}_3$. By the straightforward calculation, we see that $(r_0, r_1) = (-1, -3)$ is one of solutions. Its simultaneously $\mathcal{A}_3$-stabilizing controller is $c = (-1+\sqrt{-5})/(1+\sqrt{-5})$. Thus, p_0 and p_1 are simultaneously $\mathcal{A}_3$-stabilizable.

Thus these p_0 and p_1 satisfy (ii) of Theorem 2. Therefore, (ii) of Theorem 2 does not imply (i) of Theorem 2 in general. □

By the result of this section, we observe that the local-global principle will not play the role to investigate the relationship between the simultaneous stabilizability and the strong stabilizability of the multidimensional system.

IV. One-Side Coprime Factorization

Without considering the doubly coprime factorizability, the relationship between the simultaneous stabilizability and the strong stabilizability of the multidimensional system cannot be given yet. Alternatively, we loose the condition, that is, we here give a assumption below.

Assumption 1 A plant admits one-side coprime factorization (at least one of right- or left-coprime factorization).

From this assumption, we can use the following theorem.

Theorem 3 *(Theorem 1 of [19]) If there exists a right-(left-)coprime factorization of the plant $P \in \mathcal{P}^{n\times m}$, then the plant $[\,P^t \;\; O^{m\times m}\,]^t \in \mathcal{P}^{(m+n)\times m}$ (the plant $[\,P \;\; O^{n\times n}\,] \in \mathcal{P}^{n\times(m+n)}$) has both right- and left-coprime factorizations.*

Thus, once we have Assumption 1, the plant $[\,P\ O\,]$ or $[\,P^t\ O^t\,]^t$ admits a doubly coprime factorization. To generalize our discussion, we consider a block diagonal plant $\mathrm{Diag}(P, O^{y\times x})$ rather than $[\,P\ O\,]$ or $[\,P^t\ O^t\,]^t$, and suppose, without loss of generality under Assumption 1, that $\mathrm{Diag}(P, O^{y\times x})$ admits a doubly coprime factorization . Even so, we do not consider the doubly coprime factorizability of P. If P admits a right-coprime factorization, then we apply $y = m$ and $x = 0$, so that $[\,P^t \;\; O^{m\times m}\,]^t$ admits a left-coprime factorization as well as a right-one.

Let us review the following theorem.

Theorem 4 *(Theorem 2 of [19]) Let* $\mathcal{S}(P)$ *and* $\mathcal{S}(\mathrm{Diag}(P, O^{y\times x}))$ *denote the sets of stabilizing controllers of the plants* P *and* $\mathrm{Diag}(P, O^{y\times x})$*, respectively. Then the following equation holds:*

$$\mathcal{S}(P) = \{[\, E_m \;\; O^{m\times x}\,]C[\, {}^{E_n}_{O^{y\times n}}\,] \mid C \in \mathcal{S}(\mathrm{Diag}(P, O^{y\times x}))\}.$$

Using this theorem, we have the following without the proof.

Proposition 1 *Let* P *be in* $\mathcal{P}^{n\times m}$*. Suppose that* $\mathrm{Diag}(P, O)$ *admits a doubly coprime factorization. Then* P *is stabilizable if and only if* $\mathrm{Diag}(P, O)$ *is stabilizable.*

Now we apply the discussion above to the simultaneous stabilizability. Let P_0 and P_1 be in $\mathcal{P}^{n\times m}$. Suppose that both $\mathrm{Diag}(P_0, O^{y\times x})$ and $\mathrm{Diag}(P_1, O^{y\times x})$ admit doubly coprime factorizations. Let N_0, D_0, $\widetilde{N}_0$, and $\widetilde{D}_0$ be matrices over $\mathcal{A}$ such that $\mathrm{Diag}(P_0, O^{y\times x}) = N_0 D_0^{-1} = \widetilde{D}_0^{-1}\widetilde{N}_0$, $\widetilde{Y}_0 N_0 + \widetilde{X}_0 D_0 = I$, and $\widetilde{N}_0 Y_0 + \widetilde{D}_0 X_0 = I$ for matrices $\widetilde{Y}_0$, $\widetilde{X}_0$, Y_0, and X_0 over $\mathcal{A}$. Analogously the symbols N_1, D_1, $\widetilde{N}_1$, and so on are introduced for $\mathrm{Diag}(P_1, O^{y\times x})$.

Analogously to Proposition 1, we have the following.

Proposition 2 *Plants* P_0 *and* P_1 *are simultaneously stabilizable if and only if* $\mathrm{Diag}(P_0, O^{y\times x})$ *and* $\mathrm{Diag}(P_1, O^{y\times x})$ *are simultaneously stabilizable.*

"Only if" parts of both propositions are obvious. "If" parts of both propositions are proved by using Theorem 4. Thus the proofs are omitted due to the space limitation.

The following is an application of Theorem 5.4.3 of [1] to the above setting.

Lemma 1 *Let*

$$A = \widetilde{X}_0 D_1 + \widetilde{Y}_0 N_1, \;\; B = -\widetilde{N}_0 D_1 + \widetilde{D}_0 N_1. \qquad (7)$$

Then $\mathrm{Diag}(P_0, O^{y\times x})$ *and* $\mathrm{Diag}(P_1, O^{y\times x})$ *are simultaneously stabilizable if and only if there exists a matrix* M *over* $\mathcal{A}$ *such that* $A + MB$ *is unimodular.*

Provided that A is nonsingular, there exists a matrix M over $\mathcal{A}$ such that $A + MB$ is unimodular if and only if the associated system BA^{-1} is strongly stabilizable. Thus Lemma 1 enables us to recast the simultaneous stabilization problem as the strong stabilization problem.

Proposition 3 *Let* A *and* B *be as in (7). Then* P_0 *and* P_1 *are simultaneously stabilizable if and only if there exists a matrix* M *over* $\mathcal{A}$ *such that* $A + MB$ *is unimodular.*

This is just a combination of Proposition 2 and Lemma 1.

Suppose that P_0 and P_1 admits right-coprime factorizations. Then by Theorem 3, both $[\, P_0^t \;\; O^{m\times m}]^t$ and $[\, P_1^t \;\; O^{m\times m}]^t$ admit doubly coprime factorizations. Then Let N_0 and $D_0, \widetilde{N}_0, \ldots, N_1$ and $D_1, \widetilde{N}_1, \ldots$, be as previously. Let A and B as in (7). Now, by virtue of Theorem 3, P_0 and P_1 are simultaneously stabilizable if and only if there exists a matrix M over $\mathcal{A}$ such that $A + MB$ is unimodular.

Now we have the result that when a multidimensional system admits one-side coprime factorization, solving the simultaneous stabilizability can be recast as solving the strong stabilizability.

V. Conclusions

In this paper, we have addressed the simultaneous stabilization problem. Primary goal of the current study is to obtain some criterion of solving the simultaneous stabilization problem of the multidimensional system. We do not obtain this yet. We have two investigation ways in the future. One is to solve the conjecture given by Lin in [4], that is, to show that any stabilizable plant admits a doubly coprime factorization. If we can show it, then solving the simultaneous stabilizability can be recast as solving the strong stabilizability (as in [1], [2]). The other is to show some criterion of solving the simultaneous stabilization problem without the coprime factorizability. If the conjecture will be unsuccessful, that is, if there exists a stabilizable plant of the multidimensional system that do not admit a doubly coprime factorization, then because we will not be able to use the local-global principle by the result of this paper, we will need to employ the later way.

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